

Enhancing Conformity Assessment Process by Outsourcing

V. Burkov

Institute of Management Problems V.A. Trapeznikova RAS,
Moscow, Russia

L. Rossikhina

Voronezh Institute of the Federal Penitentiary Service of
Russia
Voronezh, Russia

L. Rogovaya

Voronezhskintezkauchuk,
Voronezh, Russia

Abstract— This paper analyzes scheduling of expert teams work with possible partial outsourcing of conformity assessment. Each task is completed by a team of experts (generally, diversely qualified ones). The number of experts in each team is limited. All the tasks should be completed within a specific time frame. The objective is to determine the set of tasks that should be outsourced to complete all the tasks with the lowest outsourcing costs. In order to achieve this objective, the paper offers heuristic algorithms that implement four priority rules. The paper presents a numerical example including different priority rules.

Keywords— team allocation, schedule, outsourcing, cost minimization.

1. INTRODUCTION

The need to optimize innovation activity is largely The goal of conformity assessment is to eliminate potential quality non-conformity and related defects, therefore to avoid customers' and potential clients' complaints and excessive remedy costs and to reduce all possible process changes introduced at the production stage, as well as costs of corresponding tasks.

Conformity assessment is performed by a cross-functional expert team.

Now the prior objective of enhancing conformity assessment process is to create the algorithms for scheduling with minimum completion time and limited resources.

The basic goal of completion time minimization with limited resources is the project timetable creation (scheduling).

This paper considers a project as a set of interconnected tasks needed to achieve specific goals. Conformity assessment process is an example of the project where tasks have precedence relationship.

General formulation of scheduling with respect to precedence relationship and resource limits while minimizing completion time can be as follows.

Let us assume that the set of tasks is specified as $N = \{1, \dots, n\}$, and the set of renewable resources is specified as $U, u = 1, \dots, U$. In every instant of time t , Q_u units of the resource u are available. The duration of tasks is specified as $\tau_i \geq 0$ for

each task, where $i = 1, \dots, n$. For the task i , $q_{iu} \leq Q_u$ units of the resource $u = 1, \dots, U$ are needed. After the completion of the task, the released resources can be fully allocated to other tasks. Partial order of completion is specified for the set of all tasks: if the task i precedes task j , then j cannot be started before completion of i . The task starts at $t = 0$ moment of time. Interruption of tasks is not allowed. It is necessary to define such a start time for the tasks $S_i, i = 1, \dots, n$ that helps minimize the project completion time, i.e. the value

$$C_{\max} = \max_i \{C_i\}$$

where $C_i = S_i + \tau_i$.

Many papers [1-7] analyze scheduling with resource constraints.

The paper [8] considers scheduling with limited renewable resources (machines, performers etc.) and order of completion constraints, the paper [9] presents a solution for scheduling with limited non-renewable resources and focuses on completion time lag minimization.

Papers [10, 11] analyze scheduling with limited accumulative resources.

Papers [12, 13, 14] consider scheduling in independent projects management.

Papers [15, 16, 17] offer the setting of scheduling problems that, if solved simultaneously, provide for additional (synergetic) effect. These works present precise and heuristic algorithms for solving these problems.

SETTING OF THE PROBLEM

There are n tasks. Each task is completed by an expert team within the time $\tau_i, i = 1, n$. The number of experts of each kind is limited to $N_j, j = 1, m$. The objective is to schedule the teams' work in order to complete of all the tasks within the required time T . If the tasks are not completed within this timeframe, the rest is outsourced. At the same time, it is desirable that the cost of the outsourced tasks should be minimal.

2. THEORETICAL ANALYSIS

Definition 1. All the teams that can work at the same time make up a feasible set of teams.

Let Q_i refer to the teams in set i , $i = 1, m$, where m is the total number of sets; P_k is all the sets that include team k ; b_{kj} is the number of experts of type j in team k . Any feasible set of Q_i satisfies the system of inequalities

$$\sum_{k \in Q_i} b_{kj} \leq N_j, \quad j = \overline{1, m}. \tag{1}$$

It should be noted that the schedule is a sequence of sets. Let $x_i > 0$ refer to the length of set i work within the schedule.

Definition 2. The schedule is balanced if for every set Q_i included in the schedule (i.e. $x_i > 0$) the length of each team's work is x_i .

Theorem 1. For each schedule, an equivalent balanced schedule can be created.

Proof. Let us assume there is a set Q_i that includes a team $k \in Q_i$ with the length of work $\Delta_i < x_i$. In this case let us divide the set into two feasible sets, one of which contains team k with length of work Δ_i , and the other does not contain team k , and its length of work is $(x_i - \Delta_i)$. The number of teams with the length of work within the set less than the one of the whole set is reduced by 1. Proceeding with this method, we reach a balanced schedule.

For a balanced schedule, the length of any work done by k is

$$t_k(x) = \sum_{i \in P_k} x_i. \tag{2}$$

If $t_k(x) < \tau_k$, then the part of the tasks assigned to k equal to $\tau_k - t_k(x)$ is outsourced for the cost of a_k per unit of time.

The cost of the outsourced tasks will be

$$S(x) = \sum a_k (\tau_k - t_k(x)). \tag{3}$$

Goal. To create schedule x complying with minimum (3) constraints

$$\sum_{i \in P_k} x_i \leq \tau_k, \quad k = \overline{1, n}, \tag{4}$$

$$\sum_i x_i \leq T. \tag{5}$$

It is the objective of linear programming. However, a huge number of feasible sets and their geometrical growth with the increasing number of teams make this objective NP-hard.

That is why, to reach it, we will analyze heuristic algorithms, based on priority rules.

Before doing that, we will transform the problem (3), (4), (5) into an equivalent maximization problem

$$\sum_k a_k \sum_{i \in P_k} x_i = \sum_i \sum_{k \in Q_i} a_k x_i = \sum_i c_i x_i \tag{6}$$

with constraints (4), (5),

$$\text{whereupon } c_i = \sum_{k \in Q_i} a_k.$$

Let us define four set priority rules.

Rule 1. The sets with the highest c_i have priority.

Rule 2. The sets with the highest

$$T_i = \sum_{k \in Q_i} \tau_k. \tag{7}$$

have priority.

Rule 3. The sets with the highest

$$S_i = \sum_{k \in Q_i} a_k \cdot \tau_k. \tag{8}$$

have priority.

Rule 4. The sets with the longest tasks have priority

$$\Theta_i = \max_{k \in Q_i} \tau_k. \tag{9}$$

3. ALGORITHM DESCRIPTION

Step 1. Take the set with the highest priority. Define its length based on the lowest length of each team's work. Delete the teams with the lowest length of work from the set.

Step 2. Adjust the priorities of other sets and the length of the tasks left. Repeat step 1.

After a finite number of steps (not exceeding the number of teams), we end up with the time left being less than the lowest length of the priority set of work left.

Assume that the length of the teams' work is equal to the time left. Calculate the length and the cost of the outsourced tasks accordingly.

Let us analyze the following problem example and solve it using different priority rules.

Example. There are 6 tasks. Table 1 comprises the data on the composition of teams working on the tasks, the cost of the outsourced units and the length of the tasks. There are also three types of experts, and $N_1=2, N_2=2, N_3=2$.

Table 1 – Given data

Team Expert	1	2	3	4	5	6
1	1		1	1		1
2	1	1		1	1	
3		1	1		1	1
τ_k	5	7	9	8	6	3
a_k	7	5	3	4	6	8

Let us assume $T=8$.

It should be noted that the highest priorities for each rule have maximum feasible sets.

Let us define maximum sets and related priorities. (Table 2)

Table 2 – Maximum sets and priorities

I	1	2	3	4	5	6	7	8
Q_i	1, 2, 6	1, 3, 5	1, 4	1, 5, 6	2, 3, 4	3, 4, 5	4, 5, 6	2, 5
c_i	20	16	11	21	12	13	18	11
T_i	15	20	13	14	24	23	17	13
S_i	94	98	67	95	94	95	92	71
Θ_i	7	9	8	6	9	9	8	7

Let us solve the problem using rule 1.

Step 1. Choose set (1, 5, 6), $c_4 = 21$. Task 6 is completed in three days.

Step 2. Adjust the priorities of the sets (Table 3) and the length of the tasks left (Table 4). Keep only maximum sets. Exclude set 4 since it is included in set 2 and set 7 since it is included in set 6.

Table 3 – Sets priorities

I	1	2	3	4	5	6
Q_i	1, 2	1, 3, 5	1, 4	2, 3, 4	3, 4, 5	2, 5
c_i	12	16	11	12	13	11

Table 4 – Length of tasks

K	1	2	3	4	5
τ_k	2	7	9	8	3

Choose set (1, 3, 5), $c_2 = 16$. Task 1 is completed in three days.

Step 3. Adjust the priorities of the sets (Table 5) and the length of the tasks left (Table 6).

Table 5 – Sets priorities

I	1	2	3
Q_i	2, 3, 4	3, 4, 5	2, 5
c_i	12	13	11

Table 6 – Length of tasks

K	2	3	4	5
τ_k	7	7	8	1

Choose set (3, 4, 5), $c_2 = 13$. Task 5 is completed in three days.

One set (2, 3, 4) with priority 12 is left.

Table 7 – Table of the length of the tasks left

K	2	3	4
y_k	7	6	7

Length of tasks $y_2 = 7$, $y_3 = 6$, $y_4 = 7$. Length of the set is

$$x = T - 6 = 2.$$

Five days of task 2, four days of task 3, and five days of task 4 are outsourced.

Cost of the outsourced tasks is $S = 5 \cdot 5 + 3 \cdot 4 + 4 \cdot 5 = 57$.

If we solve this problem using rule 2 (7), we end up with four days of task 1, two days of task 3, one day of task 4, five days of task 5, and two days of task 6 being outsourced.

Cost of the outsourced tasks is $S = 4 \cdot 7 + 2 \cdot 3 + 4 + 5 \cdot 6 + 8 \cdot 2 = 84$.

If we solve this problem using rule 3 (8), we end up with four days of task 2, one day of task 3, five days of task 4, one day of task 5, and three days of task 6 being outsourced.

Cost of the outsourced tasks is $S = 4 \cdot 5 + 1 \cdot 3 + 5 \cdot 4 + 6 + 3 \cdot 8 = 73$.

If we solve this problem using rule 4 (9), we end up with five days of task 2, two days of task 3, five days of task 4, and two days of task 6 being outsourced.

Cost of the outsourced tasks is $S = 5 \cdot 5 + 2 \cdot 3 + 5 \cdot 4 + 2 \cdot 8 = 67$.

The lowest cost is 57, and it is achieved by using priority rule 1.

CONCLUSION

The article offers a solution to the problem of scheduling of expert teams work with a limited number of experts of each type while completing all the tasks within the requested timeframe. Partial outsourcing is possible for the tasks with the lowest cost. To solve this problem, the article offers heuristic algorithms implementing four priority rules.

The setting of the problem and the algorithms of its solution are worked out in order to enhance conformity assessment process for rubber, implemented by research center laboratory specialists of AO Voronezhskintezkauchuk, an affiliated company of Sibur Holding.

In further studies, it would be interesting to analyze a discrete scenario when tasks are fully outsourced.

References

- [1] Barkalov S.A., Burkov V.N. Minimization of lost profits in project management tasks: preprint. - M.: In-t of problems of management of RAS, 2001. 56 p.
- [2] Barkalov S.A. The theory and practice of scheduling in construction. - Voronezh: VGASU, 1999. 216 p.
- [3] Barkalov S.A., Senyushkin A.V., Yanin A.G. Construction of the calendar plan with recommendatory dependencies between works. Izvestiya KGASU. - 2011. - № 3 (17). pp. 252-256.
- [4] Brucker P., Drexel A., Mohring R. et al. Resource-constrained Project Scheduling: Notation, Classification, Models, and Methods // European Journal of Operational Research. 1999. Vol. 112. pp. 3-41.
- [5] Kolish R., Padman R. An Integrated Survey of the Deterministic Project Scheduling // OMEGA - The International Journal of Management Science. 2001. Vol. 29. pp. 249-272.
- [6] Burkov V.N., Burkova I.V., Uandykov B.K. Tasks of operational project management // Bulletin of SUSU. - 2015. - Volume 15. - № 4. pp. 129 - 137.
- [7] Lazarev, A. A., Kvaratskhelia, A. G. Processes of the schedule for the minimization of the problem of a single machine // Automation and Remote Control. 2010. 71, No. 10. pp. 2085-2092.
- [8] Servakh V.V. Effectively solvable case scheduling problem with renewable resources / Discrete analysis and operations research. - 2000. - Ser. 2. - V. 7. - № 1. pp. 75-82.
- [9] Gafarov, E. R., Lazarev, A. A., and Werner, F. Single machine problems, Mathematical Social Sciences. 2011. 62, No. one.
- [10] Servakh V.V., Shcherbinina T.A. On the complexity of one task of calendar planning with stored resources // Bulletin of Novosibirsk State University. Series Mathematics, mechanics, computer science. - 2008. - T. 8. - № 3. pp.105-112.

- [11] Icmeli O., Erenguc S. A Branch and Bound Procedure for the Resource Constrained Project Scheduling Problem with Discounted Cash Flows // *Management Science*. 1996. Vol. 42. No. 10. pp. 1395-1408.
- [12] Burkov V.N., Bondarik V.N., Nguyen H.T., Seleznev A.A. The task of developing calendar plans for the criterion of lost profits // *Management systems and information technology*. - 2013. - T. 53. - № 3. pp. 32-35.
- [13] Rossikhina L.V. The task of developing schedules of the criminal executive system according to the criterion of lost profits // *West Soviet Voronezh. in-the Russian Interior Ministry*. - 2014. - № 1. pp. 286-291.
- [14] Burkov V.N., Korobets B.N., Puzyrev S.A. Tasks of scheduling in program management // *Economics and Management Management Systems*. - 2017. - 2.2 (24). pp. 204-216.
- [15] Burkov V.N., Zimin V.V., Seleznev A.A. Building a calendar plan for a regional development program taking into account the interdependence of projects. *Control Systems and Information Technologies*. - 2014. - T. 57. - № 3. pp. 45-48.
- [16] Zenischeva V.G., Rossikhina L.V., Seleznev A.A. The task of creating a calendar plan for interdependent projects // *Economics and Management Management Systems*. - 2014. - 2.3 (12). pp. 374-382.
- [17] Rossikhina L.V. Statement of the task of forming a calendar plan with interdependent events and an algorithm for its solution // *Voronezh Bulletin. in-the Russian Interior Ministry*. - 2014. - № 3. pp. 81-89.