Investment Portfolio Optimization on the Basis of Risk Approach by Means of Economic-Mathematical Modeling

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Abstract—**The main features of formation of the investment portfolio is presented. The certain tasks which can be solved in the process of optimization are listed. The key principles in the basis of the investment portfolio optimization are stated. The stages of the optimal investment portfolio formation are analyzed. Particular attention was paid to the possibility of income taxation optimization for individual instruments that are included in the portfolio. When forming an optimal investment portfolio in the context of a risk-based approach, calculations was performed using the formulas of Markowitz, Sharpe and Treinor. The authors consider the order of practical application and calculation of the above mentioned coefficients on a practical example. In order to optimize the work process and time resource savings, all calculations on the Markowitz model are made by means of Microsoft Excel. In conclusion, it was noted that the investment portfolio optimization was the most important task of the company in the investment strategy implementation and in attracting investment capital into projects.**

Keywords — investment portfolio optimization, profitability, risks, Sharpe ratio, Treinor coefficient..

I. INTRODUCTION

In modern socio-economic conditions and in the context of adopted investment strategy, as well as the implementation of the risk-based approach by companies (due to the presence of a wide range of challenges), the most important issue is the order and methods of the investment portfolio optimization.

Thus, in recent years, both foreign and domestic authors have been studying the issues of evaluating the effectiveness and efficiency of investment projects, the order of formation and of investment portfolios optimization. Special attention should be paid to the interest of foreign authors in the study of issues and problems of public investment projects [1-3].

Thus, Erwan Morellec, Norman Schürhoff having studied corporate investments and financing in the conditions of information asymmetry, came to several conclusions, including that the time of decisions made by investors may also depend on the timing and completeness of the amount of information provided by organizations to investors [4].

A review of the works of domestic scientists shows the presence of a comprehensive approach to the formation of the investment portfolio and its optimization. So, Panarin, D.V. carries out the task of portfolio optimization based on estimation of returns of securities [5].

So, Ivanyuk V.A., Andropov K.N., Egorova N. E. we analyzed different methods of optimization of the investment portfolio, including the Nelder-Mid algorithm, which opinion is not considered to "be fast in application and unreliable" [6, p. 576].

The position of Batin B.A. is of interest. He justified the need to assess 8 types of risks when choosing alternative investment projects [7], which is especially important within optimizing the investment portfolio in the context of a risk-oriented approach. It should also be noted that study similar issues are also devoted to the publication of the S.A. Smolyak, E.B. Kibalov, A.A. Kin, A.M. Afanasyev, V.V. Yanovsky, A.S. Sidorov [8-11] etc.

The analysis of the degree of themes consideration also revealed the presence of studies with regard to various economic activities and sectors of the economy: the banking sector (Babushkin V.I., Prokoptsov P.A.), energy (Buharbaeva L.Y., Gabidullina G.Z.), industry (Zubareva V.D., Zaitseva M.A., Bespalova V.E.), agriculture (Strantsov I.A., Kniga

A.S., Strelnikov I A.), retail trade (Perevoshchikov A.V.), and even a taxi (Matveev D.E., Matveev E.R.) [12-20].

The consideration of this topic in the framework of stimulating investment activity and building an innovative economy in China [21], Germany [22], Peru and Colombia [23], the Russian Federation [24, 25] is very important.

II. PROBLEM STATEMENT

The formation of the investment portfolio is determined by a number of factors, among which are the following: investment objectives, the current economic situation in the state and the world, the development of the stock market, etc.

So, the investment portfolio optimization is referred to a set of investor's targeted actions to modify its structure. Therefore, the optimization goals depend on the principal position of the investor, including in terms of the ratio of risk and return, the type of chosen investment policy. At the same time, the following tasks can be solved in the process of optimization:

1) The increase of the investment portfolio profitability.

2) Reduction of investment portfolio risks.

3) Changing the number of elements in the investment portfolio.

4) Ensuring the internal stability of the investment portfolio.

5) Ensuring the growth of the company's capitalization.

The following key principles are in the basis of the investment portfolio optimization:

1) investment portfolio optimization should be carried out within the framework of the developed and adopted investment strategy;

2) optimization should focus on a combination of profitability and risk in an acceptable passage;

3) compulsory consideration of 3-5 options of different levels of profitability and risk;

4) taking into account the investment climate of the country and the region;

5) the results of the assessment consideration of the situation on the stock market;

6) assessment of systematic and non-systematic risks;

7) responsibility, person's professionalism and competence taking a decision on the investment portfolio optimization.

An effective investment portfolio is understood as a portfolio, the composition of which fully provides one of two options:

1) maximum profitability at minimum (specified) risk;

2) minimum risk at maximum (specified) profitability.

At the same time, the optimal investment portfolio is usually understood as a portfolio, the composition of one meets the needs and interests of the investor, taking into account the combination of profitability and risk on invested funds.

The stages of the optimal investment portfolio formation are the following:

1) assessment of financial stability of the company;

2) development (adjustment) of the company's investment policy taking into account the current socio-economic situation in the country, the state;

3) formation of the general purpose of investment, the choice of a suitable type of investment portfolio;

4) analysis of existing investment opportunities;

5) implementation strategy development of cash investments;

6) selection of investment directions corresponding to the general purpose of the company;

7) overall and comprehensive business plans examination of investment projects;

8) company optimal investment portfolio formation on the basis of special methods;

9) constant control implementation over the investment portfolio achievement of previously planned profitability level and socio-economic effect (if the latter was indicated in the investment strategy);

10) optimization of the investment portfolio in real time, taking into account the changing levels of profitability and risk (including in the context of individual elements);

11) successful implementation evaluation of investment projects and portfolios.

III. RESEARCH METHODS

Particular attention should be paid to the possibility of income taxation optimization for individual instruments that are included in the portfolio. This is due to the different level of taxation, the availability of benefits and preferences for different instruments that make up the investment portfolio.

When forming an optimal investment portfolio in the context of a risk-based approach, calculations should be performed using the formulas of Markowitz, Sharpe and Treinor:

$$CSharpe = \frac{R - R_f}{\sigma_p} \qquad (1);$$

$$CTreinor = \frac{R - R_f}{\beta} \qquad (2)$$

where:

R – investment portfolio profitability;

R_f – risk-free investment portfolio profitability;

β – standard deviation of portfolio profitability (systematic risks);

σ – standard deviation of portfolio profitability (systematic and non-systematic risks).

β- coefficient is calculated by the following formula:

$$\beta = \frac{COV_{pm}}{\sigma_m^2} \qquad (3)$$

where:

cov_{pm} – covariance between the investment portfolio profitability and the average one on the market;

σ_m^2 - variance of the average profitability of the market.

Important! When calculating the Sharpe ratio, the average profitability deviation of the investment portfolio (or its element) is used, and when calculating the Treinor coefficient, the variance (square of the standard deviation) of the average profitability of the market is used.

Criteria for optimal portfolio formation by Sharpe ratio:

CSharpe > 1 – the portfolio is optimal, and ensures profitability and optimal aggregate risks

1 > CSharpe> 0 – the portfolio can be considered as optimal, because profitability is provided, but the level of risk is higher than one of profitability.

CSharpe < 0 –the portfolio is not optimal, because the profitability is negative, the level of risk is high.

Criteria for the optimal portfolio formation by Treinor coefficient:

CTreinor> 0 – the portfolio is optimal, but the closer the value to zero, the lower the investment efficiency;

CTreinor> 0 – the portfolio is not optimal.

IV. FINDINGS.

Let's consider the order of practical application and calculation of the above mentioned coefficients on a practical example. For instance, it is necessary to assess how optimal is the investment portfolio, which includes the shares of three companies (table.1) with a profitability of risk-free asset in 6%.

TABLE I. COMPOSITION OF THE INVESTMENT PORTFOLIO

Date	Quotation V1, d.e.d	Quotation V2, d.e.d	Quotation V3, d.e.d	The average profitability of the market, %
01.01.2017	1025,00	1097,00	1120,00	3,54
01.02.2017	986,00	999,00	957,00	6,57
01.03.2017	1254,00	1365,00	1025,00	9,24
01.04.2017	1296,00	1469,00	1325,00	2,25
01.05.2017	1312,00	1582,00	1425,00	3,45
01.06.2017	1456,00	1896,00	1698,00	-9,81
01.07.2017	1564,00	1968,00	1456,00	16,71
01.08.2017	1642,00	1354,00	1542,00	-12,01
01.09.2017	1354,00	1245,00	1289,00	21,00
01.10.2017	1457,00	1369,00	1659,00	10,87
01.11.2017	1596,00	1785,00	1785,00	4,37
01.12.2017	1798,00	1965,00	1896,00	9,05
01.01.2018	1998,00	2036,00	2010,00	3,54

The solution algorithm includes 7 stages:

1. Calculation of profitability of each company shares.

2. Determination of the profitability of the investment portfolio.

3. Calculation of standard profitability deviations for each element of the investment portfolio.

4. Calculation of the standard deviation of the profitability of the investment portfolio.

5. Calculation of the Sharpe ratio.

6. Calculation of β-coefficient.

7. Calculation of the Treinor coefficient.

The results of calculating of shares profitability of each of the companies and the investment portfolio as a whole are shown in table. 2

It is recommended to calculate the standard deviation using Microsoft Excel using the standard deviation function. Results are the following: B1 - 10,62%; B2 - 18,02%; B3 - 15,29% (average value of investment portfolio risk - 14,64%).

On this basis, $CSharpe = \dfrac{6,35 - 6,00}{14,64} = 0,0237$

Computation of β-coefficient for calculation CTreinor:

$$\beta = \frac{COV_{pm}}{\sigma_m^2} = \frac{0,004739}{0,008939} = 0,530154$$

Hence: $CTreinor = \dfrac{0,0635 - 0,0600}{0,530154} = 0,0065$

(the effectiveness of investments is minimal, it is recommended not to form this investment portfolio).

TABLE II. EVALUATION OF THE COMPANY'S INVESTMENT PORTFOLIO PROFITABILITY, IN %

Date	Profitability V1	Profitability V2	Profitability V3	Average portfolio profitability
01.01.2017	x	x	x	x
01.02.2017	-3,80	-8,93	-14,55	-9,10
01.03.2017	27,18	36,64	7,11	23,64
01.04.2017	3,35	7,62	29,27	13,41
01.05.2017	1,23	7,69	7,55	5,49
01.06.2017	10,98	19,85	19,16	16,66
01.07.2017	7,42	3,80	-14,25	-1,01
01.08.2017	4,99	-31,20	5,91	-6,77
01.09.2017	-17,54	-8,05	-16,41	-14,00
01.10.2017	7,61	9,96	28,70	15,42
01.11.2017	9,54	30,39	7,59	15,84
01.12.2017	12,66	10,08	6,22	9,65
01.01.2018	11,12	3,61	6,01	6,92
On average	6,23	6,79	6,03	6,35

Source: it is compiled and calculated by the author.

In practice, investors are often faced with the need to optimize the investment portfolio, taking into account the specific weights for each of the assets. When calculating the Sharpe and Treinor coefficients, it is necessary to adjust the profitability indicators and the standard deviation of the profitability of each of the elements of the investment portfolio by equity values.

If it is necessary to form an investment portfolio under maximizing profitability conditions and minimizing risk (the first option) or a portfolio with a given profitability and the minimum possible risk (the second option) , the model of G. Markowitz (Nobel prize winner 1990) is most commonly used by investors and financial analysts. The calculation is carried out within one:

A) investment portfolio profitability:

$$\text{(4)}$$

where:

R – investment portfolio profitability, $w_i - specific\ weight$ of i – portfolio element

r_i _ i – portfolio element profitability;

Б) risk of the investment portfolio:

$$\sigma = \sqrt{w_i * w_j * COV_{ij}} \qquad \text{(5)}$$

where:

σ – risk of the investment portfolio, w_i – specific weight of i-element of portfolio, w_j – specific weight of j-element of portfolio; cov_{ij} - profitability covariance of i-element and j-element of investment portfolio.

In order to optimize the work process and time resource savings, all calculations on the Markowitz model are recommended to be made through Microsoft Excel. In particular, the calculation algorithm is as follows:

The first block (common part for the two options) is the following:

Calculation of the profitability of each investment portfolio element (it is recommended to use at least monthly for the last year). To do this, use the average function with the putting in parentheses of the number range with the quotations of the corresponding element of the investment portfolio.

Calculation of each investment portfolio element risk (it is recommended to use at least monthly for the last year). To do this, you need to use the function of standard deviation with the cell interval in brackets with the quotes of the corresponding element of the investment portfolio.

Risk levels and profitability determination of the entire investment portfolio. Thus, when calculating the level of risk, first of all, it is necessary to carry out an additional calculation of the profitability covariance of the investment portfolio elements, that's why it is necessary to use "Data Analysis" configuration with the subsequent choice of the "covariance" key. In order to obtain the final value of the risk level of the investment portfolio, it is necessary to carry out the calculation according to the formula (5), for which you should use the functions ROOT (to calculate the square root) and MULTIPLY (for the matrix product of two arrays). To calculate the investment portfolio profitability, it is necessary to sum up the products of the shares and returns of the portfolio elements.

The second block (specific features for each of the two options) is following:

4.1. Determination of the optimal portfolio at a given level of profitability. To perform the calculation, you should

use "Solver" key in Microsoft Excel. Thus, when minimizing the value in the cell with the level of risk, it is necessary to introduce restrictions (including on the signs of specific weights and their amount) and set the required level of profitability. If there is a solution, the system will give the most suitable for the given criteria.

4.2. Determination of the optimal portfolio at a given level of risk. To perform the calculation, you should also use "Solution Search" key in Microsoft Excel. However, in this case, it is necessary to maximize the values in the cell with the level of profitability under the imposed restrictions (including the signs of specific weights and their amount) and set the acceptable level of risk. If there is a solution, the system will give the most suitable for the given criteria.

Thus, its practical application will be considered in given example and the data in table 1. Points 1, 2 and partially 3 (concerning calculation of profitability of an investment portfolio) of the first block of the decision are already executed at calculation of Sharpe and Treinor coefficients (see table. 2). With reference to the calculation of the investment portfolio risk, it will be 11.25% under strict sequence of with algorithm described above. Herewith, the calculations showed that the optimal portfolio at a given 6% profitability should be formed at the expense of the first element. At the same time, under the second option (the portfolio risk is not more than 10%), the maximum profitability will be 6.27% with the following investment portfolio structure: element 1 – 87%; element 2 – 9%; element 3 – 4%.

One of the most popular recommendations for the investment portfolio optimization (stated by both scientists and investors-practitioners) is its diversification, widely used in the practical application of any methods. However, sometimes (recently – increasingly) there is excessive (excessive) diversification, which poses additional problems, rather than promoting the solution of existing ones. Thus, in particular, excessive optimization can contribute to the following:

1) lack of real opportunities for the investment portfolio management on a qualitative level;

2) investment portfolio formation in the process of optimization through the application of a large number of high-risk assets (portfolio elements) in order to extract the highest profitability;

3) significant costs for the development of the optimization plan, search for new assets to be included in the portfolio and direct obtain;

4) discrepancy of the t investment portfolio structure and composition of the company's investment strategy;

5) implementation of diversification, taking into account the influence of only 1-2 factors (their real realization will lead to significant losses).

V. CONCLUSION

In conclusion, it should be noted that the investment portfolio optimization is the most important task of the company in the investment strategy implementation, investment capital in projects. At the same time, the study showed that there are many approaches, methods and principles of its implementation in the context of the practical realization of the risk-based approach.

However, the success of optimization depends on strict sequence of basic principles described and presented algorithms, as well as the accuracy of calculations, which can be achieved by using software products (for example, Microsoft Excel).

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