Critical Thinking of Pre-service Teachers in Generalizing Mathematical Idea on the Topic of Set

Abdul Haris Rosyidi
Universitas Negeri Surabaya
Surabaya, Indonesia
abdulharis@unesa.ac.id

Shofan Fiangga
Universitas Negeri Surabaya
Surabaya, Indonesia
shofanfiangga@unesa.ac.id

Tatag Yuli Eko Siswono
Universitas Negeri Surabaya
Surabaya, Indonesia
tatagsiswono@unesa.ac.id

***Abstract*— It is undeniable that the ability of critical thinking is important as mentioned in 21st century skills. Teachers, as the main educator at school, are required to think critically in every decision while they are teaching a mathematical topic. However, a number of empirical field observation have pointed out that most of the teacher unable to response to the students' difficulty due to inability to analyze the situation and think critically. This study aims to find out the critical thinking of pre-service teacher and to what extend they are able to present their idea, especially on the topic of set, in the class. 32 pre-service teachers were given the set of generalizability problems and situational problems from which the pre-service teachers' critical thinking were elicited. The findings suggest that a number of pre-service teachers are able to generalize the mathematical idea, especially by using factual generalization. This suggest that generalizing the set concept deductively using symbol only become difficulty for the pre-service teachers. In addition, however, regardless the generalization, some of them were having difficulties in presenting an understandable content.**

***Keywords*— critical thinking, generalize mathematical idea, set, pre-service teacher**

I. INTRODUCTION

Mathematics is one of the science subject that is taught in mostly all level of education. This fact is based on the reason that mathematics could develop student's logical-deductive reasoning and abstraction from which several 21st century skills can be originated [1][2]. One of the important skills is critical thinking. Several studies have defined what critical thinking is [3]–[6]. Some of them use a broad indicator of what critical thinking may involve. In summary, the definition of critical thinking presented by Rott and Leuders is used in this paper due to its flexible range of critical thinking indicators and its example in pre-service teacher case [6]. They define critical thinking as the ability that involve reflective mind in the process management of problem solving and consideration of more complex problems. Therefore, we can categorize the critical thinking as the thinking process on (1) clarification, (2) assessment, (3) inference, and (4) strategies [7].

The form of critical thinking may appear as flexible as its context. However, in mathematics learning, finding pattern and developing generalization are fundamental abilities from which mathematics activity is done [8]. Mathematics generalization involves formalizing the mathematical properties appeared in the context [2], [9]. Nevertheless, most of students experienced difficulty in generalizing mathematical idea due to the fact that they were unable to access the mathematical concept [2], [9]. The inaccessibility of the mathematical concept may occur due to the failure of the contextual delivery or teacher's delivery. Those situations are mainly resulted from how the teacher delivers the material in critical way.

Generalization in mathematics can be categorized as follows: (1) Factual generalization, where the generalization is still depended on the situation when the subject faces the process, (2) Contextual generalization, where the generalization is still depended on the thinkable context when the subject face the process, and (3) Symbolic generalization, where the generalization independent of the situation when the subject face the process [2]

To develop a student's critical thinking skill in the class, the teacher itself should be able to perform critical thinking during their teaching. However, it would be too late if they are just recently aware on how they should implement critical thinking ability when they are already teaching in the class. They should be invited to experience critical thinking since they were in pre-service teacher. Rott and Leuders suggested that the experience of pre-service teachers could impact their believe and critical ability during their future teaching [6].

Eliciting pre-service teacher critical thinking during the mathematics class may describe to what extend the teacher could response critically on generalizing mathematical fact. The topic of set is selected because it may ignite fundamental understanding on mathematical idea. Therefore, in this study, an analysis on how the pre-service teacher use critical thinking to develop their generalization on the topic of set would be presented.

II. METHOD

This qualitative study conducted to third year pre-service teacher as the research subjects in the course of School Mathematics. The data collection was using validated students worksheet that was developed from several practical cases in the real mathematics classroom. The third year students as pre-service teachers were selected because of the fact that they had already taken basic mathematics courses and pedagogical courses. In fact, they will conduct teaching immersion program in either local middle schools or International school in some places in Indonesia.

The data analysis was conducted through parallel process from tasks raters. These process were implemented to improve the interrater reliability on the data interpretation [10]. In summary, the data, then, are summarized into cross-categories both critical thinking and types of generalization formulated in the Table 1. The classification on critical thinking activities as clarification, assessment, inference, and strategies described based on the level of generalization using indicators suggested in [7].

TABLE I. FRAMEWORK ON CRITICAL THINKING ANALYSIS ON GENERALIZING MATHEMATICAL IDEA

Critical Thinking	**Generalization**		
	Factual	***Contextual***	***Symbolic***
Clarification	Analysis or discuss the fact	Analysis or discuss the scope of the context	Analysis or discuss the scope of the formalization
Assessment	Decide the presented fact is valid or relevant	Decide the presented context is valid or relevant	Decide the presented formalization is valid or relevant
Inference	Makes generalization from the basic facts	Makes generalization from the given context	Makes generalization from the formalization
Strategies	Flexible in presenting alternative facts	Flexible in presenting alternative context	Flexible in presenting alternative formalization

III. RESULT AND DISCUSSION

A. Result

The problem of generalization presented to the pre-service teachers is relating to the generalization on the cardinality of the union of two sets. The problem is presented as follow.

Diketahui A dan B adalah himpunan dengan $n(A) = 5$ dan $n(B) = 8$.
Seorang siswa bertanya, "apakah mungkin $n(A \cup B) = 8$?"
a. Susun jawaban terhadap pertanyaan tersebut.
b. Buatlah generalisasi kemungkinan $n(A \cup B)$, jika $n(A) = k$ dan $n(B) = m$ dengan k, m bilangan bulat positif dan $k \geq m$.

Translated:

Given A and B are sets respectively $n(A) = 5$ and $n(B) = 8$. A student ask whether it is possible that $n(A \cup B) = 8$

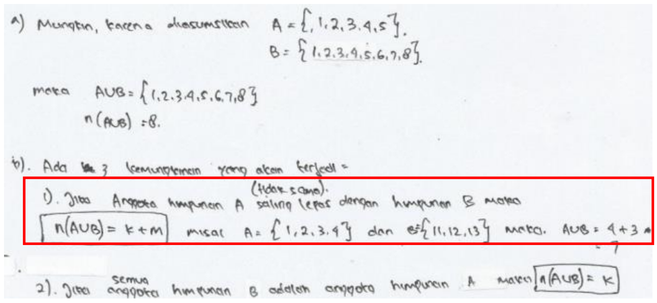
a. Arrange an appropriate answer to the student
b. Generalized the possibility of $n(A \cup B)$, if $(A) = k$ and $n(B) = m$ with k, m are positive integer $k \geq m$

Fig. 1. Problem on generalizaing in the topic of set

From the given information, we can conclude that the situation is already in the contextual information. The information on cardinality of A and B, which is the context, can be described in factual situation by presenting examples of sets which satisfy the cardinality. For example, the subjects may present A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6, 7, 8} from which the subject conclusion on the problem is dependent on the example. Therefore, the formal generalization is, then, asked to the subject as the main question in the problem.

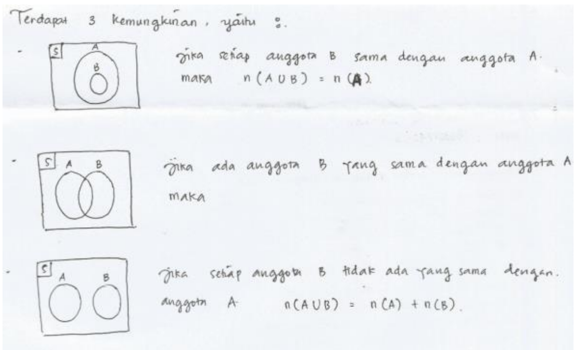
The pre-service teachers were asked to solved the given problem in the class, then, the answers were categorized and analyzed using the framework presented in Table 1. In detail, in this section we presented the results based on the type of generalization which are factual, contextual, generalization.

The first category is the factual generalization. In this generalization, the pre-service teachers provide reasoning toward their generalization using particular examples of case. In this set concept generalization, like the example in the Figure 2, the pre-service teacher used specific sets as the background. Set $A = \{1,2,3,4\}$ and set $B = \{11,12,13\}$ were used for showing that if we had $n(A) = 4$ and $n(B) = 3$, we can have $n(A \cup B) = 4 + 3 = 7$. Nevertheless, in this case, the subject did not provide an idea to what extend this statement can be applied. Furthermore, regarding the critical thinking skill, the pre-service teacher only showed the clarification indicator. This finding indicates that most of them tend to do one stage only viewing the problem. There was no effort to review using assessment, inference, and strategies in order to completely improve the generalization.



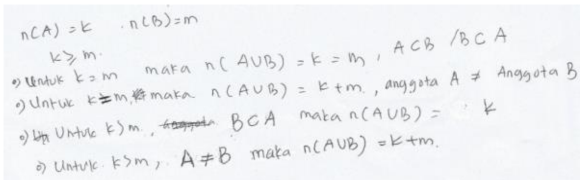
Red box:
The pre-service teacher explained the generalization using example of sets. The use of particular sets indicate that factual generalization was formed

Fig. 2. Factual generalization



(a)

The subject explained the generalization three categorization of the applied context. This context is based on the relation between set A and B.



(b)

The subject explained the generalization using two categorizations of the applied context. This context is based on the distinction of the cardinality of set A and B.

Fig. 3. Contextual generalization

The second category is the contextual generalization. In this generalization, the subjects provide reasoning toward their generalization using the categorization of context. In the Figure 3a, the pre-service teachers use the context of possible two sets relation which are B is a subset of A, B and A is intersect, or B and A have no common members. In eliciting the generalization, the context helped he/she to understand the situation (clarification). However, from this work, a further critical thinking cannot be seen.

In the Figure 3b, it can be seen that the subject tried to simplify the given generalization in the context of the set cardinality. However, he/she was failed to understand correctly the problem, which was he/she already failed in clarification as their initial critical thinking. For instance, in the line 3 when he/she discuss the case of $k = m$, they forgot the fact that if the cardinality of two sets is equal, they do not necessarily have the same members.

The third category is the symbolic generalization. In this generalization, the subjects provide reasoning toward their generalization using the symbolic terms used in the concept of sets. In the Figure 4, the subject used relation theorem for two sets relationship of cardinality (for every set A and B it applies $n(A \cup B) = n(A) + n(B) - n(A \cap B)$). The use of this theorem is, actually, could become an important starting point in generalized the concept. However, he/she failed to employ the meaning of those two sets relationship.

b. $n(A) = k$

$n(B) = m$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$k \leq n(A \cup B) \leq m+k$

The subject explained the generalization theorem in set relating to cardinality of union sets.

Fig. 4. Symbolic generalization

B. Discussion

The result on analyzing critical thinking in pre-service teacher generalization shows that they are able to perform such three different generalizations on the mathematics concept. In detail, on factual generalization, we only see the clarification action as a form on critical thinking. The other three, assessment, inference, and strategy as critical thinking, cannot be seen. This also the matter of fact suggests for the two others generalization which are contextual and symbolic generalization. Most of the subject tend to finish the generalization problem in one stop only. They did not even try to recheck the solution or the strategies used in the generalization. Another finding may also support the inability of the subjects to critically response to the problem that they incorrectly answer regardless they have already presented the correct facts to form the generalization.

IV. Conclusion

In summary, in can be concluded that a number of pre-service teachers are able to generalize the mathematical idea, especially by using factual generalization. A further form of generalization, which is mostly presented by the subjects is contextual generalization, where they divide the case into relevant context. This suggest that generalizing the set concept deductively using symbol only become difficulty for the pre-service teachers. The fact that the subjects mostly do not have experience in proofing more than one way is significant. It seems that, in the previous courses, when they were asked to prove a certain claim, they just did it as they know without even recheck the solution after they done. This situation makes the subject did not accustomed to the critical thinking especially for assessment, inference, and strategy of critical thinking. In addition, however, regardless the generalization, some of them were having difficulties in presenting an understandable content..

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