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Modified Production Functions in Modeling Economic Growth of Russia

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Abstract—**The paper studies the features of Russia's economic growth in 1990-2014. Production functions (PF) are used as a tool for estimating the effectiveness of economic growth. Traditional PF owing to the constancy of the parameters do not allow studying the variable production efficiency. To overcome this problem, the author developed a method of transformation of PF with variable parameters into a modified production function with constant parameters and a variable effect on the economic growth of technological progress and others unaccounted factors. The modified PF describes more adequately real economic processes than traditional PF with constant parameters. It some cyclicity in the production efficiency of the Russian economy was revealed on the basis of the worked out toolkit. A sharp decline in the production efficiency of the Russian economy was observed at the beginning of the analyzed period in 1990-1994. The unstable growth of the Russian economy corresponds to the period 1995-1997. The decline in the production efficiency of the Russian economy in 1998-1999 was a result of a negative impact of consequences of the Asian financial crisis of 1997. The highest and most stable rates of economic growth due to the impact of intensive factors were observed for the Russian economy during 2000-2008. The decline in the production efficiency of the Russian economy in 2009-2010 was a result of the impact of the global financial and economic crisis of 2008. The growth in the production efficiency of the Russian economy was observed in 2011-2014.**

Keywords— modeling, production functions, parameters, technological progress, Russia

I. Introduction

The modeling of Russia's economic growth is mainly based on the use of traditional production functions (PF) with constant parameters [7, 9, 10, 16, 18-21]. The static Cobb-Douglas' PF is the most commonly used in modeling [1]. Arrow K. J. , Chenery H. B., Minhas B. S., Solow R. M. developed a PF with a constant elasticity of substitution (CES) in 1961 [2, 3]. However, and this function, like the Cobb-Douglas PF, allows us to estimate only the average elasticity of factor substitution for the entire period, and not for each year.

To overcome the limitation of the production function with constant parameters, it is necessary to transit to PF with variable parameters that have a variable elasticity of factors substitution [6, 8, 12-15, 17, 22].

The most difficult problem in modeling of economic growth is the problem of estimating the impact of technical progress on economic growth. The dynamic Tinbergen's PF [1] is based on the hypothesis of the average annual rate of economic growth due to technical progress. To study Russia's economic growth we will consider traditional PF and the modified PF developed by the author with variable technical progress [6, 8, 12-15, 17, 22].

II. Production Function In Modeling of Technical Progress

Depending on account of the impact of technical progress on economic growth, PF are classified into static and dynamic functions. Cobb-Douglas' production function is an example of a static production function with constant parameters A, α and β [1]:

$$Y_t = A \cdot K_t^{\alpha} \cdot L_t^{\beta} \ (\alpha<1, \beta<1). \tag{1}$$

An example of a dynamic PF is the Tinbergen's PF with constant parameters A, α, β, λ:

$$Y_t = A \cdot K_t^{\alpha} \cdot L_t^{\beta} \cdot e^{\lambda \cdot t} \tag{2}$$

The hypothesis of a constant λ average annual rate of economic growth due to technical progress in the Tinbergen's PF is not always valid not only for different economic objects, but even for different periods of the same object.

In fact, due to the influence of technical progress and unaccounted factors parameters in PF are variable:

$$Y_t = A_t \cdot K_t^{\alpha_t} \cdot L_t^{\beta_t}. \tag{3}$$

PF (3) with variable parameters A_t, α_t and β_t can be transformed to the modified PF with constant parameters A_0, α_0, β_0 and variable rate of economic growth e^{Θ_t} due to technical progress and other unaccounted factors [6, 8, 12-15, 17, 22]:

$$Y_t = A_0 \cdot K_t^{\alpha_0} \cdot L_t^{\beta_0} \cdot e^{\Theta_t}, \tag{4}$$

$$e^{\Theta_t} = \frac{A_t \cdot K_t^{\alpha_t} \cdot L_t^{\beta_t}}{A_0 \cdot K_t^{\alpha_0} \cdot L_t^{\beta_0}}, \tag{5}$$

$$\Theta_t = ln\left(\frac{A_t \cdot K_t^{\alpha_t} \cdot L_t^{\beta_t}}{A_0 \cdot K_t^{\alpha_0} \cdot L_t^{\beta_0}}\right). \tag{6}$$

Correlation (6) gives the possibility of determining the variable Θ_t only after estimating the parameters A_t, α_t and β_t. Specifically if $t=0$ we have:

$$\Theta_0=0. \quad (7)$$

Correlation (7) of the base year gives the possibility to demonstrate Θ_t in the form:

$$\Theta_t=\Delta\Theta_1+\Delta\Theta_2+\ldots+\Delta\Theta_t=\Theta_{t-1}+\Delta\Theta_t,\ \Delta\Theta_t=\Theta_t-\Theta_{t-1}.$$

If the quantity $\Delta\Theta_t$ are constant for the whole analyzed period and are equal to quantity λ, power PF*(4)* is transformed into Tinbergen's PF (2):

$$Y_t=A_0\cdot K_t^{\alpha_0}\cdot L_t^{\beta_0}\cdot e^{\Theta_t}=$$

$$=A_0\cdot K_t^{\alpha_0}\cdot L_t^{\beta_0}\cdot e^{(\Delta\Theta_1+\Delta\Theta_2+\cdots+\Delta\Theta_t)}=A\cdot K_t^{\alpha}\cdot L_t^{\beta}\cdot e^{\lambda\cdot t};$$

$$\Delta\Theta_1=\Delta\Theta_2=\ldots=\Delta\Theta_t=\lambda;\ \Theta_t=\Delta\Theta_1+\Delta\Theta_2+\ldots+\Delta\Theta_t=\lambda\cdot t. \quad (8)$$

If the quantities $\Delta\Theta_t$=0 for the whole analyzed period, power PF*(4)* is transformed into Cobb-Douglas' PF(1):

$$Y_t=A_0\cdot K_t^{\alpha_0}\cdot L_t^{\beta_0}\cdot e^{\Theta_t}=$$

$$=A_0\cdot K_t^{\alpha_0}\cdot L_t^{\beta_0}\cdot e^{(\Delta\Theta_1+\Delta\Theta_2+\cdots+\Delta\Theta_t)}=A_0\cdot K_t^{\alpha_0}\cdot L_t^{\beta_0};$$

$$\Delta\Theta_1=\Delta\Theta_2=\ldots=\Delta\Theta_t=0;\ \Theta_t=\Delta\Theta_1+\Delta\Theta_2+\ldots+\Delta\Theta_t=\lambda\cdot t=0\cdot t=0.$$

We consider the method of evaluation of parameters of PF (4).

Differentiating (4) with respect to t, we obtain:

$$\frac{\Delta Y_t}{Y_{t-1}}=\alpha_0\cdot\frac{\Delta K_t}{K_{t-1}}+\beta_0\cdot\frac{\Delta L_t}{L_{t-1}}+\Delta\Theta_t. \quad (9)$$

Taking the logarithm (4) and summing both sides of (9), we obtain the following expressions, considering correlation (7):

$$lnY_t=lnA_0+\alpha_0\cdot ln\,K_t+\beta_0\cdot ln\,L_t+\Theta_t; \quad (10)$$

$$y(1,t)=\alpha_0\cdot k(1,t)+\beta_0\cdot l(1,t)+\Theta_t, \quad (11)$$

$$y(1,t)=\sum_{i=1}^{t}\frac{\Delta Y_i}{Y_{i-1}},\ k(1,t)=\sum_{i=1}^{t}\frac{\Delta K_i}{K_{i-1}},\ l(1,t)=\sum_{i=1}^{t}\frac{\Delta L_i}{L_{i-1}}. \quad (12)$$

Excluding the value Θ_t from (10) and (11), we have:

$$ln\left(Y_t^{(0)}\right)=\lambda\nu A_0+\alpha_0\cdot ln\left(K_t^{(0)}\right)+\beta_0\cdot ln\left(L_t^{(0)}\right); \quad (13)$$

$$Y_t^{(0)}=Y_t\cdot e^{-y(1,t)};\ K_t^{(0)}=K_t\cdot e^{-k(1,t)};\ L_t^{(0)}=L_t\cdot e^{-l(1,t)}.$$

Thus, the parameters A_0, α_0, β_0 of the PF (4) are estimated by the method of least squares from (13).

In practical calculations, we can use instead of Θ_t the value of Θ_t^* from (11):

$$\Theta_t^*=y(1,t)-(\alpha_0\cdot k(1,t)+\beta_0\cdot l(1,t)). \quad (14)$$

Substituting the value Θ_t^* instead Θ_t into (4), we determine the calculated value

$$Y_t^*=A_0\cdot K_t^{\alpha_0}\cdot L_t^{\beta_0}\cdot e^{\Theta_t^*}. \quad (15)$$

To compare values $\Delta\Theta_t^*$ and λ we can calculate of value $\overline{\Delta\Theta_t^*}$:

$$\overline{\Delta\Theta_t^*}=\Delta\Theta_t^*(\text{average})=\frac{\sum_{i=1}^{n}\Delta\Theta_i^*}{n}. \quad (16)$$

III. Experimental Modeling of Economic Growth of Russia In 1990-2014

For modeling of Russia's economic growth in 1990-2014 we use traditional and modified PF (2) and (4), where Y_t – gross domestic product (GDP) (2000 Prices, bln. roubles), K_t – fixed assets (2000 Prices, bln. roubles) и L_t– average annual number of employed in the Russian economy (mln. persons) (table1) [23].

TABLE I. PF (2) And (4) of the Russian Economy, ν=(α₀+β₀) = 1

TABLE II.

Period	$Y_t=A\cdot K_t^{\alpha}\cdot L_t^{1-\alpha}\cdot e^{\lambda\cdot t}$				$Y_t=A_0\cdot K_t^{\alpha_0}\cdot L_t^{1-\alpha_0}\cdot e^{\Theta_t}$			
	lnA (t_{lnA})	α (t_α)	λ (t_λ)	R^2 (s)	***lnA₀*** (t_{lnA})	α_0 (t_α)	$\overline{\Delta\Theta_t^*}$ (16)	R^2 (s)
1999–2009	2.247 (7.55)	0.506 (8.13)	0.033 (12.01)	0.996 (0.013)	0.575 (1.66)	0.862 (11.73)	0.019	0.939 (0.002)
1999–2014	1.971 (1.970)	0.574 (2.745)	0.018 (2.197)	0.957 (0.044)	2.588 (11.283)	0.434 (8.889)	0.022	0.849 (0.004)
1990–2014	1.487 (6.56)	0.659 (13.86)	0.012 (7.19)	0.961 (0.048)	2.925 (30.52)	0.398 (20.97)	0.008	0.950 (0.004)

Note: R^2 is the uncorrected coefficient of determination; s is standard error; t is t-statistic.

The obtained estimates of λ и $\overline{\Delta\Theta_t^*}$ PF (2) and (4) of the Russian economy for the periods 1999-2009, 1999-2014, 1990-2014 are close respectively (table 2):

λ=0,033; $\overline{\Delta\Theta_t^*}$ = 0,019; λ=0,018; $\overline{\Delta\Theta_t^*}$ = 0,022; λ=0,012; $\overline{\Delta\Theta_t^*}$=0,008.

Therefore, the estimates of the parameters $\overline{\Delta\Theta_t^*}$ = 0.019 and λ = 0.033 of the functions (2) and (4) of the Russian economy indicate an overestimated value of the parameter λ, since the value of λ = 0.033 exceeds the value $\overline{\Delta\Theta_t^*}$ = 0.019.

Thus, the average annual rate of GDP growth in the Russian economy in 1999-2009 due to the aggregate effect of all the factors considered and unaccounted for within the framework of the modified PF (4) were averaged 1.9%, and 3.3% in the traditional dynamic production function of Tinbergen's PF (2) (table 1, Fig. 1 - 3).

We consider the dynamics of the quantities Θ_t^* and λ·t for the periods 1999-2009, 1999-2014 and 1990-2014 (see Fig. 1-3).

The dynamics of Θ_t^* and λ*t* values (see Fig. 1-3) shows that the hypothesis of the average annual rate of economic growth due to technical progress on the whole meet for the periods 1999-2009, 1999-2014. The dynamics of value Θ_t^*

does not have a definite trend of decline or growth for for 1990-2014.

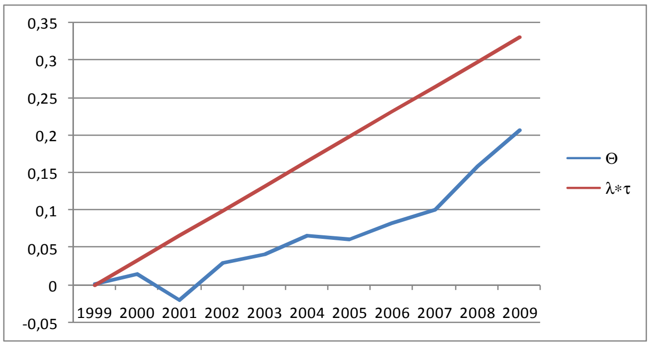

Fig. 1. Dynamics of the values Θt* and λ · t of the Russian economy in 1999-2009

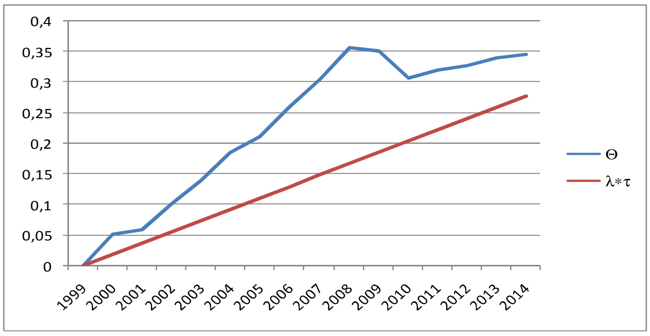

Fig. 2. Dynamics of the values Θt* and λ · t of the Russian economy in 1999-2014

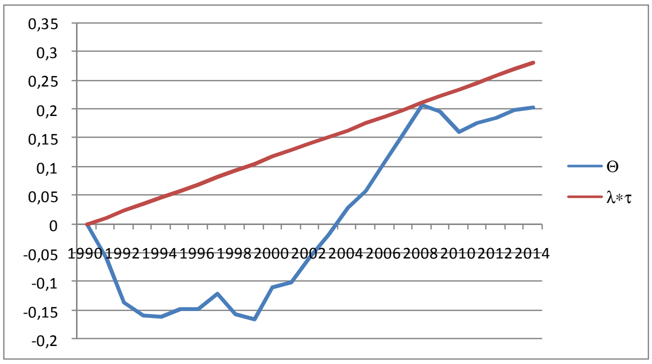

Fig. 3. Dynamics of the values Θ_t^* and $\lambda \cdot t$ of the Russian economy in 1990–2014

Therefore, Tinbergen's PF cannot be used for modeling Russia's economic growth in 1990-1994. At the same time, the values of the Tinbergen's PF parameters in 1990-2014 are economically reasonable. This is due to the fact that the choice of an incorrect hypothesis leads to errors in the values of the PF's parameters.

The efficiency of production of the Russian economy in the early 1990s fell sharply. This contradicts the hypothesis of a constant economic growth due to technical progress within the framework of the Tinbergen's PF. Undoubtedly, the violation of the hypothesis of constant economic growth due to technical progress leads to a bias of estimators of the PF's parameters.

The level of production efficiency of the Russian economy in 1994 amounted to 85.0% of the level of production efficiency in 1990 (see Fig. 3):

$$\frac{e^{\Theta_{1994}^*}}{e^{\Theta_{1990}^*}}=\frac{0,850}{1,000}=0,850.$$

Thus, in 1994, the production efficiency of the Russian economy decreased by an average of 15.0% relative to the level of production efficiency in 1990.

The growth rate of production efficiency in 1995 to the previous year was about 1.5%, and in 1997 to the level of 1996, the production efficiency increased by 2.8%:

$$\frac{e^{\Theta_{1995}^*}}{e^{\Theta_{1994}^*}}=\frac{0,863}{0,850}=1,015;\ \frac{e^{\Theta_{1997}^*}}{e^{\Theta_{1996}^*}}=\frac{0,886}{0,862}=1,028.$$

As a result of the 1997 crisis, the efficiency of the Russian economy in 1998 decreased by an average of 3.6% compared to 1997:

$$\frac{e^{\Theta_{1998}^*}}{e^{\Theta_{1997}^*}}=\frac{0,855}{0,886}=0,964.$$

Steady rates of Russia's economic growth due to technical progress have been observed since 1999 before the crisis of 2008:

$$\frac{e^{\Theta_{2008}^*}}{e^{\Theta_{1990}^*}}=\frac{1,230}{1,000}=1,230;\ \frac{e^{\Theta_{2008}^*}}{e^{\Theta_{1999}^*}}=\frac{1,230}{0,847}=1,452.$$

The crisis in 2008 influenced on the decline in production efficiency by an average of 4.6% compared to 2008. So, the level of production efficiency of the Russian economy in 2010 to the level of production efficiency in 1990 increased by 1.173 times, and to the level of 2008 it decreased by 4.6%:

$$\frac{e^{\Theta_{2010}^*}}{e^{\Theta_{1990}^*}}=\frac{1,173}{1,000}=1,173;\ \frac{e^{\Theta_{2010}^*}}{e^{\Theta_{2008}^*}}=\frac{1,173}{1,230}=0,954.$$

In subsequent years, the production efficiency began to increase, increasing in 2014 by 4.4% compared to 2010:

$$\frac{e^{\Theta_{2014}^*}}{e^{\Theta_{2010}^*}}=\frac{1,225}{1,173}=1,044.$$

The calculation accuracy using modified PF (4) is higher than using traditional Tinbergen's PF (2) (Table 2).

TABLE III. COMPARISON OF THE ACTUAL GDP Y_T OF THE RUSSIAN ECONOMY IN 1999-2014 WITH THE CALCULATED VALUES OF Y_T^* (2000 PRICES, BLN. ROUBLES)

Year	Y_t	**Tinbergen's PF (2)** $Y_t^* = A\cdot K_t^{\alpha}\cdot L_t^{1-\alpha}\cdot e^{\lambda\cdot t}$		**Modified PF (4)** $Y_t^* = A_0 K_t^{\alpha_0} L_t^{1-\alpha_0} e^{\Theta_t}$	
		Y_t^*	Y_t - Y_t^*	Y_t^*	Y_t - Y_t^*
1999	6641.50	6879.89	-238.39	6604.83	36.67
2000	7305.65	7424.72	-119.07	7284.78	20.87
2001	7677.60	7991.94	-314.34	7650.10	27.50
2002	8041.80	8166.93	-125.13	8021.31	20.49
2003	8632.70	8673.92	-41.22	8623.88	8.82
2004	9249.40	9118.23	131.17	9257.24	-7.84
2005	9841.36	9732.88	108.48	9855.86	-14.50
2006	10648.35	10329.15	319.20	10687.26	-38.90
2007	11553.46	11026.71	526.75	11621.29	-67.83
2008	12154.24	11212.71	941.53	12241.27	-87.02
2009	11194.06	10353.99	840.07	11251.98	-57.93
2010	11697.79	11734.28	-36.49	11677.78	20.01
2011	12200.80	12411.09	-210.30	12179.81	20.99
2012	12627.82	13064.47	-436.65	12605.52	22.31
2013	12791.99	13316.80	-524.82	12770.45	21.54
2014	12881.53	13609.05	-727.52	12860.02	21.51

IV. CONCLUSION

The author developed method of transformation of a power PF with variable parameters into modified PF.

To evaluate the modified PF the author developed a new approach, based on the assumption that it is necessary to use the equations in absolute values and rates of growth simultaneously.

The traditional approach in estimating the parameters of the PF assumes a separate estimation by the least squares method of the indicated equations, but different estimates of the same parameter are obtained. If the equations in absolute and relative values describe one economic process, then the corresponding values of the parameters of these functions must coincide. And this is possible only when these equations are simultaneously used for evaluating of parameters.

The use of modified PF allowed finding some cyclicity in the dynamics of production efficiency of the Russian economy. For the Russian economy in 1990-2014 the hypothesis of constant economic growth due to technical progress is not fulfilled.

The accuracy of calculations using modified PF is higher than using traditional PF [6, 8, 12-15, 17, 22].

The developed methodology for studying economic growth with the help of modified PFs not only does not contradict modern methodology, but also complements with new research tools.

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