Mechanism of Reverse Priorities in Distributing Financial Resources

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Abstract— **The article analyses distributing limited resources in probabilistic in-definiteness of the amount of a resource being distributed. An active system consists of a Centre and consumers (agents). The Centre has a resource that is distributed between consumers in accordance with their requests. When the requests are announced, the consumers only have information about the function of resource distribution by the Centre. The article studies priority mechanisms of resource distribution based on reverse priorities principle (function of the priority is a decreasing function of requests) in a determined case when the amount of a resource that the Centre has is known to the consumers and when there is probabilistic indefiniteness. When consumers' objective functions of the resource amount rise monotonously, Nash equilibrium situations are described for various priority functions.**

Keywords— resource distribution, reverse priority mechanism, probabilistic in-definiteness, Nash equilibrium.

1. INTRODUCTION

Objectives of distributing limited resources (regional development programmes funding [1, 2, 3], research and development priorities [4, 5]) are the ones most often set in planning, that is why it is a very timely topic.

Classic pattern of resource distribution is the following [6]. The Centre has a certain amount of resource R that it distributes between consumers (agents). Every consumer makes a request to the Centre for a resource S_i, $i = \overline{1, n}$. The Centre distributes the resource according to the established mechanism of resource distribution.

Theory of active systems suggests priority mechanisms that can be described by the following resource distribution procedure [7]:

$$x_i = \begin{cases} S_i, if \sum_i S_i \le R \\ \min\left(S_i, \gamma\eta_i\left(S_i\right)\right), if \sum_i S_i > R, i = \overline{1, n} \end{cases}$$

whereon x_i is the amount of a resource obtained by the ith consumer;

$\eta_i\left(S_i\right)$ is the function of the ith consumer priority in relation to their request.

Minimum operation determines the condition under which the consumer may obtain the resource in an amount fewer than stated in the request.

Parameter γ is determined by the condition

$$\sum_{i=1}^{n} \min\left[S_i, \gamma\eta_i\left(S_i\right)\right] = R.$$

In the case of these mechanisms, the resource is distributed proportionately to the values of consumers' priorities functions. Three types of priority mechanisms were identified: in absolute priority mechanisms the resource is distributed in direct proportion to a certain identified value of the priority (but not more than requested); in direct priority mechanisms the priority function is an increasing function of consumers' requests; in reverse priority mechanisms the priority function is a decreasing request function.

Assume that effect functions $f_i\left(x_i\right)$ are strictly increasing ones x_i. In this case, the consumers' objective is to obtain as many resources as possible; in other words, within the mechanism of direct priorities the dominant strategy of any consumer is the largest bet on the resource. That is why direct priority mechanisms result in a rise in the number of requests for a resource that constitutes their serious drawback [8].

Mechanisms of reverse priorities have advantages in comparison with these ones. For the first time in the research of reverse priorities mechanisms [9], the priorities function was presented like this $\eta_i\left(S_i\right) = A_i / S_i$, $i = \overline{1, n}$, whereupon A_i characterises the effect of the resource being used by the consumer. Then $\eta_i\left(S_i\right)$ defines the effectiveness of utilizing the resource.

In the past priority mechanisms were used when the Centre's resource was known [10-16]. In practice in resource distribution, its total amount is often not known. The study [17] shows several results of the analysis of equilibrium situations and guaranteeing situations in reverse priority mechanisms in case of the Centre having a random amount of a resource.

The article demonstrates the analysis of reverse priorities mechanisms when consumers know the Centre's resource distributing function.

2. GOAL SETTING

Let us consider an active system which consists of a Centre and consumers. The Centre has a resource that is distributed between consumers in accordance with their requests. When requests are being made, the consumers only know the distribution function $F(R)$ of the resource R that is being distributed by the Centre. Consumers' objectives functions $f_i(x_i)$ are increasing functions of the resource obtained x_i. Let is mark an ith consumer's request for a resource as S_i. While distributing a resource the Centre uses reverse priority mechanisms. Below we examine the objective of defining a Nash equilibrium for reverse priority mechanisms.

3. THEORY

Let us first consider the determined case, when the amount of the Centre's resource is known to the consumers.

Let the priority function be presented as follows:

$$\eta_i(S_i) = \frac{A_i}{S_i}, \ i = \overline{1, n},$$

whereupon A_i is the parameter limiting the consumers' priorities.

Resource distribution mechanism is as follows:

$$x_i(S) = \min\left(S_i, \frac{A_i \cdot R}{S_i \cdot Y}\right), \tag{1}$$

whereupon $Y = \sum_j \frac{A_j}{S_j}$.

We adopt that the consumers' objective functions are increasing in x_i.

In equilibrium $S_i = \frac{A_i \cdot R}{S_i \cdot Y}$ or $S_i = \sqrt{\frac{A_i \cdot R}{Y}}$ is evident.

Under conditions of $\frac{1}{S_i} = \sqrt{\frac{Y}{A_i \cdot R}}$, $Y = \sum_j \sqrt{\frac{Y \cdot A_j}{R}}$

we get the equilibrium situation $Y = \frac{1}{R}\left(\sum_j \sqrt{A_j}\right)^2$,

$$S_i = \frac{\sqrt{A_i} \cdot R}{\sum_j \sqrt{A_j}}.$$

Herewith $x_i = S_i$ is $i = \overline{1, n}$ in all cases.

Let us proceed to the analysis of probabilistic case. Let us mark $F(R)$ function of distribution amount of the resource that is a continuously differentiable function R.

The ith consumer's mathematical expectation of a resource amount equals

$$M(S) = \int_0^{R_i} \frac{A_i R}{S_i Y} dF(R) + S_i\left[1 - F(R_i)\right], \tag{2}$$

whereupon $R_i = \frac{S_i^2 \cdot Y}{A_i}, i = \overline{1, n}$.

Let us define the maximum of this variable by S_i when proposing the condition of a weak effect of the evaluation S_i on the variable Y. In other words, when considering the amount of the request, agents do not take into account the effect of S_i on Y regarding it as just another parameter.

It should be noted that

$$\int_0^{R_i} R dF(R) = R_i F(R_i) - \int_0^{R_i} F(R) dR.$$

Let us make another calculation:

$$\frac{dM}{dS_i} = -\frac{A_i}{S_i^2 Y}\left[R_i F(R_i) - \int_0^{R_i} F(R) dR\right] + \frac{A_i}{S_i Y} R_i F'(R_i)\frac{dR_i}{dS_i} + 1 - F(R_i) - S_i F'(R_i) \cdot \frac{dR_i}{dS_i}$$

,

After simple manipulations we get

$$\frac{dM}{dS_i} = 1 - 2F(R_i) + \frac{1}{R_i}\int_0^{R_i} F(R) dR.$$

It is worth noting that this formula does not depend on A_i.

Let us adopt the convention that $M(S)$ is convex downwards and therefore has a maximum. The equilibrium value R_i is one and the same for all the consumers, that is to say that $R_i = R^*$ for all i.

Under the condition

$$2F(R^*) - \frac{1}{R_i}\int_0^{R^*} F(R) dR = 1, \tag{3}$$

we define the maximum R^* and therefore

$$S_i^* = \sqrt{\frac{A_i \cdot R^*}{Y}}.$$

Further

$$Y^* = \sum_j \frac{A_j}{S_j^*} = \sqrt{\frac{Y}{R^*}}\sum_j \sqrt{A_j}, \ Y^* = \sqrt{\frac{\sum_j \sqrt{A_j}}{R^*}}.$$

Finally, we get

$$S_i^* = \frac{\sqrt{A_i}}{\sum_j \sqrt{A_i}} \cdot R^*. \tag{4}$$

Let us consider another priority function

$$\eta_i(S_i) = A_i - S_i, \ i = \overline{1, n}.$$

Let us consider a determined case.
We have

$$x_i(S) = \min\left(S_i, \frac{A_i - S_i}{Y} R\right),$$

whereupon $Y = \sum_{j}\left(A_j - S_j\right)$.

In equilibrium situation

$$S_i = \frac{A_i - S_i}{Y}R \text{ or } S_i = \frac{RA_i}{Y+R}, \ A_i - S_i = \frac{A_i Y}{Y+R}.$$

Under the condition $Y = \sum_{j}\left(a_j - s_j\right) = \frac{Y}{R+Y}\sum_{j}A_j$

we obtain $Y^* = A - R$, $S_i^* = \frac{A_i}{\sum_{j}A_j}R$.

For an equilibrium situation, the condition of A>R must be met.

Let us proceed to the analysis of probabilistic case. By analogy with the previous case, we have

$$M\left(S_i\right) = \int_0^{R_i}\frac{\left(A_i - S_i\right)}{Y}RdF\left(R\right) + S_i\left(1 - F\left(R_i\right)\right),$$

whereupon $R_i = \frac{S_i Y}{A_i - S_i}$, $\frac{dR}{dS_i} = \frac{YA_i}{\left(A_i - S_i\right)^2}$.

Calculating

$$\frac{dM}{dS_i} = -\frac{1}{Y}\left[R_i F\left(R_i\right) - \int_0^{R_i}F\left(R\right)dR\right] + 1 - F\left(R_i\right) = 1 - \left(1 + \frac{R_i}{Y}\right)F\left(R_i\right) + \frac{1}{Y}\int_0^{R_i}F\left(R\right)dR$$,

$$\frac{d^2M}{dR} = -\frac{1}{Y}F\left(R_i\right) - \left(1 + \frac{R_i}{Y}\right)F^{'}\left(R_i\right) + \frac{1}{Y}F\left(R_i\right) = -\left(1 + \frac{R_i}{Y}\right)F^{'}\left(R_i\right) < 0$$
.

Therefore, the function $M\left(S_i\right)$ is convex downwards and the maximum is defined by an equation that is identical for all consumers.

$$\left(R+Y\right)F\left(R\right) - \int_0^{R}F\left(x\right)dx = Y.$$

4. CONCLUSION

The article analyses game-theory model of non-cooperative interactions between agents (consumers). We assumed that consumers make decisions simultaneously and independently of one another. Every consumer chooses an action S_i that belongs to an admissible set $\sum_{i}S_i \leq R$, $i \in N = \left\{1,2,...,n\right\}$ of consumers. The gain of the ith consumer depends on his own action, on other consumers' actions vector and on the priority function. The game solutions (Nash equilibrium conditions) are defined in two priority functions under the condition of probabilistic indefiniteness of the resource being distributed.

Reverse priority mechanism in probabilistic indefiniteness considering the amount of the distributed resource is applied for funding planning in administrative support for the penitentiary system under FSIN of Russia territorial body given the rise in effectiveness of subdivisions' activities (the final evaluation [18]).

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