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Random Event and Probability in Mathematical Modeling of Economic Processes

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***Abstract*— The article substantiates the necessity of including future random variables in the model of economic processes, considers various concepts of types and descriptions of their probability that are different from the frequency probability. An example of the use of Bayesian subjective probability for assessing the risks of insurance is considered, and the improvement of the parameters of auto-regression models is estimated with including f-lag (future expected) variables in them.**

***Keywords*— Random event; Probability; Bayesian networks; Econometric model; Lag variables; f-lag variables**

1. INTRODUCTION

Mathematical modelling of economic processes can‟t do without taking into consideration an existence of contingency and uncertainty. Thus, a key element in mathematical modelling of economic processes is a concept of random event that includes random future events affecting the economic process.

Inferences resulted from mathematical modelling of economic processes and based on the assumption that future will have no other events except happened in past and consequently defined the dynamic of processes and parameters due to mathematical models are not correct according to many scientists‟ researches [1-7]. Accounting for the effects of possible future events on referable characteristics of social-and-economic processes is a present area of a study. The most actual is the problem of accounting and assessment of random event probability in insurance business.

It‟s necessary to expand classical econometric models of adaptive expectations and semi-corrections with f-lag variables that reflect the future events expectations on explaining economic processes [8] as well as to conduct a feasibility study and reassessment of probability concept in economic processes.

We suppose that unlike well-known adaptive expectation models and a partial or semi-correction, probabilistic characteristics of these variables should be analyzed not only from the positions and notions of frequency probability but from the probability positions different from it. Further in this article we provide the examples of including f-lag variables in a classical adaptive expectation model as well as an impact of random event probability on rates insurance.

2. HISTORICAL RESEARCH ANALYSIS

The concept of mathematical probability of random events was set in science in the middle of XVII century owed to researches of such French scientists as B. Pascal and P. Fermat, and a Dutch scientist Ch. Huygens. It referred to calculations of different possibilities in gambling. All events in some extend are occasion, so we‟ll use just an event later on.

An event may be defined as a change of an object or process‟s attributes that lasts for some period of time. For instance, the event „drank a cup of coffee‟ for a clerk going to work this morning is in a measurement of his condition before this event and after. This exact event is a consequence of many foregoing happenings as well as a cause for many forthcoming events, e.g. „hasn‟t drunk another cup of coffee‟ at work or the event such as „blood pressure has risen‟.

In physical science all events consist of point-events, that are events with zero lasting. In social-and-economic processes and their models there are maybe both events with zero lasting (black swans by N. N. Taleb) and expected events that may occur with some level of possibility during some time. The event may have occurred, and it means that the possibility of its occurrence became maximum at the period of time, and during forthgoing period this possibility fluctuated at current time. Such possibility of an event occurrence is usually determined as an opportunity. The peculiarity of an event occurrence in a social-and-economic sphere is its attribute belonging to oscillatory processes as well as a relationship with other random events.

Insertion of other content in every notion of a triad „event – uncertainty – quantity" bares different awareness of probability. For instant, in case of so-called classical or frequency probability the likelihood of an event is determined as a relation of the number of conducive outcomes to the number of possible outcomes [4] on an unlimited number of trials.

In case of supposition that in future nothing will change in repetition of the same real experiment, the same frequency (or probable) characteristics will take place. However, if it"s possible not to account for the impact of future events in some physical experiments (included expected events) then in creating economic models it leads to incorrect conclusions [7].

As an example of such an effect let"s consider a case of purchasing a share on a financial market. Investor"s ask price is changed under influence of events that may be external or internal to him. These events may take place at present or occurred in past or belong to future. The investor is supposed to make a decision based on a mental and later on a formal model. As everything in the world is interconnected all events of past, present and future make a graphical probability model. Fig. 1 presents an example of graphical net model where share"s characteristics are presented as well as events influencing them.

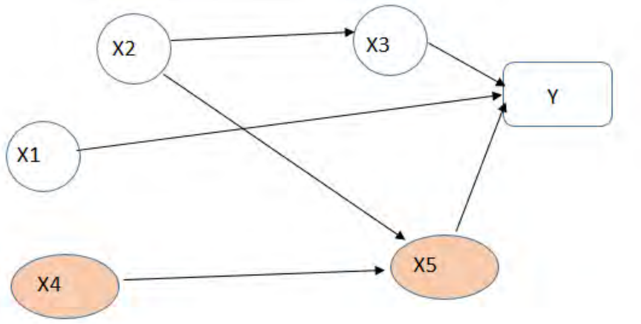

Fig. 1 Events and characteristics manifolds. Where:
X1, X2, X3 are the events taken place;
X3, X4 are some evens may take place in some circumstances;
Y is a trade deal characteristic that changes under circumstances.

In the above scheme, past and future events are absolutely equal in the field of their influence on this variable. This idea was obvious for Einstein who said that "the distinction between the past, present and future is only a stubbornly persistent illusion" [9].

In the above example only probable valuation of results may be derived and they in return depend on probable characteristic of samplings of past events and possibilities of future (expected) events. Classical (or frequency) probability plays not necessarily a dominant role. Statistics of past events (changes in factors X1, X2, X3) are the samplings used for assessing model"s parameters in classical econometrics (e.g. using ordinary least squares estimator). Future events are not always submitted to frequency probability as they may be unique, e.g. president elections in Russia or the USA.

Thus, unique random events go far beyond the scope of frequency probability application. Moreover, there are a great number of so-called theory of frequency probability paradoxes that are correct reasoning"s at the first sight but those later leading to conclusions that contradict either experience or other arguments that are plausible too. On the one hand, a wide rage of probability methods" using exists. On the other hand, there are real difficulties appearing under it.

These difficulties originated vivid discussions, sometimes bore the rejection of using some traditional probability methods. The reason of the above problems" appearance are both in a field of exact mathematical modeling and in obstacles connected with justification of different model applications to solving a task. The former problem is solved with setting an exact (as usual, axiomatic) base of mathematical theory. The most well-known and widely-used one is axiomatics worked out in the beginning of 30s last century by A.N. Kolmogorov [10].

For the first time axiomatic task of setting probability theories as a logically perfect science was challenged in 1917 by a famous mathematician S.N. Berstein. Yet at the same time S.N. Berstein"s idea proceeded from qualitative comparison of random events according to their probability that may be more or less one [11].

A concept of logical probability arose from and developed in works by Keynes, Jonnson and Jeffrey [12, 13]. The most systematic research of such a concept was conducted by R. Carnap [14]. His formulation of logical probability started with a construction of a formal language. In 1950 he considered a type of very simple languages consisting of finite number in logically independent monadic predicates named attributes, and computing quantities of constants. For getting more complex proposals logical links were used. Further R. Carnap compiled a description of all possible universe conditions.

3. Probabilities and Problems with Their Interpretations

A concept of logical probability is one of interpretations connected with probability notion [13]. Formally logical probability is a proposal function in any language. Exact logical probability values for every of its artificial argument *H* depend on another argument *E*, which may be interpreted as a description of a subject"s knowledge. Accordingly, logical probability is named as an epistemic (i.e. knowledge dependent) probability. In some extend it may be explained as a variation of subjective probability. Meanwhile, logical probability values are identified with predetermined knowledge system, thus they have objective character. There is an opinion that if a subject"s knowledge may be provided in total evidence then logical probability is able to serve as a reasonable study of subjective probability.

Dissatisfaction of limited possibility of frequency probability in case of real event modeling led some researchers to implementing such a notion as „inclination" in dealing with objective probability problems. The first provided this idea was Ch. S. Pierce 1910 [17], but the most developed it became due to the works by K. Popper (1959) [18]. A hypothesis of „inclination (or vulnerability)‟ connected with objective probability claims that probability is a vulnerability or a tendency of a nature that tend to provide sub product while a single trial, what is more there is a necessary association with a great amount of testing‟s. What is more, such „inclination‟ is supposed to be objectively existing, even if in a metaphysic area. Width of assumptions suppose that formalization might be intricate in some extend.

There are some other kinds and definitions of probability. For instance, inductive probability occurs while considering judgement as a quantitative estimate in correctness of conclusion subject to validity of drives [19]. If classical or frequency probability are the fixed datum, inductive and subjective probabilities are said on the level of „more – less‟. There is a certain parallel with numerical and ordinal scales that are examined in a measurement theory.

In 1960s L. Zadeh implemented another, different from classical concept for quantitative characteristic of uncertainty, i.e. ambiguity (or fuzziness).

Bayesian subjective probability characterizes the level of a subject‟s confidence in realizing an event or as „a gage of knowledge condition‟. Subjective probability may be formally presented in different ways: with density distribution on a variety of events, with a partial probable allocation or binary relation and some other ways. Most often subjective probability is a probability measure derived from empirical ways.

Many contemporary methods of machine learning are based on objectivistic Bayes‟ principles [21]. Since 1950s, Bayes‟ theorem and Bayesian probability have been widely used by virtue of Cox‟s theorem and James H. Justice Principle of Maximum Entropy [22, 23]. In many applications Bayesian methods are more general and seem to gain better results than frequency probability. Bayesian factors were used for Okama‟s Razor [24].

Taking into consideration a network structure of relationships in random events it‟s possible to judge that Bayesian networks (BNs), also known as the web of trust Bayes suit in the best way to economic processes modeling if future random events are considered as well.

Well-known Bayes‟ equation for stating a conditional probability of an event contains possibilities of fixed at a time random events:

$$P\{A_i \mid B\} = \frac{P\{B \mid A_i)P\{A_i\}}{\sum_{i=1}^{n} P\{B \mid A_i\}P\{A_i\}}$$

4. Probability problem solving in insurance

As an example, let‟s consider the results of applying Bayesian networks in insurance business where the assessment of random event probability influences the company‟s performance.

A form of insurant was taken as a base for BNs in calculating a net rate for individual risk health policy. This form contains general parameters influencing the possibility of a loss occurrence that are: health condition of an insurant, kind of his/her occupation, lifestyle, including such parameters as smoking, alcohol drinking, risk sportsmanship. Moreover, a net rate is affected by such factors as the time of insurance coverage and an insurant‟s age.

All abovementioned criteria were used for creating BNs and are presented as peaks of the network. These peaks have cause-and-effect relations.

BNs (Fig. 2) that determine possible losses of an insurance agreement contains following peaks: Smoking, Dangerous sports, Work, Alcohol, Diseases, Age, Quantity of insurants, Time coverage, etc. Each of variables may have probability from 0 to 1.

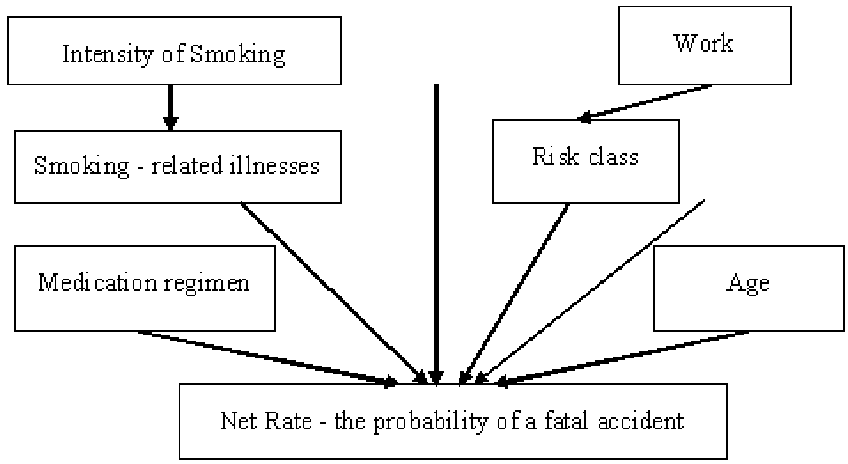

Fig. 2 A fragment of the web of trust Bayes to calculate the net rate

For working in this BN, it‟s necessary to set marginal probabilities for its every marginal peak and conditional probabilities for each related peak that are found out by expertise. Marginal probabilities depend on parameters of an exact signed agreement and contain consolidated return (data) on insurants. These data are got from insurants‟ forms.

Conditional probabilities between related peaks are worked out by expertise. For example, one of the factors determining the insurant‟s health condition and consequently a risk of a fatal accident from any reason, is smoking frequency. To identify the risk of a disease from smoking it‟s necessary to set intensity of smoking in a percentage point (where 0% is „non-smoker‟, 100% is „a person smoking 20 or more cigarettes a day‟) and later identify a disease probability based on conditional probabilities.

Depending on possible smoking intensity of an insurant his smoking-dependent illnesses probability is identified. This probability influences the possibility of a risk event and thus, a loss on an insurance policy on a whole.

A structure of BN that is a system of marginal and conditional probabilities is set on the database of expertise system. BN for an expertise system in diagnosing policy‟s losses may be exercised in Netica [25].

Arrowed lines are used to show which peak of a network is a parent one and which is a secondary with the help on an arrow sign on the toolbar. After that tables which contain data of marginal probabilities for secondary nodes are set.

Depending on parameters of an insurance agreement and data on an insurant‟s form, an insurance policy may have different net rate. For example, if a 24-hour coverage agreement is signed with intensity of smoking is 35%, alcohol drinking intensity is 73%, an insurant‟s going in for sport, a net rate for calculating an insurance premium will equal to 0.0003457.

The model for calculating a net rate based on BN and realized in a program complex Netica was adopted by Russian insurance company “Alliance Life” for preparing a suggestion on real insurance agreements. At the moment there are no programmers that use BN in insurance on Russian market. A model running described in this article approves its adequacy and possibility of its practical usage in conducting risk assessments in insurance companies.

To confirm a hypothesis about influence of future random events, a time range characterizing monthly milk production in the UK (Jan 1962 – Dec 1975) may be used as an example. Changes in volume of milk production are random events (see the Fig. 3).

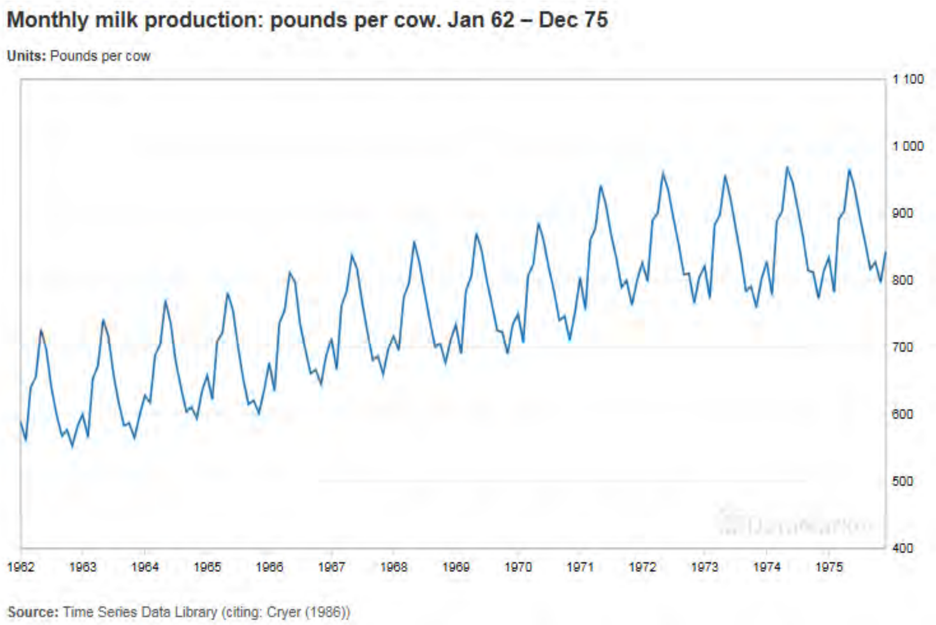

Fig. 3 Dynamic of milk production

To evaluate the quality of three econometric models describing this process it needs using:

1- line model $Y_t = a_0 + a_1 * t + e_t$,
2- lag model $Y_t = a_0 + a_1 * t + a_2 * Y_{t-1}$
3- a model with included f-lag variables
$Y_t = a_0 + a_1 * t + a_2 * Y_{t-1} + a_3 * Y_{t+1}$

In the last presented model besides lag variable Y_{t-1}, f-lag variable Y_{t+1} is included.

Evaluating parameters of these three models with a help of function *Regression* from *Data analysis by Excel* the following results have been received (Tab.1).

From Table 1 data it‟s obvious that the best model is the one where both lag variables and f-lag variables are counted. Moreover, they both are really peering as they have practically the same coefficients and Student fractions.

TABLE 1 Parameters‟ evaluations of three models

Model #	Coefficients	Fisher criterion	Standard error	t-statistics
1	611.6823 1.692615	306	60.7	t1 = 64.9 t2 = 17.5
2	173.0139 0.458869	395	42.4	t1 = 5.1 t2 = 3.9 t3 = 13.2
3	8.82816 -0.02601 0.508091 0.506214	499	32.1	t1 = -0.3 t2 = -0.27 t3 = 11.1 t4 = 11.2

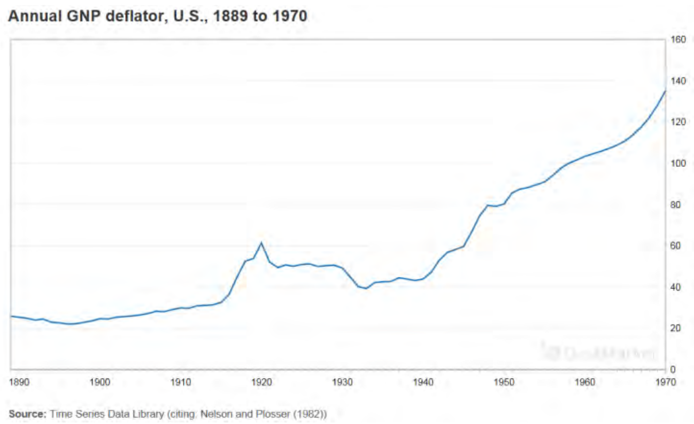

Fig. 4 Annual GNP deflator, U.S., 1889 to 1970

Let us consider another example to compare the models of the US GNP deflator dynamics with lag (model №1) and f-lag variables (model №2). The results of model comparison are given in table. 2.

Model № 2, as it is seen from the table. 2, is more preferable than model No. 1, which speaks in favor of adding to the model description of the f-lag variable.

TABLE 2 Parameters" evaluations of two models

Model #	Coefficients	Fisher criterion	Standard error	t-statistics
1	-0,4 1,03	11034	79	t1 = 105 t2 = 0,7
2	0,51 0,492	16731	77	t_1 = 12,8 t_2 = 13,3

CONCLUSION

On the basis of the aforesaid, the following conclusions may be put forward.

1. Subjective statistics and BNs allow researching models of economic processes where probability of future expected events may be set and their influence on inaccuracy and risk management decisions may be evaluated.

2. Models of economic processes have a net structure thus Bayes" concept of probability is the most suitable for modelling future events probability.

3. Using f-lag variables improve characteristics of economic models with lag variables.

4. Due to complexity of network economic models, very reliable results may be derived only in case of using self-learning neural networks and big data technology.

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