

# Analytical Notes on Growth of Economic Indicators of the Enterprise

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**Abstract** — A brief analysis of the distribution regularities of economic indicators is presented. The role of the exponential distribution law of random variable and its direct connection with the measurement of information about the state of the economic system is determined. The process of additive and multiplicative growth of economic reliability indicators is briefly substantiated. The probability and intensity of the system state are singled out among these indicators. These indicators were considered in relation to the exponential distribution law of random variables associated with multiplicative growth. The growth of economic reliability indicators is expressed through the natural logarithm, which allows to determine the amount of information belonging to the system element. The additive growth is due to the event occurrence with some probability, which transferring an element from one state to the opposite, and the multiplicative growth is due to the simplest streaming process of the states of system elements. Considering the total growth, the mathematical expressions for determining the information entropy are obtained. These expressions are based on logarithmic calculations. Quantitative values of entropy are a measure of assessing the state and level of economic reliability of the system.

**Keywords** — *economic system, economic indicators, information entropy; additive and multiplicative growth, integrated reporting*

## I. INTRODUCTION

Manufacturing enterprises of the world economy are in conditions of dynamically developing and accelerating business processes. These processes require improvement of economic science, which includes the tasks of identifying their properties and possible states. The problem of assessing the possible future state of the enterprise and its value is relevant in terms of the needs of suppliers of financial capital in a timely and reliable integrated information about the enterprise. The public reporting of companies today is aimed at revealing its business model, the process of creating the company's value and future opportunities and risks. At the same time, a wide range of diverse information on financial, industrial, intellectual, human, social-reputational and natural capital should be taken into account and is analyzed. The solution of

practical problems of work with such information is seen, including in use of methodology of the system and information content analysis.

“Substantially information Paradigm” of social development is one of the promising directions in this area [1]. The methodology of system-information analysis is based on this paradigm [2]. This analysis requires the development of tools to identify patterns of growth of economic indicators.

This methodology doesn't exclude the existence of unity of such phenomena as production and sales, cost, price, money and inflation that have the general information content. In turn, laws of the organization and development of production systems are caused by laws of the information theory [3]. Such multidimensional concepts as order, chaos, organization, inefficiency, control and self-organization are related to the research of the behavior of complex systems. These concepts (considered as some probabilistic states) don't exclude from consideration of informational correlations and interactions between the considered subjects (elements of the system).

Since the dynamics of indicators is due to structural internal and external correlations, the probabilistic nature of the impact of various factors, the question of analyzing the dynamics of indicators is often associated with information entropy. In fact, according to the works of K. Shannon [4], N. Wiener [5], L. Brillouin [6] etc., entropy is a measure of information uncertainty. Information entropy is a measure of uncertainty of an open production and economic system. The state of this system is characterized by a number of signs: instability, disorderliness, instability, disorganization at all levels of the structural and functional organization.

Understanding the role and importance of the tasks of the system-information analysis, let's further limit ourselves to consideration of the growth of economic indicators of the enterprise, without excluding the role of information entropy.

## II. ANALYSIS OF THE PROCESSES OF DYNAMICS OF ECONOMIC INDICATORS

The unpredictability of indicators inevitably arise when economic systems (competing with each other) complicate the process of accurate modeling of market exchange. There is information uncertainty, which should be removed in the process of analyzing the states of the production and economic system. In addition to the above-mentioned, it should be noted that the analysis of market indicators growth doesn't exclude probabilistic modeling.

Considering the issues of dynamics of economic indicators from the standpoint of the system analysis, additive growth (involves addition operation) and multiplicative growth (based on multiplication operation) are selected. The process of growth is connected to time and if to consider the system from the standpoint of such properties as emergence, the presence of elements and connections, then it is important to consider the structurality. It is directly related to the orderliness of the system, characterized by a certain set and arrangement of elements with connections between them. Its topology is not constant in time as the content changes.

Let's consider growth or decrease as a process in time. It is important to distinguish the form of dynamics, that is, how time is calculated. The linear growth reflects the occurrence of a single event on the set at each moment of time. This event increases the value of the set. The linear growth of the total value is not related to what process is modeling: additive or multiplicative.

If the value of the set increases proportionally with each time step, then the growth rate (or change) of the total value of the set  $M$  is directly proportional to its current value:

$$\Delta M = C \cdot M, \quad (1)$$

where  $C$  is a constant. Considering the growth dynamics, the total value of the set in time is written as:

$$M(t) = M(0)e^{Ct}, \quad (2)$$

where  $M(0)$  is the value of the set at the zero instant of time. If  $C > 0$ , then one have the growth dynamics of the set,  $C < 0$  - decrease of the set. For example, as value can be considered the volume of production and sales.

Expression (2) reflects a simple exponential dynamics, which is common in solving the problems of the economy [7]. In this case, calculation of time is connected to course of physical processes and, therefore, the intensity of events in the system is proportional to the value (size) of the system. In cases where the intensity of the events in the system is a constant and doesn't depend on the value /size of the system, the calculation of time is directly related to the occurrence of the events. Thus, the total value of the set changes linearly in event time, and exponentially in physical time [8].

Further, let's turn to the exponential distribution law of a random variable. Suppose that a histogram is constructed in coordinates  $(p; x)$ , and  $p(x) \cdot dx$  is a fraction or probability of occurrence of the event  $x$ , lying in the interval between  $x$  and  $x+dx$ . Such histogram having a power-law distribution of a random variable can be converted into a histogram of the same data, but presented in a double logarithmic scale. If this histogram is a straight line of the form:  $\ln p(x) = -\alpha \ln x + C$ , where  $\alpha$  and  $C$  are constants, this means the presence of the power-law. This discovery is associated with a name G. K. Zipf [9]. If one takes the exponent of each part of the Zipf equation, then obtain:

$$p(x) = Cx^{-\alpha}, \quad (3)$$

where  $C = e^C$ .

Distribution of the type (3) corresponds to the power-law distribution, which in most cases approaches the exponential distribution (not excluding the probability distributions) [10-12]. Here the constant  $\alpha$  is an exponent of the power-law distribution and has a fixed value. The constant  $C$  doesn't play an essential role because it is determined from the requirement: the amount of distribution  $p(x)$  must be equal to 1.0 [13].

The exponential distribution (3) can be found in various problems of the economy considered as a closed system. Therefore, expression (3) agrees with the exponent of the probability distribution of an element  $i$  of the system:

$$p_i(t) = e^{-\lambda_i t}, \quad (4)$$

where  $\lambda_i$  is the intensity of the occurrence of the events in the interval  $t$  related to an element  $i$ .

The value  $p_i(t)$  is determined based on the availability of statistical data on the occurrence of events with an intensity  $\lambda_i$ . In particular, if to consider the structure of production as a consistent connection of elements (for example, "logistics – material logistics – production – sale"), then in the output, one can obtain the probability of events occurrence of the entire system by the expression:

$$P(t) = \prod_i p_i(t). \quad (5)$$

As the occurrence of events is a streaming process in time, their intensity will be proportional to a size of the system:

$$\Lambda = \sum_i \lambda_i, \quad (6)$$

where  $\Lambda$  – the intensity of events occurrence in the system.

According to (4), the probability obtained according to (5) can be presented in the following form:

$$P(t) = e^{-\Lambda t}. \quad (7)$$

Another statistical parameter can be considered the average time of the element  $i$  in the considered state (after the occurrence of the event):

$$T_i = 1 / \lambda_i, \quad (8)$$

and the average time of finding of the system in the considered state:

$$T = 1 / \Lambda \text{ or } T = \frac{\prod_{i=1}^n T_i}{\sum_{i=1}^n \left( \prod_{\substack{j=1 \\ j \neq i}}^n T_j \right)}, \quad (9)$$

where  $n$  – the number of elements considered.

Application of the model (4)-(9) allows to determine the parameters of the system state having connected them with the output parameters, and also to pass on measuring information entropy. However, this model can only take into account the system states and to measure the information of the changing structure. As for the analysis of the growth of enterprise indicators, let's pass to the consideration of event and physical time.

Considering the exponential processes in the economy, the number of events in time is proportional to the production parameters. For example, the probability of growth of a production indicator on certain time intervals is a constant. It means that the set of small increments of the indicator generates a stream of events. The more increments (particles) are added, the more intensive the flow will be.

According to work [8], let's consider the features of additive and multiplicative growth of indicators. If to analyze the indicators of a single element, then it is difficult to distinguish between additive and multiplicative growth as both of them look exponential. Let's turn to the generalized process of growth for which the probability of an increase for element  $i$  can be determined.

Let's consider two simplest examples.

1. Additive growth. Let within time, the increase of indicators is the same for all elements of the system. It is possible that all elements have the same properties. Over time, the difference in the volume of indicators between them will be remain little discernible. Therefore, the absolute difference between the quantitative values of the indicators will be remain approximately constant, and the relative difference will

tend to zero. If to consider the value of the element indicator as a proportion in the total amount of the values of the system indicators, then probability of the increase for each element  $i$  will be equal to  $j$ :  $p_i = 1/k$ , where  $k$  is the number of elements of the system. Such simple process of additive growth indicates that the role of the element in the structure of the system has been levelled out, since the participation rates of the elements in the total amount of the indicators are close. It becomes obvious: there is a simplest process of quantitative growth. Statistics of such events indicate the independence of functioning of the element in the considered system, and the occurrence of events doesn't depend on the prevailing external and internal circumstances.

2. *Multiplicative growth.* Let's consider the nature of events, when several indicators are added to the already existing ones, the number of which is determined each time by multiplying the available indicators by a certain number. Such nature of the stream of events involves the influence of external and internal factors on the element state. In this case, the values of each element will grow unevenly, then at each considered time interval the relative difference between the values will remain unchanged, and the absolute difference will increase. However, the correlations between the elements aren't taken into account here, and they are considered as independent. It is obvious that according to such growth, the probability of increase for element  $i$  will be:  $p_i = n_i / (n_1 + n_2 + n_3 + \dots + n_k)$ , where  $n_i$  is the quantitative value of the element. The expression shows that the probability is increased with the increase of  $n_i$ , that is, the proportion of  $p_j$  in the total value of probabilities will increase. In this case, there is multiplicative growth. For this growth, the absolute difference between failures of elements is not preserved. On the other hand, the relative difference remains (the proportion of failures of each of the elements), since growth is inherent in all elements of the system. In fact, multiplicative growth reflects the complexity and diversity of the processes occurring in economic systems.

Let's sum up the above-mentioned arguments. Considering the additive growth, summation is associated with the presence of the principle of growth and as a consequence the independence of correlation between the elements of the system. On the contrary, multiplication operation of multiplicative growth supports the structure of these events with the appearance of external and internal factors. However, it is worth remembering that above-mentioned information about additive and multiplicative growth refers to ideal cases. Nevertheless, probabilistic multiplicative growth in the limit leads to exponential frequency distribution of probability irrespective of the initial form of distribution of the set, which confirms the validity of the use of the mathematical expressions (4) and (7).

### III. THE USE OF MATHEMATICAL EXPRESSIONS TO DETERMINE THE AMOUNT OF INFORMATION

Application of a measure of information measurement of additive and multiplicative growth is possible in terms of the information theory. In this theory the principles of calculating

the amount of information using the Shannon's model [14] are justified. Here the questions of measure of information uncertainty, which is quantitatively expressed through the entropy of the system states, are touched.

The exponential distribution includes an extensive class of distributions, which include: 1) the normal distribution, the binomial distribution, the Poisson distribution, and 2) the exponential distribution and its discrete analogue – the geometric distribution. For these distributions, the probability density determines the probability that the value  $x$  will be in the interval  $[a; b]$  and is directly related to the determination of entropy.

The entropy of the exponential distribution of a random variable can be calculated by applying the Shannon's model:

$$H = -\sum_{i=1}^n p_i \ln p_i \text{ (nats)}, \text{ by } \sum_{i=1}^n p_i = 1. \quad (10)$$

where  $i$  accepts a set of discrete values corresponding to  $n$  different signal states.

The expression (10) is valid for determining the amount of information in the system, provided that the elements function independently, that is, without taking into account the connections and the occurrence of joint events. Thus, it is about consideration of a closed system, without taking into account, for example, correlation (which can be taken into account with both the use of the conditional probability and using the multidimensional entropy model of Gaussian systems [15]).

The natural logarithm is used in the expression (10) because its content part is close to the consideration of the exponential distribution law of a random variable.

In the case of multiplicative growth consideration, entropy  $H$  is calculated in nats and, in fact, by the same Shannon's formula (10). This model doesn't exclude from consideration of use of the logarithm, for example, with the base of 2, as confirmed in [16-20]. Let's show the connection between entropies whose values can be obtained for different bases of the logarithm. A convenient number to perform the analysis is taken as the base of the logarithm (2 or the Napier's number  $e$ ). The logarithm base has no effect on the result of obtaining the value of entropy.

For a set of event probabilities, the entropy can be calculated by formulas:

$$H = -\int_0^1 p \log_2 p dp = -\left(\frac{p^2 \ln p}{2} - \frac{p^2}{4}\right) / \ln 2; \quad (11)$$

$$H = -\int_0^1 p \ln p dp = -\left(\frac{p^2 \ln p}{2} - \frac{p^2}{4}\right). \quad (12)$$

The difference between expressions (11) and (12) allows to establish equality between the entropies:

$$1,443 \cdot p \ln p \approx p \log_2 p. \quad (13)$$

As for Napier's number  $e$ , it is considered convenient for the analysis, since it has no qualitative impact on the entropy.

According to [21], let's select a part of the expression (10) in order to substantiate the role of entropy in the analysis task of growth of the indicators:

$$h_i = -\ln p_i(t), \quad (14)$$

where  $h_i$  is the private entropy inherent in element  $i$ .

The private entropy (its quantitative characteristic)  $h_i$  in exponential law reflects the number of events per unit time  $t$ . When the formula (4) is substituted in the expression (14) for  $t=1$ , one obtain the intensity of the element events –  $\lambda_i$ . If function  $h_i(\lambda)$  is determined on some set  $\Lambda$  having a limit value equal to 0, then one is equal  $\lim_{\lambda \rightarrow 0} h_i(\lambda) = 0$ . With increasing  $p_i(t)$  the private entropy tends to zero. The higher the economic reliability of the system [22, 23], the higher the probability and average time to ensure the effectiveness of its sustainable development, the less private entropy indicating the completeness of the information which the company in the process of using all types of resources and satisfying of all stakeholders in conditions reasonable economic risk.

For the system as a whole, the private entropy will be determined by expression:

$$h = -\sum_{i=1}^n \ln p_i(t) = -\ln \prod_{i=1}^n p_i(t) = \Lambda. \quad (15)$$

In fact, the private entropy (in relation to exponential distribution) is determined by a set of intensity of events associated with ensuring the economic reliability of the system.

Let's turn to the Shannon's formula (10), in which there is the private entropy  $h_i = -\ln p_i(t)$  and  $p_i$  is the probability of occurrence event leading to change in state of an element  $i$ . The probability of occurrence of an event is determined as:

$$p_i = \frac{\lambda_i}{\Lambda}, \text{ therefore, } \sum_{i=1}^n p_i = 1. \quad (16)$$

In turn, the expression (10), which serves to determine the entropy of the system, will take the form:

$$H = \sum_{i=1}^n H_i = -\sum_{i=1}^n p_i \ln p_i(t), \text{ by } \sum_{i=1}^n p_i = 1. \quad (17)$$

The expression (17) is valid only if the elements of the system are independent. Considering the stream of events inherent to the elements as the simplest (satisfying the conditions of stationarity, absence of consequences and ordinarity), the entropy of the system is determined by the expression:

$$H_{\Sigma} = -P \ln P(t), \quad (18)$$

where  $P(t) = \prod_{i=1}^n p_i(t)$  is probability of occurrence of events in the system,  $p = \prod_{i=1}^n p_i$  is probability of occurrence of  $n$  joint (crossing) events in the system.

In case of uniform distribution of probabilities  $p_i = \frac{1}{n}$  of discrete random variables, according to (18), the maximum amount of entropy, that is, the maximum uncertainty of events, is determined.

Applying the mathematical expression for determining of entropy is the following:

1) The additive growth of states increases the entropy to a maximum value corresponding to the entropy of equally probable states (10) and has connection with change of structure of the system;

2) the multiplicative growth doesn't change the entropy of the set of states (in relation to growth of organized systems) that means the invariance of their structure.

Expression (18) implies consideration of both additive and multiplicative growth.

#### CONCLUSION

The process of development of an economic system can be considered from the standpoint of the system analysis of its structure. The indicators of this structure are changing and correlating with additive and multiplicative growth. A number of variables, for example, such as the intensity of occurrence of events, obey the exponential law of distribution of a random variable, which allows to use the information theory to determine the amount of information entropy for the purpose of the system state assessment. The exponential law served as the basis for the creation of a mathematical model for describing the correlations between economic indicators and entropy, allowing to measure the structural content of the economic system. The logarithmic basis for measuring entropy is provided in the model, taking into account feature: separation of entropy into two components. The first component includes the private entropy, the second – the distribution of private entropies between events related to elements. This approach to the determination of the

information allows to identify trends in growth of the system using the information entropy, to estimate the level of its structural and functional organization and to build the simplest trends.

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