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Numerical Calculation of the Penetration of Parametric Beams into Sediment

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***Abstract*—KZK equation is used as the base of calculation. When the grazing angle of the source is smaller than the critical angle, the Snell's law can not be used to determine the progressive direction of the evanescent wave in the sediment that is necessary for solving the KZK equation by using the finite difference method. In this paper, a phase-lag method is used to solve this problem. Numerical results are compared with the experimental results published by Muir et al. [J.Sound.Vib. 64, 539-551(1979)] and are shown in good agreement.**

***Keywords*—parametric array; bottom penetration; KZK equation**

I. Introduction

The behavior of directional acoustic beams penetrating into sediment from water has been investigated, both theoretically and experimentally, for a long time 2~5, 10 since Muir et al. presented their experimental results. Some of the theories 2, 10 did not consider the nonlinear effects when tried to explain the abnormal phenomena found by Muir et al. in their experiment. We here use the KZK equation, which has been widely used as the base of calculating the nonlinear sound field numerically in a single-phase medium, to calculate the sound field in the two-phase media. So the least simplification need to be made for the physical model.

To solve the KZK equation numerically in the Cartesian coordinates, the cross-section of the sound beam at any acoustic axial range must be perpendicular to the acoustic axis 6. It is difficult to select the progressive direction, when part of the sound beam has penetrated into the sediment, if the grazing angle of the source is smaller than the critical angle (since the sound speed is different in sediment with that in water, and Snell's laws can not be used to adjust the progressive direction of the evanescent wave for the post critical incidence). To solve this problem, a position-dependant phase lag is assumed for each field point and this will be discussed in detail in section I.

The method is proved to be effective by the good agreement between the numerical results and the experimental data presented by Muir et al..

II. Theory

The normalized parabolic equation (KZK equation) of the sound field is [6]:

$$(4\frac{\partial^2}{\partial\tau\partial\sigma}-\frac{\partial^2}{u_x^2}-\frac{\partial^2}{u_y^2}+4\alpha r_0\frac{\partial^3}{\partial\tau^3})\overline{p}=2\frac{r_0}{l_D}\frac{\partial^2}{\partial\tau^2}\overline{p}^2 \quad (1)$$

Where

$$\sigma = z/r_0,\qquad \tau=\omega(t-z/c_0),$$

$$\overline{p}=p/\rho_0 c_0 u_0,\qquad l_D=\frac{1}{\beta\omega u_0}$$

$$u_x^2 = x/a,\qquad u_y^2 = y/a.$$

here $r_0 = \pi a^2/\lambda$ is the Rayleigh distance of the projector, a is the radius or the half length of the broad side of the source, ω is the angular frequency, β is the nonlinear coefficient, ρ_0 is the ambient density, c_0 is the ambient sound speed and u_0 is the velocity at the source surface.

The water and the sediment are assumed as two kinds of liquid with different densities, ambient sound speeds and nonlinear coefficients. The KZK equation is valid both in water and in sediment, but the respective coefficients are different.

The equation (1) was originally used to solve the harmonic sound field. When it is used to solve the parametric sound field, some changes in the initial condition and normalizing parameter for the convenience of calculation should be made [7].

The KZK equation will be solved in the frequency domain. So the pressure can be expanded as:

$$\overline{p}=\sum_{n=1}^{\infty}\overline{p}_n=\sum_{n=1}^{\infty}(g_n\sin n\tau+h_n\cos n\tau) \quad (2)$$

Substitute (2) to (1), the following partial equations of g_n and h_n can be obtained:

$$\frac{\partial g_n}{\partial\sigma}=-n^2\alpha r_0 g_n+\frac{1}{4n}(\frac{\partial^2}{u_x^2}+\frac{\partial^2}{u_y^2})h_n+n\frac{r_0}{2l_D}(\frac{1}{2}\sum_{p=1}^{n-1}(g_p g_{n-p}-h_p h_{n-p})-\sum_{p=n+1}^{\infty}(g_{p-n}g_p+h_{p-n}h_p))$$
$$\frac{\partial h_n}{\partial\sigma}=-n^2\alpha r_0 h_n-\frac{1}{4n}(\frac{\partial^2}{u_x^2}+\frac{\partial^2}{u_y^2})g_n+n\frac{r_0}{2l_D}(\frac{1}{2}\sum_{p=1}^{n-1}(h_p g_{n-p}+g_p h_{n-p})+\sum_{p=n+1}^{\infty}(h_{p-n}g_p-g_{p-n}h_p)) \quad (3)$$
$$n=1,2,...$$

The above equations can be solved step by step from the boundary $\sigma= 0$ along theσ direction using finite difference method (see Fig. 1), but the length of the progressive step must be small enough[8]. Equation (3) is used for solving the sound field in water. When the sound field in sediment is considered, the n^2 of the first item in the right-hand side of equation (3) should be replaced by n if the absorption in sediment is assumed to be linear with frequency.

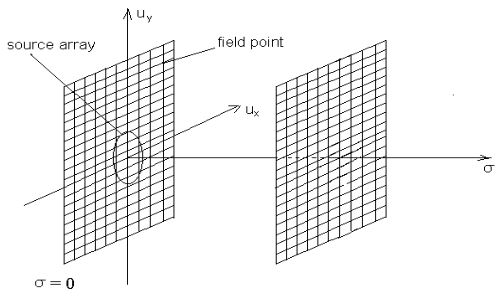

FIGURE I. COORDINATE FOR CALCULATION

When a part of the sound beam reaches the water-sediment interface, for the pressure within this part of sound beam, the continuity of each harmonic is assumed:

$$p_{ni}+p_{nr}=p_{nt} \qquad n = 1, 2 ... \quad (4)$$

where p_{ni}, p_{nr}, p_{nt} is the nth harmonic of the incident, reflective and refractive wave respectively. So the p_{nt} at the water-sediment interface can be obtained [9] from the calculated p_{ni} in the last step if the length of progressive step is small enough. The following procedures of calculating sound field in sediment will be based on the KZK equation in sediment.

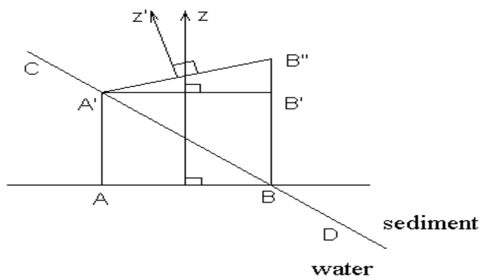

FIGURE II. DIRECTION CHANGE FOR BEAM ACROSS THE WATER-SEDIMENT INTERFACE

Equation (3) was obtained through eliminating variable τ.This means that at each step of calculation, pressure at any point in the cross-section must be of the same moment. Then some difficulties occur, when part of the sound beam has been in the sediment. For example, in Fig. 2, A, B are two points in the cross-section of one calculation step and B is just on the interface CD of two different media. The next step A will travel to A', but B will travel to B" instead of B', because the sound speed in sediment is a bit larger than that in water. In order to validate the finite difference method, either the point B" is chosen and the progressive direction is adjusted from z to z', or the B' is chosen and a phase lag is introduced to keep the moment of the pressure at B' equals to that at A'. If the grazing angle is smaller than critical angle, adjusting the progressive direction of the evanescent wave by Snell's law is not correct. So, B' is the only choice. The different points in the cross-section will be subject to a different phase lag when they are in the sediment because they reach the interface in different progressive step. The earlier the point reaches the interface, the more the phase changed. This method is valid only if the source last long enough that when A has progressed to A', there is still sound pressure at B'.

Since the sound field in sediment is concerned, the reflective wave is neglected in calculating the sound beam in water while part of the sound beam is in sediment. This is, of course, not accurate for the sound field in this part of water volume, but if the nonlinear interaction between the incident and reflective wave is not very strong, the sound field in sediment will not be influenced seriously.

III. NUMERICAL RESULTS AND DISCUSSION

As an example, calculations corresponding to the experiments presented by Muir et al. were made. The following parameters were chosen according to Ref. 1: sound speed is 1465 m/s in water and 1675 m/s in sediment, absorption coefficient α is 0.082dB per wavelength in sediment, the specific gravity of sediment is 1.96 and the porosity is 40.3%.

For the convenience of calculation, 200kHz and 180kHz were used as two primary frequencies (in Ref. 1, they are 210kHz and 190kHz). The nonlinear parameter was assumed 3.5 in water and 6.5 in sediment [11], and the absorption in water was assumed to be 0.00019dB/m for the difference frequency and to be proportional to the square of frequency.

For saving the computing time, up to the 3rd harmonic of the primary frequency were considered. When the nonlinearity is not very strong, and the aim is to calculate the difference frequency sound field only, this arrangement is considered to be reasonable.

The water-sediment interface makes the sound field unsymmetrical with respect to the acoustic axis, so the source can not be treated as having only one dimension even though it is a circular array, which was the condition in the experiment of Muir et al., and equation (1) needs to be solved in the half space $u_y \geq 0$ instead of the quarter space $u_x \geq 0$ and $u_y \geq 0$ in which it is usually solved in the condition of a two-dimension source in a single-phase medium.

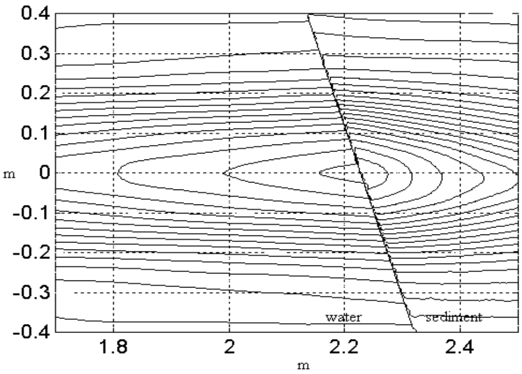

FIGURE III. PRESSURE CONTOUR PLOT, GRAZING ANGLE 76.8 DEG

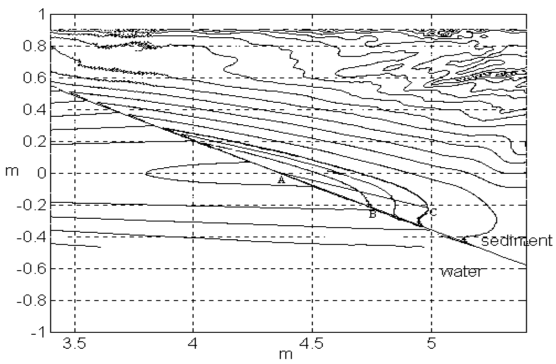

FIGURE IV. PRESSURE CONTOUR PLOT, GRAZING ANGLE 29.8 DEG.

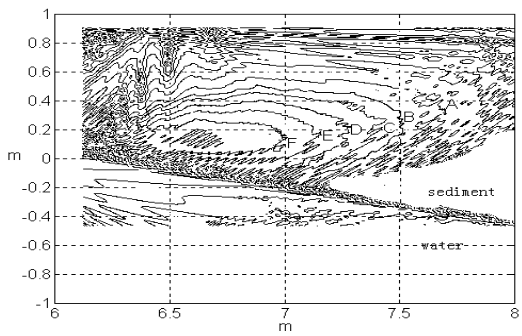

FIGURE V. PRESSURE CONTOUR PLOT, GRAZING ANGLE 14.0 DEG.

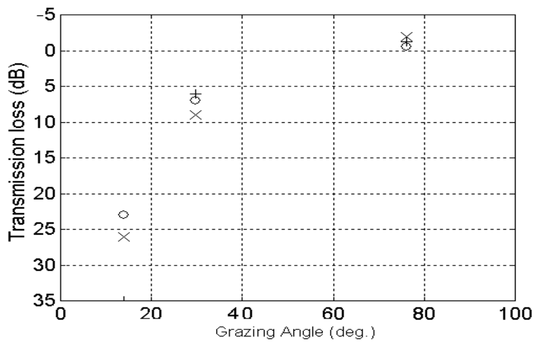

FIGURE. VI. X…EXPERIMENTAL DATA IN REF. 1 (PARAMETRIC); +…EXPERIMENTAL DATA IN REF. 1 (LINEAR); O…NUMERICAL DATA BASED ON KZK EQUATION

Figure 3, Figure 4 and Figure 5 are contour plots of pressures in sediment. The sound beams travel from the left-hand side along the horizontal direction.

The grazing angle of Figure 3 is 76.8°, corresponding to 4a in Table 1 of Ref. 1. Results which are coincident with Snell's law can be seen. Figure 4 corresponds to 2a in Table 1 of Ref. 1, the grazing angle is 29.8°, which is close to the critical grazing angle. We can see the sound beam penetrating into the sediment, which can not be explained simply by Snell's law. The angle ∠CAB is about 15°. This angle is not the transmission angle, it just tells the maximum amplitude position in the cross-section through point C. In fact, the angle ∠CAB in Figure 11 of Ref. 1 is 14.9°(calculated through the position of the three point A, B, and C), which is almost the same as our calculated results.

Figure 5 is the contour plot of the pressure in sediment at the grazing angle of 14°. The penetration of the sound beam into sediment is also very clear. The angle of the maximum amplitude axis is closer to the transmission angle measured by Muir et al. at positions where the sound beam penetrated into sediment relatively deeply (for example, the point A, B, C). The reason for that is when the sound beam has traveled in the sediment for a quite long distance, then the absorption at various points in a wave front tends to be the same.

The pressure contour in Figure 5 is much more complicated than that in Figure 4 and Figure 3. The smaller the grazing angle, the more complicated the pressure field in sediment. This trend was shown by the experiments of Williams et al., but not by their theory (see Figure 6, 7 in Ref. 3).

Figure 6 shows the transmission loss of our calculation comparing with measured data shown in Figure 10 of Ref. 1.

REFERENCES

[1] T. G. Muir, C. W. Horton, Sr. and L. A. Thompson "The penetration of highly directional acoustic beams into sediment", J. Sound. Vib. 64(4), 539-551 (1979).

[2] J. N. Tjøtta and S. Tjøtta "Theoretical study of The penetration of highly directional acoustic beams into sediment", J. Acoust. Soc. Am, 69, 998-1008 (1981).

[3] K. L. Williams, L. J. Satkowiak and D. K. Bugler "Linear and parametric array transmission across a water-sand interface—Theory, experiment,

and observation of beam displacement", J. Acoust. Soc. Am, 86, 311-325 (1989)

[4] D. J. Wingham "A theoretical study of the penetration of a water-sediment interface by a parametric beam", J. Acoust. Soc. Am, 76, 1192-1200 (1984).

[5] Wen-sen Liu and Zhen-xia Xu "Propagation of the difference frequency wave generated by a truncated parametric array through a water-sediment interface", J. Acoust. Soc. Am, 89, 92-97 (1991).

[6] S. I. Aanonsen, T. Barkve, J. N. Tjøtta and S. Tjøtta "Distortion and harmonic generation in the nearfield of a finite amplitude sound beam", J. Acoust. Soc. Am, 75, 749-768 (1984).

[7] E. H. Verfring, J. N. Tjøtta and S. Tjøtta "Nonlinear effects in the sound field of a bifrequency source", Proceeding of 12th ISNA, 251- 256, edited by M. F. Hamilton and D. T. Blackstock, Elsecier Science Publishers Ltd., London, 1990.

[8] S. I. Aanosen "Numerical computation of the nearfield of a finite amplitude sound beam", Report no. 73, Sept. 1983, University of Burgen, Burgen, Norway.

[9] M. A. Brekhovskikh 1960 "Waves in layered media", New York: Academic Press, 1960.

[10] F. B. Jesen and H. Schmidt "Subcritical penetration of narrow Gaussian beams into sediments", J. Acoust. Soc. Am. 82, 574-825 (1987).

[11] L. Bjørnø "Finite-amplitude wave propagation through water-saturated marine sediments", ACUSTICA, Vol. 38, No. 4, 1977.