

# Argumentation in Covariational Reasoning: Middle School Student's Solving Covariation Problem with Different Cognitive Style

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**Abstract-**The aim of this research is to describe the process of covariational reasoning and how to build argumentation in solving covariation problems for middle school students who had verbal and visual cognitive style. This qualitative descriptive research investigated the differences in covariational reasoning and how to build argumentation process for students with verbal and visual cognitive style, whether they have different covariational reasoning processes in solving covariation problem or not. The participants of this research are 2 students, one with verbal cognitive style and other with visual cognitive style. The participant selected from the result of the cognitive style test, the mathematics performance test and the teacher's recommendation. Data were obtained using covariation problems and interviews. The result of this study pointed out that student with verbal cognitive style were able to construct images of function's dependent variable changing in tandem with the imagined change of the independent variable, and in some situations, were able to construct images of rate of change and were able to construct images of rate of change for contiguous intervals of a function's domain. Student with verbal cognitive style was constructing her argumentation of mental action deductively. While, student with visual cognitive style was able to exhibit mental action of initial coordination, direction of changes coordination and had some difficulties in doing mental action of amount of change coordination. The student with visual cognitive style was constructing her argumentation of mental action inductively. We discuss implications of these results for perspectives on argumentation in covariational reasoning, students' understandings of graphs and discuss curricular treatments of function again with a covariational lens, and areas of future research

**Keywords:** Reasoning, Covariational Reasoning, Argumentation, Cognitive Style

## I. INTRODUCTION

One of the mathematical concepts taught in school is function. It is one of the concepts in mathematics that requires reasoning and argument in understanding it. The concept of function is often suggested as the main reference in mathematics. The function concept is an important and unifying concept in modern mathematics, central to many different branches of mathematics, and essential to related areas of the sciences [1].

The concept of function is centre of a student's ability to describe the relationship between change variables, explain parameter changes, and interpret and analyze graphs [2]. The development of the concept of function raises two approaches in teaching function, namely the correspondence approach and covariational approach [3]. "The correspondence approach is based on the definition of function as a relation between two sets of domains and ranges", so that for every member of the domain there is exactly one  $y$  member of range". While "the covariational approach refers to the ability to form a picture of two varied quantities and coordinate and their change in relationships with one another". The covariational approach emphasizes the expression "relationship" between two structured quantities that can be expressed algebraically, visually in the graph or in the real world [4].

Unfortunately, learning of function in schools is currently dominated by correspondence approaches which reveals that most classroom functional learning is done by means of students being asked to manipulate algebraic equations and compute answers for specific types of functions. This is also expressed by several studies in Turkey that "The concept of function is not concerned with the covariational approach to function" [5]. The emphasis on the correspondence approach make students to focus more on rules and formulas (many emphasize operating procedures), so students cannot describe how to obtain the output value of the known input value, since the students are given more notation, manipulation, and function formula[6]. Consequently, students have difficulty in translating between different representations and applying basic concepts at different levels of abstraction. Ayes et al., Vinner and Dreyfus [7] report that many students think a function must only be represented by a single algebraic rule describing a continuous, one-to-one function. However, even high performing students have difficulties in modeling dynamic function situations.

In particular, students' difficulties are associated with lack of imaging and coordinating the simultaneous changes of variables, namely their covariational reasoning abilities [8].

Furthermore, Carlson et al. [9] define covariational reasoning as “to be the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other”. In this study, Carlson et al. proposes a covariation framework encompassing detailed mental actions that students exhibit as they engage in mathematical activities when applying covariational reasoning in the context of representing and interpreting a graphical model of a dynamic function event.

**Table 1. Covariation Framework**

MA1	An image of two variables changing simultaneously;
MA2	A loosely coordinated image of how the variables are changing with respect to each other (e.g., increasing, decreasing);
MA3	An image of an amount of change of one variable while considering changes in discrete amounts of the other variable;
MA4	An image of rate/slope for contiguous intervals of the function's domain;
MA5	An image of continuously changing rate over the entire domain

Therefore researchers emphasize that the concept of function should be introduced as covariation, that is coordinate changes in one variable with the other variable, and student should be provided with more opportunity to explore the concept of rate of change in earlier grades [10].

Brodie [11] states that "when students reason, they develop arguments to convince others or themselves of a particular claim; to solve problems, or to integrate a number of ideas into a more coherent whole". To find out students' arguments in constructing covariational reasoning, the Toulmin argumentation model developed by Toulmin was used. Toulmin's model of the argument [12] explains that "the aim of argumentation is to construct an explanation (a warrant) for the information concerning the initial state (the data) necessitates the statement which is argued (the conclusion)".

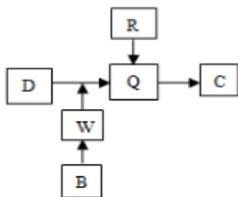


Figure 1. Argumentation of Toulmin

Everyone's covariational reasoning process is different depending on the nature of the information. When solving a problem, the process of student's work is influenced by the ways he prefers in composing what is seen, remembered and thought about, how to compile and process information, as well as experiences that have been obtained, these ways are called cognitive styles [13]. One of the cognitive styles that many students have is the verbal and visual cognitive style. This verbal and visual cognitive style uses a person's habits use his senses as the difference. The students with visual cognitive style tend to have the ability to see so it is easier to receive, process, store or use information in the form of images; the student with verbal cognitive style is more likely to say and would prefer to communicate to someone by showing how they do it [14]. Thus, students with verbal cognitive styles and visual cognitive styles have a different understanding of information, so that their argumentation and covariational reasoning different according to the information they possess and understand.

## II. METHOD

### 2.1. Participants

This study is a qualitative research that will produce descriptive data in the form of a picture of covariational reasoning and argumentation of students in completing the covariate tasks. The purpose of this study is to describe the process of covariational reasoning and how to build argumentation in solving covariation problems for middle school students who had verbal and visual cognitive style. To choose the participants, 36 students from grade 8 were administered Verbalizer-Visualizer Questioner (VVQ) (developed by Richardson [15]) and mathematics performance test to determine students' cognitive style and their ability in mathematic. VVQ contained 20-item in which each item contained 10 verbal items and 10 visual items and mathematics performance test consisted of 6 questions about integers, algebra, geometry and statistics. Based on the VVQ results, we got 12 students in verbal cognitive style group (33.3%) and 18 students in visual cognitive style group (50%). One student selected from each group who had the same sex and similar score in mathematics performance test.

### 2.2. Data collection and analysis

For this study, first we used Verbalizer and Visualizer Questioner (VVQ) to measure students' verbal and visual cognitive style. The questioner used VVQ. Then, to get argumentation in covariational reasoning process from participants, we used covariation test and interview. The Covariation Test (CT) was designed to promote students' ability to attend to the covariant nature of dynamic functional relationships. Each activity contained a collection of prompts that encouraged students to coordinate an image of the two variables changing and to attend to and represent the way in which the independent and dependent variables changed in relationship to one another and participants has to finish it in one hour. Each activity also included prompts that encouraged students to imagine a set of input values being acted upon by a

process (or function-machine) to produce a set of output values [16]. Then, we used interview to get deeper information about covariational reasoning process of students by understanding the process of student covariational, this will help explain the reasons students have trouble working function problems, especially in students with verbal and visual cognitive styles. Interview was conducted after the participants finished their answer. It needed 30 minutes for each participant, to answer 10 questions about any information that they build in covariational reasoning and how they did that. The credibility test approach used in this study is the triangulation of time, with one week length.

This section presents the analysis of data which were collected through clinical interviews. Considering the proposed research questions, qualitative data analysis techniques were employed. Students' words and their responses supporting their reasoning about a covariant aspect of real-world dynamic function situations were examined. Coding procedures were used by the researcher after collecting the interview data to compare one unit of information about covariational reasoning with for recurring regularities.

**III. RESULTS AND DISCUSSION**

**3.1 Results**

Based on the results of data analysis including data condensation, data display and data checking, we found students covariational reasoning process while solving covariation problem and argumentation while covariational reasoning process. That is presented in Table 2.

**TABLE II. STUDENT'S COVARIATIONAL REASONING PROCESS.**

Visual Participant's Covariational Reasoning Process	Verbal Participant's Covariational Reasoning Process
<ul style="list-style-type: none"> <li>The student expresses an understanding of the two quantities involved by labelling the axes with time and height (MA1);</li> <li>Understand that the volume is decreasing with respect to time and determines a decreasing tv-graph for each cooler (MA2);</li> <li>Understand that the constant rate of decrease of the volume with respect to time determines a linear graph (MA3) (Figure 2)</li> </ul>	<ul style="list-style-type: none"> <li>The student expresses an understanding of the two quantities involved by labelling the axes with time and height (MA1);</li> <li>The student correctly drew graphs of piecewise linear functions, by associating the narrower part of the coolers with a line of greater slope (MA2);</li> <li>Understand the rate of change of height for the first part of the container would be different from the rate of change of height in the second part of the container, and since the rate of volume is same, however, the width is different, the height would decrease(MA3);</li> <li>more rapidly in the first (upper) part than the second (lower) part The student drew graphs for Cooler 1 and Cooler 2 that had the same vertical intercepts and different horizontal intercepts, with the one closer to the origin belonging to the graph corresponding to Cooler 2 (MA4).</li> <li>Furthermore, over the entire domain, the graph for Cooler 2 was steeper than the graph for Cooler 1, and the seam points for these graphs had the same vertical coordinate (MA5). (Figure 3)</li> </ul>

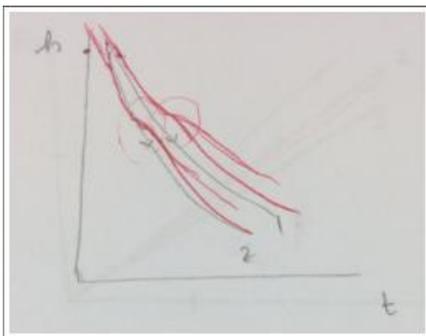


Figure 2. Visual Participant's Covariational Reasoning Process Graph

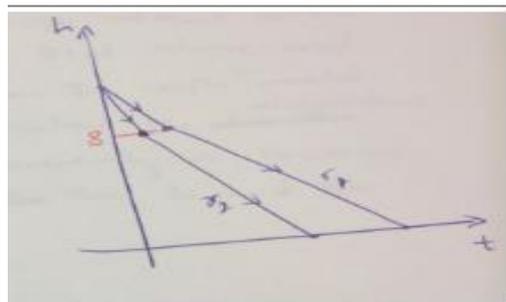


Figure 3. Verbal Participant's Covariational Reasoning Process Graph

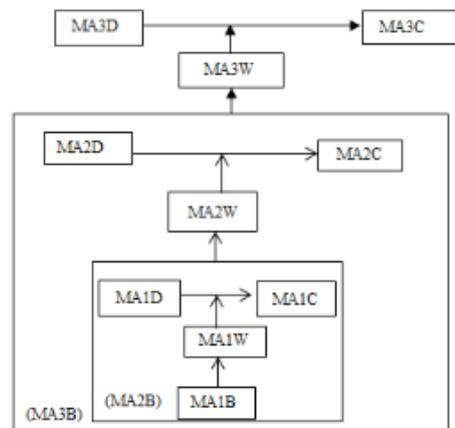


Figure 4. Argumentation visual student in covariational reasoning

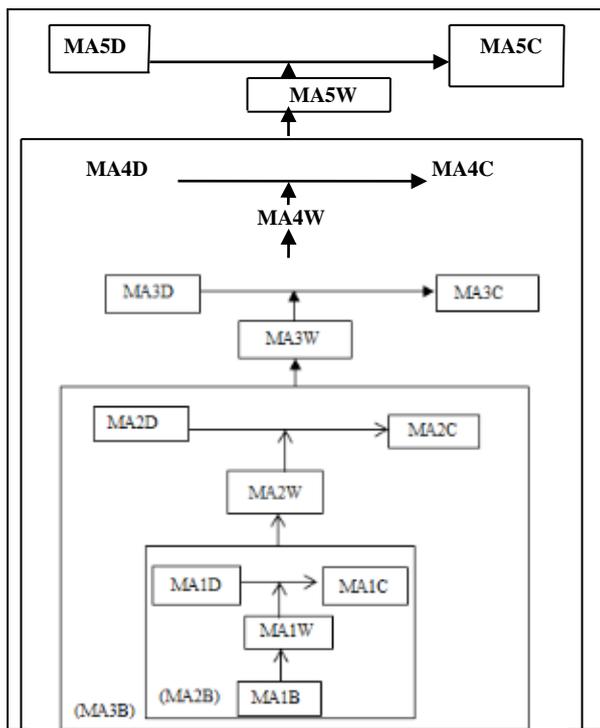


Figure 5. Argumentation verbal student in covariational reasoning

### 3.2. Discussion

Carlson et al. defines covariational reasoning in terms of mental action. This action is closely related to behaviour. While this behaviour is in turn related to Cartesian graphs. Carlson et al. explain that student difficulties have initially been observed in the context of interpreting and representing graphic function information. However, we can argue that covariational reasoning is defined by graph support [18].

Analysis of student responses to covariance tests finds that students can represent the direction of change in one variable over another, whether it is volume over time, or high against time. However, a verbal graph draws a graph reflecting the number of changes in one variable against another, while even fewer students draw a graph that accurately reflects the rate of change in one variable against another. Similar findings were reported by Carlson et al. on student difficulties by sketching charts that accurately reflect the rate of change. On the other hand, the current study found that the issue of the rate of change of one variable on the other for one cooler does not negatively impact the ability to consider the relative level of change of the same variable for other coolants. For example, in parts of question 2 corresponding to volume versus time and high against time, students who draw graphs for Cooler 2, only steeper.

In our view, the students in our experiments showed that the high water drops slower at the cooler 1 with hole  $r_1 < r_2$ , indicating the form of covariant reasoning. However, they were unable to express their covariational

reasoning in conventional charts. The main reason is that students do not have basic graphics skills. In addition, they also do not have a good understanding of the speed; for speed (whether in the context of the dropping water or moving objects) has not become a variable for them. They can reason about speed qualitatively, using adjectives like "fast" and "slow", but they are not yet accustomed to speed as a quantity.

### IV. CONCLUSION

Different cognitive style has a different way to process the information. Visual students have difficulty in drawing graphs rather than verbal students. The verbal student can build his argument perfectly and can explain well but the visual students can't draw graphics correctly, and cannot explain or cannot embrace arguments well. From the finding, students may have the wrong understanding while solving covariation problems that resulted in them choosing the wrong way and getting the wrong answer as well. This is very dependent on the experience and their habit of studying a problem. So, it is important we know how students understand a mathematics problem so that teachers can provide interventions in accordance with the needs of students. Especially on material that deals with covariation problem.

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