

Mathematical Modeling of Droplet Injection Process of Indirect Piezoelectric 3D Printhead for Casting Sand Mold

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Abstract—Aiming at the problem of small size of ejected ink droplet and easy damage to contact between piezoelectric sheet and liquid in the process of casting sand mould 3D printing, an Indirect piezoelectric actuated diaphragm-driven micro-droplet jetting technology is presented to overcome the defects of the existing micro jetting technology. The primary principle of the piezoelectric actuated diaphragm-driven micro jetting technology is introduced and physical model is made to establish the mathematical model. As a result, through the force - displacement equation of piezoelectric actuator was simplified, the force - displacement equation of piezoelectric actuator was obtained. When the fluid movement is considered the law of mass and energy conservation is employed, which greatly simplifies the analyzing procedure, the mathematical model of piezoelectric actuated micro-droplet jetting process on casting sand mould 3D printing is built. The research will provide theoretical basis for revealing the spraying behavior of droplet injection device and realizing the optimal design of system structure for 3D printing of casting sand mold.

Keywords—3D printing; printhead; casting sand mold; mathematical molding

I. INTRODUCTION

As one of the most promising intelligent manufacturing technologies, three-dimensional (3D) printing technology reflects the current situation of industrial manufacturing in a country[1], which is widely used in casting sand mold printing[2], fine casting[3] and other fields[4, 5]. The key technology is the micro drop injection technology, and printhead is one of the key components.

It has been the basic process of foundry production of sand casting process for a long time due to it is simple, cheap and easy to obtain. However, the foundry casting process of large complex metal castings will take a long cycle, which seriously affects the timeliness of products on the market. In order to change this situation, the incremental manufacturing technology has been introduced in casting industry to realize the rapid forming of sand mold without the mold, that is to say, the industrial 3D printing technology has effectively improved the forming efficiency of large castings [1]. The key technology of industrial 3D printing is the micro injection technology, and the key components is the inkjet printhead, the jet stability is the key factor to ensure the reliable operation of the whole machine, and the mathematical model

of the jet process can be used to study the reliable work of the print head. This problem has been paid close attention to by many scholars.

At present, domestic and foreign scholars carried out many related researches on the printhead performance and its influence on the precision of sand forming. Herman wijishoff studied the dynamic characteristics of piezoelectric sprayer from the aspects of drive, droplet formation and satellite droplet[8], a good agreement between voltage waveform and drop volume based on the piezoelectric vibration method was obtained[9]. Research shown waveform was the main factor determining the droplet spray characteristics, its parameters were closely related to the behavior of micro droplet injection[10]. On this basis, more and more people were beginning to pay attention to the production quality and dimensional accuracy of the casting sand mold, new research focus was on the interaction between micro injection performance and waveform as well as the print quality fluctuations caused by the micro drop spray characteristics[11, 12].

All Industrial 3D printing technology is a process of making a three-dimensional sand solid object of virtually any shape from a digital model. It is achieved using an additive process, where successive layers of material are laid down in different sand shapes. The foundry mold 3D printing is considered distinct from traditional sand mold techniques, which mostly rely on the removal of material by methods such as cutting or drilling [2]. The micro droplet ejection process of the printhead is affected by voltage fluctuation, jet viscosity, inlet velocity, positive and negative pressure of the cavity and combined effects of various factors. Based on the fluid volume method, a numerical analysis and simulation of a piezoelectric inkjet printhead are carried out in Chen's research, the influence of the driving parameters on the droplet ejection behavior is investigated [3], Wei Dazhong et al. Made a droplet ejection device with a piezoelectric chip as the driving system. The mathematical relationship between the deformation of the piezoelectric wafer and the on demand ejection was analyzed, and the indirect on-demand ejection with the piezoelectric material was distinguished [4], Cheng adopted the electromagnetic valve to control the opening and closing of compressed gas, so that the pressure direct effect on the material level by injection pulse, the relationship between the droplets and the injection pressure is analyzed, but this

pressure injection device working chamber, droplets are difficult to control accurately and the droplet size consistency, However, because of the large pressure and large working chamber of the injection device, the droplet formation is difficult to control accurately and the droplet size is poorly consistent [5]. One such analyzed the numerical simulation of uniform droplet ejection process, and the mathematical model of uniform droplet injection was validated experimentally [6]. These micro droplet jetting process modeling research provide some reference to solve the micro jet printing head stability and state monitoring, but the injection process stability of the casting mould 3D printhead is not only closely related to micro jet control parameters but it is also restricted by the complicated working condition. So the stability problem of sand mold 3D printhead is very complex.

Motivated by above works, this paper present a new indirect-piezoelectric 3D printhead conducted to research on the mechanism of the influence between the droplet ejection behavior and the structural parameters and working conditions of foundry sand mold 3D printhead, on this basis, the feasible optimal control method is studied. In view of that, this study is based on the conservation of mass and momentum conservation, and takes the foundry sand 3D printhead as the object of study, the relationship between the structure parameters and the droplet ejection behavior is studied.

II. OPERATING CONDITION OF CASTING SAND MOLD 3D PRINTEAD

Figure I shows the diagram of large casting sand mould 3D printer acquired by main research and development, the paper takes this as the object of study. The main parameters are as follows, the width of the printhead is 2200mm*1500mm, the printing liquid is furan resin, the viscosity is 14cp, the excitation voltage is 110V, and the print head runs at the speed of 700mm/s.



FIGURE I. THE SCHEMATIC DIAGRAM OF 3D PRINTER

III. MICRO DROPLET JETTING PRINCIPLE OF INDIRECT-PIEZOELECTRIC 3D PRINTEAD

All The principle of new indirect-piezoelectric 3D printhead is shown in Figure II, it is controlled by voltage signal applied on PZT all the work process, and the ink will be spited under the influence of deformation driven by piezoelectric ceramics, due to the cyclic extrusion of piezoelectric chamber and direct contact between the ink and piezoelectric ceramics, the fluid at the nozzle mouth is always subject to fluid pressure and surface tension, when the fluid

pressure is sufficient to overcome the surface tension, periodic droplets are formed. The ejection of a droplet by a piezoelectric actuator is exactly corresponding to a periodic change of the piezoelectric control signal, so that the micro droplet can be accurately controlled by the control signal of the piezoelectric ceramic.

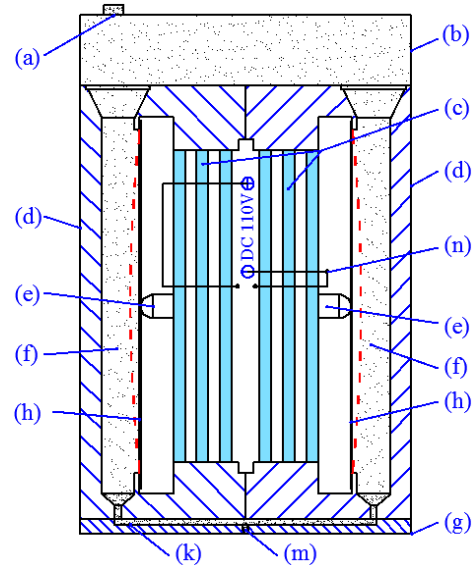


FIGURE II. MICRO DROPLET JETTING PRINCIPLE OF INDIRECT-PIEZOELECTRIC 3D PRINTEAD

IV. MICRO MODELING OF DROPLET EJECTION PROCESS

Under the influence of electric field, the piezoelectric ceramic diaphragm will produce expansion or contraction of the shape variables, with a result in changes in the volume of the sealed liquid storage chamber and pressure fluctuations. When the piezoelectric diaphragm causes negative pressure in the chamber, the ink fluid is conveyed from the ink inlet to the cavity, conversely, when the positive pressure is applied, the fluid is ejected from the cavity nozzle, that means a complete cycle of ink absorption and ink jet is completed. This period includes the electromechanical conversion process of the piezoelectric diaphragm, and also the rheological process of the liquid separated into droplets under the excitation of the cavity.

A complete ink jet process in a cycle is simplified to model shown in Figure III in order to facilitate theoretical analysis, in order to avoid repeated labeling, the parameters of the same model are only marked once. If both sides of inkjet ink inlet working chamber pressure is the same, you can work on both sides of the chamber into the ink outlet, through an indirect type simplified piezoelectric driven micro droplet jetting apparatus includes an ink inlet, a working chamber, a driving thin film and inkjet nozzle four part. Among them, the total quality of rectangular piezoelectric ceramic plate group named m_0 , the rectangular driving membrane area is A_0 , the rectangular piezoelectric ceramic plate rigidity coefficient is k_0 , The inlet of ink jet chamber is $v_1(t)$, the inlet section area is A_1 , the nozzle velocity is $v_2(t)$, and the nozzle section area is A_2 .

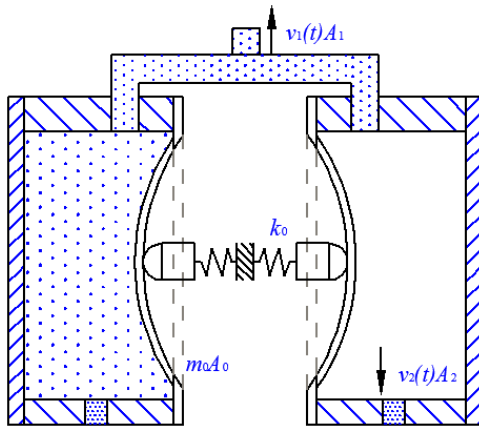


FIGURE III. SIMPLIFIED THEORETICAL MODEL OF JET PROCESS

V. FORCE ANALYSIS OF MICRO DROPLET EJECTION PROCESS

According to the mechanical properties of the piezoelectric material, piezoelectric group can be simplified as spring, and assuming the structure as a center of symmetry, the indirect type piezoelectric drive device in the film can be simplified as elastic system shown in Figure IV.

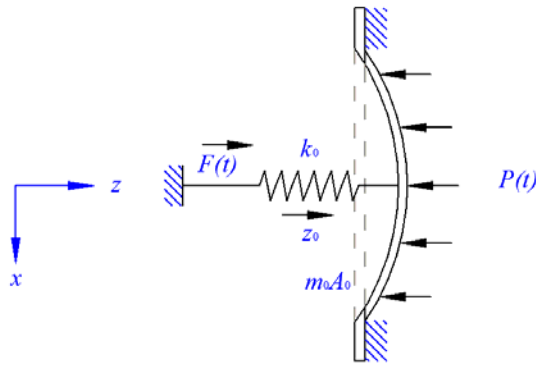


FIGURE IV. SCHEMATIC DIAGRAM OF FORCE ACTING ON INJECTION DEVICE

In this coordinate system, The Z_0 shape variable will occur when the piezoelectric plate is subjected to an external varying electric field, the driving film with an area A_0 is subjected to an extrusion force of $F(t)$ and this can be accelerated by the piezoelectric plate:

$$F(t) - k_0 z_0 - P(t) A_0 = m_0 \frac{d^2 z_0}{dt^2} \quad (1)$$

According to the piezoelectric equation of piezoelectric material [1]:

$$S_1 = s_{11}^E T_1 + d_{31} E_3 \quad (2)$$

$$D_3 = \epsilon_{33}^T E_1 + F_1 \quad (3)$$

In the formula, D_3 and E_3 are respectively to the electric stress and electric field, S_1 and T_1 were the strain and stress of X direction, ϵ_{33}^T is a constant under the constant external force, d_{31} is a piezoelectric constant, s_{11}^E is the coefficient of elastic compliance when the electric field is equal to zero or constant.

According to the principle of indirect piezoelectric micro droplet injection, the piezoelectric chip group is driven by DC power supply. Assuming that the piezoelectric field is applied with a uniform electric field E_3 and the piezoelectric plate excitation voltage U_0 , the electric field E_3 in the formula (2) can be expressed as a linear relationship of voltage U_0 , that mean:

$$E_3 = \frac{\sigma}{l} U_0 \quad (4)$$

In the formula, σ is the coefficient of conductivity medium.

We can get the following results when translating the equation (4) into equation (2)

$$S_1 = s_{11}^E T_1 + \frac{\sigma}{l} d_{31} U_0 \quad (5)$$

According to the schematic diagram of the ejector in Figure IV coordinate system, we can get the following relationship:

$$\begin{cases} S_1 = Z_0 \\ T_1 = F(t) \end{cases} \quad (6)$$

We make the following relationship true, that $\alpha = \frac{\sigma}{l \cdot s_{11}^E} d_{31}$, $\beta = \frac{1}{s_{11}^E}$, according to formula (5) can be drawn:

$$F(t) = \beta z_0 - \alpha U_0 \quad (7)$$

In the formula, α and β are all certain It can be seen that the maximum deformation amount of the driving film increases with the voltage applied on the piezoelectric plate and the external force of the driving film. With the formula (7) substituting into formula (1), the force equation of the driving device film droplet jet device can be obtained:

$$m_0 \frac{d^2 z_0}{dt^2} + (\beta - k_0) z_0 + \alpha U_0 + P(t) A_0 = 0 \quad (8)$$

The relation between the force acting on the piezoelectric driving plate and the displacement of the driving film is shown in the equation (8).

VI. ANALYSIS OF LIQUID MOTION IN DROPLET EJECTION PROCESS

Under the excitation of the voltage signal, the piezoelectric actuator generates deformation displacement Z , the pressure P_0 is generated by pushing the push rod to the inside of the cavity, and the working cavity generates a pressure of $P(t)$ in size. In this paper, the assumption of micro droplet ejection in double chamber is done without affecting the accuracy of the model.

The actual movement of working chamber of liquid in the complex, without affecting the accuracy of the model, the micro droplet jetting process make the following assumptions on the double chamber: Simulation of the liquid is incompressible Newton fluid; two spray cavity structure and piezoelectric group displacement are axisymmetric: the fluid in the simulation is incompressible Newton fluid; the two inkjet cavities and the displacement of the piezoelectric plate are axisymmetric.

In the injection process, the fluid between the two micro jet chamber and the nozzle channel need to satisfy the Navier-Stokes continuity equation:

$$m_0 \frac{d^2 z_0}{dt^2} + (\beta - k_0) z_0 + \alpha U_0 + P(t) A_0 = 0 \quad (9)$$

In the formula, g is the acceleration of gravity.

We can know according to the working principle, the liquid is ejected from the nozzle of the ink jet working cavity when the driving film is excited by the piezoelectric plate set, and the flow rate of the nozzle and the ink outlet can be considered the same without taking into account the influence

of gravity. $\int_1^2 \frac{dv}{dt} dz$ is the work done by the liquid to overcome the resistance when passing through the nozzle. With L_0 as the distance from the liquid working chamber to the final composition of export, R_0 is the ink radius, then $\int_1^2 \frac{dv}{dt} dz$ can be expressed as equation (10) is shown in the form:

$$\int_1^2 \frac{dv}{dt} dz = (2L_0 + 4R_0) \frac{dv^2}{dt} \quad (10)$$

Similarly, there is a cavity in the ink jet cavity:

$$\int_1^2 \frac{dv}{dt} dz = (2L_1 + 4R_1) \frac{dv_1}{dt} \quad (11)$$

In the formula, L_1 and R_1 are respectively the ink length and inlet radius.

Thus far, the indirect droplet ejection process can be described according to equation (8), equation (10) and equation (11).

VII. CONCLUSION

In this paper, starting from the existing problems of micro droplet jet printhead in 3D printing process of foundry sand mold, an indirect piezoelectric jet device is proposed to solve the problem that the ink head of the existing printing head is too small and the piezoelectric sheet is in direct contact with the jet liquid.

The principle of duplex cavity micro jet synthesis is described, and the physical model of indirect piezoelectric driving micro jet process is established.

A simplified equation of force displacement between the piezoelectric plate and the actuating cavity is derived. Based on the conservation of mass and the conservation of energy, a mathematical model of fluid motion in the micro injection process of a duplex cavity injection device is established.

The research will provide theoretical basis for revealing the spraying behavior of droplet injection device and realizing the optimal design of system structure for 3D printing of foundry sand mold.

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