Application of production function framework for the integral assessment and forecast of industry development in a region

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Abstract— **The article describes the main directions of applying the production functions as a tool of analysis, assessment and forecast of production processes in the economic systems of different levels. Using the production function framework, the authors present the studies of regularities and tendencies in the development of the industry branches in Astrakhan region (Russia). The application of the Cobb-Douglas production function for the analysis of industry at the regional level is reasoned. Relying on the analysis, the authors draw the conclusion on a limited application of the Cobb-Douglas function because of the high level of indicator aggregation and prove the direction of the production function modification, enabling to consider the characteristics of fixed assets of industry in the conditions of the contemporary information restrictions. The proposed modification of the Cobb-Douglas production function is tested by the authors for the series of dynamics of the main indicators, characterizing the industry of Astrakhan region.**

Keywords—production function; Cobb-Douglas function; industry; mathematical models; factors of economic growth; coefficients of elasticity; coefficient of scale.

I. Introduction

One of the elements of economic models is production functions, allowing to obtain the generalized quantity characteristics of economic system which can be used for the analysis, assessment and forecast of tendencies of the development and changes in a production process. L. L. Terekhov defines a production function as an economic-mathematical expression of the dependence of results of production activity on the indicators-factors which caused these results [26]. The production function establishes and quantitatively measures the relation between the production result (production rate, gross domestic product, profit, etc.) and technical, natural, social, and economic factors which provide the result [5].

II. Literature Review

The analytical review of scientific works on the issues of creation and application of production functions enabled to conclude on the considerable interest of a wide range of scientists in this area [5]. Many authors, L. L. Terekhov [26], G. B. Kleiner [14], E. Heady and J. Dillon [11], N. B. Barkalov [4], M. K. Plakunov and A. Rayatskas [19], and A. I. Shapiro [22], focus on the economic essence of a production function and consideration of the methodology of its creation and assessment of parameters. The scientists in the field of econometrics and economic modeling, A. I. Anchishkin [2], A. G. Granberg [10], A. S. Emelyanov [8], B. N. Mikhalevskiy [18], E.M.Chetyrkin and A.Klass [7], determine the role of a production function in the quantitative analysis of the development of economic systems, its use in the models of economic growth. The studies of the foreign authors such as R. Solow, K. Arrow, R. Hoffman, R. Sato, S. Klemkhout, L. Johansen, Yu.Paiestcki [22, 7], R. Stone [25], Ya.Stal and G. Shakolshai [24] and some others propose the modification directions of a production function and the specification of its variables. In contemporary research, as G. B. Kleiner [13] points out, the development of the neoclassical economic theory contributed to the application of production functions for any economic systems of micro, meso -, macro - and mega-economic levels. The scientific works of many Russian researchers are devoted to the use of production function tools at the micro-level for the analysis and forecast of economy of an enterprise or a complex, assessment of innovative technology influence, and arrangement of rational and effective production [1, 3, 17, 20, 21, 23]. One of the directions of applying the production functions is their use for the determination and assessment of quantitative and high-quality components of economic growth, determination of the contribution of growth factor groups to the production dynamics of the economic system [6].

III. METHODOLOGY

The article reports on the results of the study of the regularities and tendencies of the development in the industry branches of Astrakhan region (Russia) by using the framework of a production function.

The industry is the leading branch of the economy in Astrakhan region. Following the results of 2014, the industrial output is equal to 173012.2 million Russian rubles (RUB) according to the current prices which make about 60% of the gross regional product (288951.65 million RUB) [9]. Owing to this fact, the industry considerably determines the efficiency of resources utilization (natural, material, and labor resources) and the potential, scientific and technical production of the region. Table 1 describes the dynamics of indicators of the industry in Astrakhan region.

Over the last decade, the industrial production output of the region grew by 4.2 times, the number of employees of the industry had significantly decreased by 36% by 2005 and the size of the employed fixed assets grew almost by 2.5 times at the same time. The extraction of mineral resources (58.3%) forms the greatest specific part in the production volumes of Astrakhan region. The processing plants and production and distribution of electric power, gas and water make 28.1% and 13.6% respectively. In the extraction of mineral resources, the region took the 19th place across the Russian Federation (in the Southern Federal District – the 1st place) in 2013, among the processing plants – the 64th (4th), in production and distribution of electric power, gas and water – the 53th (4th).

TABLE I. DYNAMICS OF INDICATORS OF THE INDUSTRIAL PRODUCTION IN ASTRAKHAN REGION [9]

Indicators	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Industrial production output in actual prices (mln. RUB), including:	40,922	45,598	51,960	84,692	62,969	76,689	95,724	125,356	162,313	173,012
- extraction of mineral resources	9,504	13,371	13,687	9,319.4	18,548	22,508	32,246	68,392.9	93,638.4	100,786
- processing plants	21,550	24,041	29,939	67,243	33,631	40,938	47,863	39,812,1	48,550.8	48,602.5
- production and distribution of electric power, gas and water	9,869	8,156.5	8,323	8,129.5	10,790	13,238	15,615	17,150.5	20,123.6	23,623.6
Industrial production output in comparable prices (mln. RUB)	40,922	38,153	35,537	83,143	42,545	37,871	39,104	50,764	56,337	53,835
Average number of employees (person), including:	68,900	69,600	55,400	55,159	50,581	48,681	46,221	45,410	45,635	44,211
- extraction of mineral resources	5,400	5,400	4,900	5,183	4,889	5,007	5,071	4,946	4,941	5,133
- processing plants	49,400	49,400	36,900	36,457	32,117	30,115	28,261	27,643	27,941	26,421
- production and distribution of electric power, gas and water	14,100	14,800	13,600	13,519	13,575	13,559	12,885	12,820	12,752	12,657
Fixed assets at the end of the year (mln. RUB), including	73,196	91,446	79,207	101,878	119,945	178,530	217,954	241,877	268,273	182,235
- extraction of mineral resources	29,673	41,632	43,883	61,005	68,930	117,929	143,260	158,722	169,791	116,364
- processing plants	8,903	12,291	14,589	18,178	21,333	24,97	26,232	29,701	30,161	13,006
- production and distribution of electric power, gas and water	34,620	37,523	20,735	22,695	29,682	36,104	48,462	53,454	68,321	52,865

On the basis of the characteristics of the production function types [12, 15, 16, 22], the conducted research enables to conclude that there is practicability in the use of the Cobb-Douglas production function for the analysis of industry which has a number of advantages over others: 1) allows to get more adequate assessments of the parameters of the function if the restriction to the existence of a single replacement of factors is refused; 2) contains a small number of parameters which have clear economic interpretation; 3) allows to take into account the unidentified factors of economic growth which represent the technical progress in a broad sense; 4) gives the possibility to analyze the change of the total economic efficiency of production and assess the segregated influence of extensive and intensive factors of growth. So, the analysis of the industry of Astrakhan region is based on the Cobb-Douglas production function for which variables the exponential law works:

$$Y = e^{Ao} * K^{\alpha} * L^{\beta} \quad (1)$$

where Y is the volume of output products; K – expenses of fixed assets; L – number of employees, working in the production; A – the coefficient of scale which brings the both parts to measurement units of gross domestic product, α and β – the coefficients of elasticity for expenses of fixed production assets and labor. The parameters of α_0, α, β result from the processing of dynamics series *Y*, *K*, *L* for the period from 2005 to 2014.

The parameter assessment of the function of this type is complicated because it is a power function therefore it is impossible to apply the linear least-squares method. However, if their logarithms are used instead of variables, then the function becomes linear and logarithmic:

$$\ln Y = a_o + \alpha \ln K + \beta \ln L. \quad (2)$$

The parameters of such a function can be assessed with reasonable facility.

The correlation and regression analysis was applied as a tool of processing the series of dynamics and it results in the creation of regression equations – production functions – for the industry branches of Astrakhan region.

TABLE II. THE PRODUCTION FUNCTIONS FOR THE INDUSTRY BRANCHES of Astrakhan region (the standard errors of coefficients are given in the parentheses)

Branch of industry	α^*	β^*	$e^{a0\,*}$	R^2	F	**Production function**
1	*3*	*4*	*5*	*6*	*7*	*8*
The whole industry including:	0.474* (0.346)	-1.469 (0.997)	$e^{21,65}$ (14.516)	0.808117	14.74031*	$Y=e^{21.65}\cdot K_t^{0.474}\cdot L_t^{-1.469}$
extraction of mineral resources	1.299* (0.358)	1.963 (6.038)	$e^{-21.268}$ (53.958)	0.718	8.89*	$Y=e^{-21.268}\cdot K_t^{1.299}\cdot L_t^{1.963}$
processing plants	0.110 (0.335)	-0.934 (4.602)	$e^{19.196}$ (9.02)	0.521	3.81	$Y=e^{19.196}\cdot K_t^{0.11}\cdot L_t^{-0.934}$
production and distribution of electric power, gas and water	0.589* (0.117)	-4.119* (0.907)	$e^{42.372}$ (9.369)	0.936	51.553*	$Y=e^{42.372}\cdot K_t^{0.589}\cdot L_t^{-4.119}$

The parameters of regression were calculated by the least-square method, using the packet of data analysis of Microsoft Excel software programme.

IV. RESULTS

The obtained results, i.e., the production functions for the industry branches of Astrakhan region, are provided in Table 2.

Now, the models of production functions, presented in Table 2, should be checked for assessment quality. Checking the adequacy of the models, created on the basis of regression equations, starts with checking the magnitude of each regression coefficient (rc). The standard errors of coefficients are calculated for all above provided models (Their values for each parameter of the regression equation are given in the parentheses in Table 2.) and statistically significant errors are marked with an asterisk. The check of the hypothesis on the relevance of regression coefficients shows that some parameters are statistically significant, and some of them are insignificant. Thus, it is impossible to deselect the models, relying on this criterion. Further, the certification of adequacy of the whole model is performed by calculating Fisher's F-test. If the following condition is met:

$$F_r > F_{rc}\ (\alpha; \nu 1 = k - 1; \nu 2 = n - k), \quad (3)$$

the hypothesis of discrepancy of the relations, expressed by equation of regression (r) is rejected by the existing equation.

This condition is met for all models except for the processing industry; in all other models for a number of industries of Astrakhan region, $F_r > F_{rc}$ which proves the adequacy of the presented models of the real situation.

The created model is in general adequate according to its check by the F-test but some coefficients of regression are not significant. Therefore, the model with similar characteristics can be used for making a certain range of decisions.

The intension of the calculated parameters of production function is confirmed by the absence of multi-collinear relation between the variables *K* and *L*. The values of the correlation coefficients for some industries are rather small (less than 0.55 by the module) that proves the absence of significant correlative dependence between the variables *K* and *L*.

The multiple correlation coefficients for the calculated equations of the production function make over 0.8 in all cases, i.e., shows quite strong relation between a dependent variable and a set of independent variables. It means that the suitable type of equations is chosen, and all major factors of interrelation are considered.

The values of determination coefficient (R^2), indicating which part of variation of a dependent sign (Y) is explained by the factors included into the model, are also rather high (from 0.71 to 0.94) and exceed normalized values. The processing plants ($R^2 = 0.52$) are an exception. Thus, from 70 to 94% of the variation of production output values depend on the change of indicators of fixed assets and average number of employees and from 6 to 30% are influenced by unidentified factors. Having regard to the above-mentioned consideration, the production functions provided in Table 2 can be used for practical purposes.

Further, the authors analyze the results of the calculation of the production function (Table 2). The α and β parameters show the size of an annual average increase in output of end products, falling by 1% of an increase of the corresponding production factor in case of the assumption of an invariance of other factor. It can be noticed that the initial prerequisite, considered by Cobb and Douglas as $\alpha + \beta = 1$ is not observed for any industry. In general, $\alpha + \beta < 1$ for industry which demonstrates the decrease in an average total indicator of efficiency for the considered period. From economic point of view, it is possible to speak about the negative effect of the expansion of production scales in this case. The production output increases more slowly in comparison to the growth of costs of production factors, their total efficiency declines,and the deintensification of economic growth takes place. The negative effect of expansion of production scales is noted in Astrakhan region in production and distribution of electric power, gas and water. In such an industry as extraction of mineral resources, the obtained parameters of the production function are $\alpha + \beta > 1$, and it means that the total average indicator of efficiency in the specified industry raised, that is the positive effect of the expansion of production scale.

For the industry as a whole, processing plants and production and distribution of electric power, gas and water, the elasticity of labor is negative that means the inverse relation between the dynamics of volumes of production output and the dynamics of the number of employees when the amount of labor resources, involved into the industries, was falling that took place in the period from 2005 to 2014; the amount of production output in the comparable prices in general grew over the above mentioned period.

The reduction of labor resources dominantly influenced the dynamics of production volumes in the industry branches of Astrakhan region as a whole and the growth of costs of fixed assets had essentially a smaller influence. It is specified by the high elasticity of production volumes by labor in all industry branches (The indicator β in Table 2 in the module is significantly higher than the indicator α.). As the indicator β has a negative value and the value of α is positive, it makes possible to draw a conclusion that in the period from 2005 to 2014 the industry of Astrakhan region was characterized by the use of capital intensive technology; from Table 2 it is seen that $\alpha > \beta$ for all industry branches apart from the extraction of mineral resources. It means that each per cent of increase in the rate of capital gain, in comparison to the labor resources, provides bigger rate growth of production volume. This fact can be considered as the indirect evidence of rather more effective use of the capital in the industry of Astrakhan region. In the industry branch "extraction of mineral resources" the used technology, on the contrary, is characterized as labor-intensive. The calculation of parameters of production functions shows that the efficiency of functioning of the industry of Astrakhan region is low, there is a production deintensification.

As the analysis shows, the production function of type (1) features some disadvantages, namely, the high level of indicator aggregating. Moreover, some researchers point out that the scientific and technical progress provided by this function is the exogenous, i.e., attracted by the factor from the outside, though it leads to growth of an indicator of total cost efficiency of factors. The disadvantage of this function is the fact of underestimation of the role of the fixed business assets as material carrier of the scientific and technical progress. With respect thereto, it is necessary to carry out more detailed specification of the factor "fixed business assets" for improving the quality and expansion of analytical potential of the used model.

The authors considered the methods of conducting effective assessment of the fixed assets involved into the production which exist in the Russian and foreign theory and practice, and made conclusions on the difficulties of their applied use and practicability of introduction of the indicators, which characterize the level of application, age and process of reproduction of fixed assets, into a production function. For this purpose, the authors propose to use the traditional indicators, describing the efficiency of use of fixed production assets, i.e., depreciation coefficient of fixed production assets (k_D) or coefficient of availability (k_{av}), complementing it to 1, coefficient of utilization of production capacities (k_{upc}); coefficient of replacement of fixed production assets (k_{rep}), and coefficient of retirement rate of fixed production assets (k_{ret}).

Then, the authors characterize the economic sense of the specified indicators. It is known that the coefficient of utilization of production capacities specifies the intensity of use of fixed assets, efficiency of functioning of potential production of the industry (enterprise) and denotes the level achieved in using the equipment capacity. The level of use of production capacity and its value are determined by various factors, but the group of technical factors including the age of fixed assets, rates of their replacement and structure enhancement has the highest value. The age of fixed assets describes the coefficient of availability (k_{av}). The enhancement of the fixed assets structure is carried out during the progress of reproduction of fixed assets. The coefficients of replacement and retirement can be mentioned among the most important characteristics. The first of the above mentioned coefficients (k_{rep}) is defined as the ratio of the cost of the fixed assets, introduced during the period, and their total cost at the end of the period. The level of replacement can decrease because of decline in the amount of the fixed assets introduced. Besides, the coefficient of retirement of fixed assets also has a certain influence on the level of replacement. This coefficient is calculated by dividing the cost of the fixed assets which were retired from the operation to their cost by the beginning of the considered period. If the rates of retirement of fixed assets are rather low, then the replacement level also reduces as there is a preservation of the technical condition of the fixed business assets at a certain time.

Thus, the authors suggest detailing the Cobb-Douglas production function of type (1), applied for the analysis of the industry of Astrakhan region, in view of all the above mentioned facts:

$$Y = e^{Ao} * K^{\alpha_1} * k_{av}^{\alpha_2} * k_{SE}^{\alpha_3} * L^{\beta} \quad (4)$$

As the processes of replacement and retirement of the fixed production assets are interdependent, the authors propose to consider these processes through the generalizing indicator, called "coefficient of structure enhancement of fixed assets" (k_{SE}) and it is necessary to understand the enhancement of structure of the fixed production assets as their "rejuvenation", that is the increase in specific rate of the equipment which is "younger" in operation and progressive in its total amount as:

$$k_{SE} = k_{rep}/ k_{ret} \quad (5)$$

The developed modification of the Cobb-Douglas function has undoubted advantages over the traditional model as it enables to receive more detailed effective assessment of the factor "fixed assets" in the conditions of imperfection of the contemporary system of statistics and can be used without essential difficulties for the applied purposes.

The parameters of the function were obtained by using the tools of the correlation and regression analysis in the assessment of dynamics series of indicators Y, K, k_{av}, k_{SE}, L. The check of the created models for the adequacy with the F-test shows that the type of the equation and the variables are selected correctly for all branches which means the provided models correspond to a real-life situation.

TABLE III. THE PARAMETERS OF THE PRODUCTION FUNCTION Y= EAO *KA1 * KAVA2 * KSEA3 * LB

Branch of industry	***α₁***	***α₂***	***α3***	β*	$e^{a0\,*}$	***R²***	***F***
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
extraction of mineral resources	1.772195	-0.33911	-0.36914	-1.418	$e^{4.023}$	0.904482	7.101951
processing branches of industry	-0.12648	-5.20258	0.348412	0.883676	$e^{22.22}$	0.954	15.4
production and distribution of electric power, gas and water	0.836343	-0.93247	0.010229	-2.66787	$e^{-29.53}$	0.987	56.328

The fact, that the proposed model of the production function of type (4) in comparison to the initial type (1) allows to obtain more exact estimates, is confirmed by the calculated coefficients of multiple correlation and determination. The values of these indicators for the proposed model considerably exceed the values of the specified coefficients for the production function of type (1). If to judge by values of determination coefficient, then 90.4-98.7% of the variation of values of production output depend on the change of indicators of fixed assets and average number of employees, and only 1.3-9.6% – on the influence of unidentified factors. Due to the improvement of values of coefficients of multiple correlation and determination, the practical importance of the proposed function is higher than the importance of the functions of type (1).

V. CONCLUSION

The obtained results allow to draw the conclusion that the solution of the problems which the economy of Astrakhan region is facing, first of all, should involve the increase in the contribution to the economic growth of intensive factors, i.e. production intensification. There is a need to increase fixed assets of the industry as their depreciation is rather high and to optimize the number of employees in the majority of industry branches. The influence of fixed assets on the change of production output is not too large in the most cases that is linked, in our opinion, with the low extent of use of available and additionally introduced production capacities in the majority of the industry branches of Astrakhan region. The low degree of the utilization of production capacities is caused by the following reasons: limited demand for the product of local producers, high depreciation of fixed assets, and achievement of the limits of production concentration. Owing to the influence of the specified and other reasons (e.g., inefficient management) there is also a production deintensification, i.e., the increase in the introduced resources leads to the decrease in its efficiency. The inflow of capital investments into the industry branches is reasonable and it is mainly linked with the need of forming the new industrial base of industrial production within the longer term.

REFERENCES

[1] Afanasiev A. A. (2009). .*Econometric research of production functions of the gas industry of Krasnoyarsk Krai*, Economy and mathematical methods, Vol. 45, No. 3.

[2] Anchishkin A. I. (1973). *Forecasting of Growth of Socialist Economy*, Moscow.

[3] Bagrinovskiy K. A., Egorova N. E. (2010). *Methods of the analysis of innovative technologies on the basis of Farrell's index*, Economy and mathematical methods, Vol. 46, No.1.

[4] Barkalov N. B. (1981). *Production Functions in Models of Economic Growth*, Moscow.

[5] Belik E. A. (2003). *A Production Function in the System of Industrial Policy*. Abstract of dissertation of Candidate of Economic Sciences. Astrakhan: AGTU.

[6] Belik E. A., Karlina E. P. (2003). *Problems of applying Cobb Douglas function for the analysis of industry*, Modern Aspects of Economy, No. 3, St Petersburg.

[7] Chetyrkin E.M. Klass A. (Ed.) (1986). *Theory and Practice of Statistical Modelling of Economy*, Moscow.

[8] Emelyanov A. S. (1980). *Social Production: Dynamics*, Tendencies, Models, Kiev.

[9] Federal State Statistics Service, www.gks.ru

[10] Granberg A. G. (1985). Dynamic Models of the National Economy, Moscow.

[11] Heady E. Dillon J. (1965). *Production Functions in Agriculture*, Trans. from English, Moscow.

[12] Ivanilov Yu. P., Lotov A.V. (1979). *Mathematical Models in Economy*, Moscow: Science.

[13] Kleiner G. B. (2014). *Bases of the theory of production functions of economic systems*, System modelling of social and economic processes, 35th Anniversary meeting of the workshop school. Voronezh: VSU, pp. 64-66.

[14] Kleiner G. B. (1986). *Production Functions: Theory, Methods, Application*, Moscow.

[15] Kotulic R., Pavelkova J., Vozarova I. K., Vavrek R. (2014) *Managerial view on the production analysis in agriculture - the application of the Cobb-Douglas production function*, International Multidisciplinary Scientific Conference on Social Sciences and Arts SGEM2014, www.sgemsocial.org , SGEM2014 Conference Proceedings, ISBN 978-619-7105-28-5/ ISSN 2367-5659, September 1-9, Book 2, Vol. 4, pp. 459-466.

[16] Kuranov G. O. and Lifanchikov A. N. (1971). *Economical and statistical modelling in industries and the analysis of intensive factors of development*, Economy and mathematical methods, issue 3.

[17] Matveenko V. D. (2009). *Anatomy of a production function: technological menu and choice of the best technology*, Economy and mathematical methods, Vol.45, No.2.

[18] Mikhalevskiy B. N. (1972). *System of Models of Mid-Term Economic Planning*, Moscow.

[19] Plakunov M. K., Rayatskas R. (1984). *Production Functions in the Economic Analysis*, Vilnius.

[20] Revazov B. V. (2007). *Application of Cobb Douglas production function as an element of effective management of sustainable development of an enterprise*, Space of Economy, No. 4-3, [Electronic source]. Retrieved from http://cyberleninka.ru/article/.

[21] Ruzanov A. I. (2011). *Production functions and their use for the description of regularities of production*, Bulletin of NNGU, No. 5-1, [Electronic source]. Retrieved from http://cyberleninka.ru/article/.

[22] Shapiro A.I. (Ed.) (1970). Forecasting of Capitalist Economy. Methodology problems, Moscow.

[23] Smagin B. I., Dachkin A.V. (2005). *Application of production functions in the analysis of efficiency of using resources of agrarian production*, Issues of modern science and practice, University named after V. I. Vernadskiy, No. 1.

[24] Stal Ya., Shakolshai D. (1967). *The generalized production function CES. Economy and mathematical methods*, Vol. 3, issue 4.

[25] Stone R. (1965). *Modelling of economic systems*. Economy and mathematical methods, Vol. 1, No. 3-4.

[26] Terekhov L.L. (1974). *Production Functions*, Moscow.