ATLANTIS PRESS

Advances in Economics, Business and Management Research, volume 68

International Symposium on Social Science and Management Innovation (SSMI 2018)

Algorithm Implementation and Research of Prospect Theory Portfolio Optimization Model

Mengying Chang [a], Yang Xu [b], Yulian Fan [c]

College of Science North China University of Technology Beijing, China.

[a]1583137386@qq.com, [b]254583759@qq.com, [c]1647443232@qq.com

Abstract. With the continuous development of society, investment has become an indispensable skill for everyone. The research on portfolio optimization has received more and more attention from the public. This paper first introduces the Prospect Theory model developed on the basis of expected utility theory, and then combines the particle swarm optimization algorithm to empirically analyze the feasibility of the two in the portfolio optimization research,and then draws corresponding conclusions. Through the research and solution of the problem, this article will provide some reasonable opinions on personal investment behavior, aiming at playing a positive role in personal investment and playing a certain role in promoting the prosperity and development of the securities investment market.

Keywords: portfolio optimization, Prospect theory utility model, particle swarm optimization.

1. Introduction

With the continuous development of financial markets, investment has become more and more popular. Investment is an activity in which income and risk coexist. The most important issue for investors is how to get the most benefit from the range of risks which can be tolerated. Portfolio optimization, popularly speaking, refers to the fact that investors allocate funds to several certain kinds of assets, so that the investment amount of each type of assets accounts for a certain proportion of the total investment, The purpose is to that the overall income of the assets held by investors is as high as possible, or make the investment risk as low as possible. The most basic kind of portfolio optimization theory is the expected utility theory, that is, the decision is made by the expected value.But investing is a very complicated issue. There are many factors to consider. There may be some situations where the expected utility is not applied. With the deepening of research and the development of theory, the theory of Prospect utility evolved on the basis of the expected utility theory. Compared with the expected utility theory , prospect theory added some psychological components and corrected some of its shortcomings.

When it comes to optimization problems, the most common idea is to find an objective function f, which can measure the result, and then, within a certain range, find the global minimum (maximum) value according to a certain rule,finnally get the result we want. One of the commonly used optimization methods is the randomness method. Random algorithms generally originate from the simulation of natural phenomena or social behaviors, It is advantageous to solve many complex problems such as high dimensionality, polymorphism, and non-differentiation. The group intelligence method is an algorithm evolved on the basis of random calculation. Its main idea is to find the optimal problem solution through the operation and competition in the individual population. This kind of algorithm usually finds the optimal solution faster than the traditional optimization method when solving complex optimization problems, and it is more efficient, so it is widely valued. Particle swarm optimization (PSO) is one of the group intelligence methods, which can be used to solve a large number of nonlinear, non-differentiable and multi-peak complex problem optimization. Many fields in science and engineering have been widely used, including financial field. However, the main content of the research of the predecessors is the particle swarm optimization algorithm combined with the most basic mean variance model, the prospect utility theory model is rarely used.

This paper study the portfolio optimization problem by combining the prospect utility theory model with the particle swarm optimization algorithm. Through the research and solution of the problem, it aims to provide some reliable opinions on personal investment behavior and play a certain driving significance on the prosperity and development of the securities investment market.

2. Prospect Theory Utility Model

The prospect theory utility model proposed by psychology professors Daniel Kahneman and Amos Tversky in 1979: people make decisions based on potential losses and benefits, not just the final results. This theory assumes that each person has a different attitude toward risk based on the initial situation (reference point location), that is, the decision choice of the person under uncertain conditions depends on the gap between the result and the prospect (expectation, assumption),not the result alone. The great contribution of prospect theory is as a descriptive decision-making model, which divides people's decision-making process into two processes of editing and judging: first in the editing stage, individuals collect and process information by means of “framework” and “reference point”. In this paper, the reference point r_0 is defined to determine the individual's return and loss in the portfolio, r_0 represents the zero return or zero loss that the individual thinks; second in the evaluation stage, the value function and the weight function of the subjective probability are used to judge the information. Similarly, this article depend on the subjective probability weight function $\pi(\mathrm{p})$ and the value function to judge the information. The actual utility functions are derived through these two phases.

The value function formula in the prospect theory given by Tversky and Kahneman (1992) is:

$$\upsilon(r)=\begin{cases}(r-r_0)^{\alpha},r\geq r_0,\\-\lambda(r_0-r)^{\beta},r<r_0,\end{cases}$$

The prospect theoretical value function $\upsilon(r)$ represents the behavioral value of the obtained or lost result. For a given reference point r_0, since the function reflects the attitudes of different investors on the gains and losses, the parameters are different so that the images of $\upsilon(r)$ are different as well. Among them, α、β respectively represent the risk aversion coefficient of income and loss ; λ is the loss aversion coefficient. The Prospect theoretical weight function $\pi(\mathrm{p})$ is used to measure the size of the event's desirability to the result. $\pi(\mathrm{p})$ is an increasing function , $\pi(0)=0$, $\pi(1)=1$ .when the probability p is very small,that is $\pi(p)\geq p$. To simplify the study, delimit the weight function $\pi(p)=p$. Therefore, the prospect theoretical utility function $\pi(p)$ and $v(r)$ can be represented by and:

$$PT_U=\sum_{s=1}^{S}\pi(p_s)\upsilon(r_s).$$

3. Particle Swarm Optimization

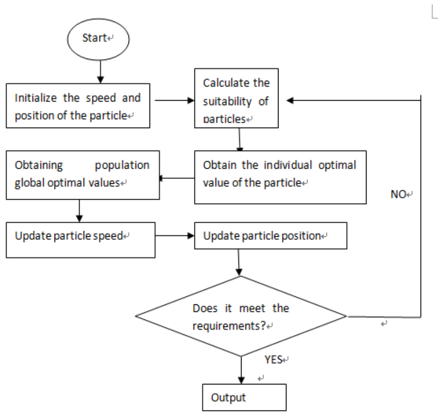
In 1995, Dr. Kennedy and Dr. Behrart proposed a basic particle swarm optimization algorithm, which is a swarm intelligence algorithm that originated from the simulation of the behavior of flock populations. If there is such a scene: a flock of birds randomly searches for a piece of food in an area. All the birds don't know where the food is, but they can judge the distance from each location to the food no matter before or they are in now., they can also determine the closest distance and the best position of the other birds in the current group. It is then relatively easy to think of the surrounding area of the bird that is closest to the food and the surrounding area where it has been closest to the food. Particle swarm optimization is inspired by this phenomenon. In the particle swarm algorithm, each bird is a "particle", which is the potential solution of the problem to be optimized in the search space. Each particle has a fitness that is determined by the function being optimized and the position of the particle itself. There is a speed that determines the direction and distance at which they move next. Then the particles search in the solution space according to the optimal position that they once and the all particles in the previous population. In each iteration, the particle determines its own acceleration based on the two data (and then determines its own speed): the first is the optimal solution found by the particle itself, and this solution is called the individual optimal solution: P_t ;Another extreme value P_g is the optimal solution currently found for the entire population, this extreme value is the global optimal solution: The individual optimal solution and the global optimal solution are updated once per iteration.

The mathematical description of the currently used particle swarm algorithm is: in an m-dimensional space, there are n potential solutions (particles) that make up a population $S=\{x_1,x_2,...,x_n\}$,where the i-th particle represents a D-dimensional vector $X_i=\{x_{i1},x_{i2},...,x_{im}\}$, $i=1,2,...,m$ that is the position of the i-th particle in the m-dimensional space. According to the objective function, the fitness of each particle is calculated (such as the distance from the thing), represents the optimal solution of the particle i iteration to t times, and represents the optimal solution experienced by the whole population iteration t, $V_i=\{V_{i1},V_{i2},...,V_{iD}\}$ represents the velocity of the particle i. The basic particle swarm algorithm iterative method is

$$V_i^{t+1}=w*V_i^t+C_1\mathrm{rand}(1)(P_i^t-X_i^t)+C_2\mathrm{rand}(2)(P_g^t-X_i^t)$$

$$X_i^{t+1}=X_i^t+V_i^{t+1}$$

Where i=1,2, . , n is the particle label, t is the number of iterations, w is the inertia weight (the algorithm did not have this parameter when it was first proposed, at that time, we saw it as 1.c1, c2 are learning factors (positive numbers) , rand (1) , rand (2) is a random number between [0, 1].
The flow chart of the particle swarm optimization algorithm is as follows:



3.1 The Flow Chart of the Particle Swarm Optimization Algorithm

The particle swarm optimization algorithm has high precision, fast convergence, parallelism, and its principle is simpler than many modern algorithms. Therefore the implementation is also easier, and there are not too much parameters to set, so once raised,it has attracted the attention of the academic community. However, it also has some defects. Because the algorithm has no mutation mechanism, when dealing with multi-peak problems, it is usually impossible to jump out of the local optimal solution, resulting in "premature" phenomenon.
Empirical analysis

3.2 Analysis and Selection of Data

The criteria for good and bad investment can be reflected in the rate of return. The rate of return is high,generally speaking, such investment is a meaningful investment; the rate of return is very low or even negative, we can avoid such investment. In addition to the rate of return, choosing the rate of return of different years makes our research more meaningful. In this paper, first we select several stocks with obvious good and bad characteristics verify that the output results, which proves that our theoretical model and optimization algorithm are feasible. Further, the yields of 7 stocks in the past three years were selected, namely Ping An Bank (stock code 000001), Vanke A (stock code 000002), Guonong Technology (stock code 000004), Century Star Source (stock code 000005). The annual yields of 2015, 2016 and 2017 for Shenzhengye A (stock code 000006), brand new good (stock code

000007) and Shenzhou high-speed rail (stock code 000008), annual return rates for 2015, 2016, and 2017, respectively.
The rate of years

Stock code	year		
	2015	2016	2017
000001	-0.0807	-0.0765	0.4789
000002	0.7935	-0.1293	0.5499
000004	1.948	-0.0218	-0.5016
000005	1.4488	-0.3267	-0.3876
000006	0.6494	-0.1685	0.0614
000007	0.6295	0.0773	-0.025
000008	1.3197	-0.2088	-0.0601

3.3 Model Setting and Calculation

Prospective theoretical value function formula given by Tversky and Kahneman:

$$\upsilon(r) = \begin{cases} (r - r_0)^{\alpha}, r \geq r_0, \\ -\lambda(r_0 - r)^{\beta}, r < r_0, \end{cases}$$

The risk aversion coefficient of our income and loss $\alpha = \beta = 0.88$: loss aversion coefficient $\lambda = 2.25$. According to the prospect theory, the optimal portfolio optimization problem can be expressed as:

$$\max PT(x) = \sum_{s=1}^{S} p_s \upsilon\left(\sum_{i=1}^{N} r_{si}\omega_i\right)$$

$$\begin{cases} \overline{r}(x) = \sum_{i=1}^{N} \overline{r}_i \omega_i \geq d, \\ \sum_{i}^{N} \omega_i = 1, \\ \omega_i \geq 0, i = 1, \ldots, N. \end{cases}$$

Which x represents a portfolio, ω represents the weight of each asset, S represents a different period, N represents the number of assets.
In this paper, the program for the above model is written in Matlab. First, we studied the effect of different iterations on the optimal solution when the population size is 50. The results are as follows:
results of different iteration times

Stock code	Weights		
	Iteration 100 times	Iteration 500 times	Iteration 1000times
000001	0.48	0.53	0.54
000002	0.03	0.03	0.07
000004	0.05	0.08	0.02
000005	0.07	0.16	0.01
000006	0.05	0.11	0.12
000007	0.20	0.13	0.12
000008	0.12	0.05	0.12
Max utility	0.47	0.53	0.52

Secondly, we study the effect of different population sizes on the optimal solution when the number of iterations is fixed (100 times). Results are as follows:
results of different population sizes

Stock code	Weights		
	Number of population 10	Number of population 50	Number of population 100
000001	0.37	0.48	0.47
000002	0.07	0.03	0.11
000004	0.07	0.05	0.09
000005	0.03	0.07	0.05
000006	0.08	0.05	0.20
000007	0.07	0.20	0.12
000008	0.19	0.12	0.02
Max utility	0.38	0.47	0.46

From the table, we can see that when the population size is fixed, the optimization effect increases with the increase of the number of iterations, but the number of iterations is not strictly proportional to the degree of optimization of the solution; Similarly, when the number of iterations is fixed, as the population increases, the optimization effect is improved, but the population size is not strictly proportional to the degree of optimization. Combining the two tables, we can conclude that Ping An Bank (stock code 000001) is worth investing, followed by Shenzhen Zhenye A (stock code 000006) and brand new good (stock code 000007), according to the actual situation , China High Speed Rail (stock code) 000008) is also conceivable. However other stocks do not recommend investment.

4. Conclusion and Suggestion

Through the above analysis, we can conclude that the prospect theory utility model and the particle swarm optimization algorithm can be combined for portfolio analysis. At the same time, investors can optimize the optimal solution based on parameters such as changing the population size or number of iterations and so on. However, investment is always accompanied by risks. Risks are always on. Any investment advice given by one person may have limitations and timeliness. Therefore, it is recommended that investors should treat investment issues rationally and make decisions based on your own situation. Remember that never follow blindly.

References

[1]. Huang Qifu. Foundation of Financial Economics. Tsinghua University Press, 2003.

[2]. Markowitz, H. "Portfolio Selection", Journal of Finance,1952, pp.77-91.

[3]. N. Grishina, C. A. Lucas, P. Date. Prospect theory–based portfolio optimization: an empirical study and analysis using intelligent algorithms. Quantitative Finance, 2017,pp. 17.

[4]. Yang Jianchi, Wang Yunji, Qian Daqing, et al. Research on Decision Model Based on Prospect Theory. Journal of System Simulation, 2009, pp. 2469-2472.

[5]. Bian Shen, Cai Zhijie. Consistency of Expected Utility Theory and Prospect Theory. Economics, 2005.pp.9.

[6]. Zeng Jin. Transnational Comparison of Enterprise Risk Tendency——Based on Prospect Theory.Science and Technology Management,2009.

[7]. Hu Wang, Li Zhiwei. A More Simplified and Efficient Particle Swarm Optimization Algorithm. Journal of Software, 2007, pp.861-868.

[8]. Zhou Chi, Gao Haibing, Gao Liang, et al. Particle swarm optimization algorithm. Journal of Computer Applications, 2003, pp. 7-11.

[9]. Liu Hao, Han Jing. MATLAB R2014a completely self-study one pass . Electronic Industry Press, 2015.

[10]. Cao Yuanyuan, Zhu Konglai. Optimization Analysis of Securities Investment Portfolio Based on MATLAB. Library and Information Service, 2006,pp.142-144.

[11]. He Junde, Chen Wei. Risk Investment Decision under the Guidance of Behavioral Finance. Science and Technology Progress and Policy, 2004, pp. 118-120.

[12]. Li Yuelin, Liu Jiqun. Research on Information Utilization Behavior in Individual Investor Investment Decision. Information and Documentation Services, 2014, pp. 29-34.