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A Long Run Risks Model with Rare Disaster: An Empirical Test in the American Consumption Data

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Abstract. Equity risk premium is a popular area of research in the past twenty years. Economists strive to find a model that can both explain the equity risk premium and achieve asset pricing. Long run risk model and risk model with disaster are two mainstream model for equity risk premium. In this paper the two model will be combined, with the assumption that the expected return rate is correlated with consumption growth rate. In this paper the explicit solution for equity risk premium is devised theoretically, and is then compared with existing combined model. The proposed model can explain equity risk premium, better than the traditional long run risk model, as well as be used for asset pricing, matching actual data.

Keywords: Equity risk premium; Economists strive; consumption growth rate.

1. Introduction

The phenomenon of equity risk premium is first discovered by Mehra and Prescott (1985). Based on their analysis of data in the United States from 1889 to 1978, the expected return on stock is 7.9%, while the risk-free return on bond is merely 0.8%, with a equity risk premium of 6.98%. The puzzle of equity risk premium is one of the most important in the field of economics and finance, and attempting to solve the puzzle has been a critial area of study. Equity risk premium is an important factor affecting investment portfolio as well as a determining factor of stock prices. Finding an asset pricing model that can explain the equity risk premium with economic relationships that does not contradict the reality becomes the focus of research of many economists.

Many research papers construct asset pricing model, to explain the equity premium puzzle, through introducing factors such as wealth preference, habit forming, loss aversion, and supervision. Kurz (Kurz, 1968) introduced utility function for the first time, defining it as utility gained by investors by owning wealth in addition to consumption. It considers risks in consumption and wealth, which helps explaining the puzzle and also significantly reduces return on long term risk-free assets. Based on wealth preference, Bakshi and Chen (1996) constructed consumption-investment model and the capital asset pricing model (CAPM). Constantinides (Constantinides, 1990) introduced habit forming into the investment-consumption model and explained the risk premium puzzle through optimization theory. Carroll (Carroll, 2000) studied the effect of habit forming on asset pricing. Adding the factor of habit forming primarily changed the construction of the utility function. The two main models are long run risk model (Bansal and Yaron, 2004) and rare disaster risk model (Barros, 2006). Long run risk model assumes that investors care about expected long term growth and uncertainty of economy, and the interaction of the two drives the risk purturbation in asset pricing. Therefore Bansal and Yaron believe that there exists a small long term factor in consumption and return rate, and a hit in expected growth rate will not only affect expected economy growth in short term, but also in long term. Bansal and Yaron further stated that the time variance in expected risk premium is cansed by change in economic uncertainty. By adding time variant volatility in consumption to simulate the uncertainty, they proved that increasing time variance can explain not only the equity risk premium, but also the volatility in the return on stock market and price to earnings (P/E) ratio. Barro held a different opinion from others. He introduced disaster into the Markov process of economic growth to explain the puzzle of high equity risk premium. Based on many major economic crises in many countries, Barros found proof for the existence of disaster and demonstrated that disaster status can explain high equity risk premium through theoretic model. After the long run risk model and rare disaster risk model were proposed, many scholars studied improved and extended models. Bansal, Dittmar, and Lundblad

(2005) considered cross-section risk using cointegration to simulate the relation between consumption and stock return. They added rare disaster to the long run risk model in 2010, reducing small discrete change and increasing small discrete total volatility, and normalized small discrete change in macroscopic level, causing large change of asset prices and subsequently economic crisis in the long run risk model. Different from Bansal, Kiku, and Yaron (2010), the jump distribution they used were gamma distribution rathen than exponential distribution.

But would it be more effective, in terms of explaining the equity risk premium, to introduce disaster risk into the long run risk model, and what is the mechanism? In this paper, based on the factor of disaster risk, a more effective long run risky capital asset pricing model is proposed. The second part of this paper is related works. The third part explains the models used in detail. The fourth part is dataset and the sources. The fifth part is result and conclusion, and the last part is future work.

2. Related Work

For the investment preference function in the long run risk model, the risk aversion coefficient and the separable utility function of intertemporal elasticity of substitution, proposed by Epstein and Zin (1989), are used to obtain the intertemoral marginal rate of substitution m_{t+1}:

$$m_{t+1} = \theta\log\delta - \frac{\theta}{\varphi} g_{t+1} + (\theta - 1) r_{a,t+1} \tag{1}$$

g_{t+1} is the continuous consumption growth rate, $r_{a,t+1}$ is the continuous asset growth rate based on total consumption. Bansal and Yaron indicated the dynamic model of consumption and stock growth rate, and simulated the fluctuating economic uncertainty through the time variace of consumption growth:

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1} \tag{2}$$

$$g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \tag{3}$$

$$\sigma_{t+1}^2 = \sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_\omega \omega_{t+1} \tag{4}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t \mu_{t+1} \tag{5}$$

$$e_{t+1}, \eta_{t+1}, \omega_{t+1}, \mu_{t+1} \epsilon N(0,1) \tag{6}$$

$g_{d,t+1}$ is the stock growth rate,σ_{t+1}^2 is the variance of consumption growth rate, x_t is a sustained measureable part contained in consumption and stock growth rates. Thus the equity risk premium from economy can be obtained:

$$E_t(r_{m,t+1} - r_{f,t}) = \beta_{m,n}\lambda_{m,n}\sigma_t^2 + \beta_{m,e}\lambda_{m,e}\sigma_e^2 + \beta_{m,\omega}\lambda_{m,\omega}\sigma_\omega^2 - 0.5Var_t(r_{m,t+1}) \tag{7}$$

$\beta_{m,n}$ is short term risk exposure, $\beta_{m,e}$ is long term risk exposure, $\beta_{m,\omega}$ is asset volatility risk exposure. Therefore, long run risk model is composed of short run risk premium, long run risk premium, and volatility of economy risk premium. The disaster risk factor, introduced into long run risk model, contains three types: dissipating impact (i.i.d. normal distributions), V type jump impact (drastic contraction in output without default), W-type jump impact (drastic contraction in output with default). In this paper mostly focuses on V-type jump impact, v_{t+1}, which depicts downward jump of GDP with low probability, satisfying:

$$v_{t+1} = \begin{cases} 0, & \text{with probablity } e^{-p} \\ log(1-b), & \text{with probablity } 1 - e^{-p} \end{cases} \tag{8}$$

b is an i.i.d. variable with probability density funciton having the same occurrence and magnitude as economic contractions.

3. The Model

The first method, proposed by Bansal, Kiku and Yaron (2010), is adding disaster impact to the periodical part of normalized long run risk model. However, even 10% economic contraction in every five years has little impact on the equity premium. Therefore, in this paper the long term expected growth rate is reduced mainly in small discrete fluctuations; they also increase total volatility. More precisely, negative jumps are added to the expected return rate, and small positive jumps are added to consumption volatility rate. Traditionally, it is assumed that expected growth rate and consumption volatility rate are mutually independent. However, in actual risk premium, they are not mutually independent. To better simulate the reality in this paper, it is assumed that consumption volatility rate is a jump function of expected growth rate. Therefore, long run risk model with disaster can be expressed as the following:

$$g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \tag{9}$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1} + v_{t+1} \tag{10}$$

$$\sigma_{t+1}^2 = \sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_\omega \omega_{t+1} + f(v_{t+1}) \tag{11}$$

$$g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t \mu_{t+1} + \pi_d \sigma_t \mu_{t+1} \tag{12}$$

g_{t+1} is consumption growth rate. x_t is the continuously changing part of consumption and stock growth rate. σ_t^2 is the conditional expectation of the variance of consumption, and the unconditional expectation is σ^2. Stock return and consumption is not completely correlated, but sharing the same continuously measurable part x_t, satisfying the function $f(v_{t+1})$. The five impact terms $\omega_{t+1}, e_{t+1}, \mu_{t+1}, \eta_{t+1}, v_{t+1}$ are mutually independent. v_{t+1} has the following distribution:

$$v_{t+1} = \begin{cases} 0, & \text{with probablity } 1-b \\ -\varphi_b log(1-b), & \text{with probablity } b \end{cases} \tag{13}$$

The random variable represent the magnitude of disaster risk, φ_b is the coefficient of the magnitude of disaster risk. Assume the logarithm of price-consumption ratio $z_t = \log\left(\frac{P_t}{C_t}\right)$ is a linear function of the mean and variance of consumption growth rate:

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \tag{14}$$

Similarly, the logarithm of price-divident ratio $z_{m,t} = \log\left(\frac{P_{m,t}}{D_{m,t}}\right)$ is a inear function of the mean and variance of stock return rate:

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} Var\big(f(v_{t+1})\big) \tag{15}$$

$Var\big(f(v_{t+1})\big)$ repersents the variance of stock return rate, which is a funcition of consumption growth rate: $f(v_{t+1})$. Based on the standard estimation method, proposed by Campbell and Shiller (1988), regarding the logarithm relation of the return on consumer ownership, consumption growth rate, and price-consumption ratio, it can be obtained that:

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \tag{16}$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1} \tag{17}$$

κ_0 and κ_1 rely on the mean price-cosumption ratio. The Endogenous solution of $\breve{z_t}$ and $\widetilde{z_{m,t}}$ can be obtained through the following equations:

$$\breve{z_t} = A_0(\breve{z_t}) + A_2(\breve{z_t})\breve{\sigma}^2 \tag{18}$$

$$\widetilde{z_{m,t}} = A_{0,m}\left(\widetilde{z_{m,t}}\right) + A_{2,m}\left(\widetilde{z_{m,t}}\right) Var\left(f(v_{t+1})\right) \tag{19}$$

In application, $f(v_{t+1})$ can be expressed as a high degree polynomial of v_{t+1}.
Then the disturbance term of market return rate can be obtained:

$$r_{m,t+1} - E_t\left(r_{m,t+1}\right) = \pi_d \sigma_t \mu_{t+1} + \varphi_d \sigma_t \mu_{t+1} + \beta_{m,e} \sigma_t e_{t+1} + \beta_{m,\omega} \sigma_\omega \omega_{t+1} + \beta_{m,h} h_{t+1} + \beta_{m,k} k_{t+1} \tag{20}$$

$\beta_{m,e} = \kappa_{1,m} A_{1,m}\, Var\left(f(v_{t+1})\right)$, and $\beta_{m,\omega} = \beta_{m,k} = \kappa_{1,m} A_{2,m}, \beta_{m,h} = \kappa_{1,m} A_{1,m}$. Assuming that stock return and consumption growth rate are related in the following way: $f(v_{t+1}) = {v_{t+1}}^2$, then the equity premium is:

$$E_t\left(r_{m,t+1} - r_{f,t}\right) = \lambda_\eta \pi_d \sigma_t^2 + \beta_{m,e} \lambda_e \sigma_e^2 + \beta_{m,\omega} \lambda_m \sigma_\omega^2 + \beta_{m,h} \lambda_h Var(v_{t+1}) + \left(\lambda_h \beta_{m,k} + \lambda_k \beta_{m,h}\right) - \left(E_t({v_{t+1}}^3) - E_t(v_{t+1}) E_t({v_{t+1}}^2)\right) + \lambda_k \beta_{m,h} Var({v_{t+1}}^2) + 0.5 Var(r_{m,t+1}) \tag{21}$$

Therefore, the disaster risk affects the equity premium through mean, variance, skewness, and kurtoisis. Moreover, the larger the volatility of disaster is the higher the equity risk preium is, so the effect of disaster risk on equity premium is proven in theory.

4. Data Set

All data of domestic production and consumption is from the US Bureau of Economic Analysis, from 1930 to 2008. Only risky assets and riskless assets are included. The return rate on risky assets is from bond prices research center, and the return rate on riskless assets is based on regression result using nominal interest rate and annual inflation rate. In this paper the result is mainly compared with the result obtained by Bansal, Kiku, and Yaron (2012). The parameters of preference, consumption, and stock return are the same. The effect of different function relation between stock return rate and consumption rate on the equity risk premium is shown. In the paper by Bansal, Kiku, and Yaron (2012), both consumption growth rate and stock return rate are 0.0015, and the actual annual growth rate is 1.8%. After introducing the disastr impact with nonzero mean, the consumption growth rate is adjusted to 0.0018. Stock return rate is a function of consumption growth rate. In this paper two functions are chosen. One is mutually independent, referring to Bansal, Kiku, and Yaron (2012), where both consumption growth rate and stock return rate are 0.0015. Another one is $f(v_{t+1}) = 0.5{v_{t+1}}^2$. When the consumption growth rate is adjusted to 0.0018, to match the actual data the stock retur rate can be hanged to 0.0024. Since the disaster risk in the U.S. stock market is low, so φ_b is 0.1789, matching the actual magnitude of disaster. The probability of disaster p can be determined using total occurrences divided by number of months. Based on Barro and Jin (2011), the probability of disaster in the previous year is 0.38, and the monthly probability is 0.0032.

Table 1. Parameters of the Proposed Model

Preference	δ		γ		ψ	
	0.9989		10		1.5	
Consumption	μ	$\delta\rho$	φ_e	σ	v_1	σ_ω
	0.0018	0.975	0.038	0.0072	0.999	0.0000028
Stock growth rate	μ_d	ϕ	φ_d	π_d		
	0.0024	2.5	5.96	2.6		
Disaster risk	φ_b	p				
	0.1789	0.0032				

5. Result

Table 2 shows that the model implies the moments of consumption growth rate and stock return rate. From the comparison data it is apparent that after introducing risk factors the long run risk model matches the Bansal, Kiku, and Yaron model well in terms of mean, vairance, 1st, 3rd, 4th, and 5th moments. However, the autocorrelation coefficient of 2nd moment is not in the implied range of the model, but is in that of the Bansal, Kiku, and Yaron (2012) model. Therefore, from the dynamics of consumptio growth rate and stock return rate, the two models mostly coincides with actual data, and there are no significant differences.

Table 2 shows the unconditional moment stock return rate, price-divident ratio, and risk-free rate. From the data it is apparent that the volatility of risk-free rate is not in the implied confidence interval of the model. On the other hand, in the Bansa, Kiku, and Yaron (2012) model, the mean price-divident ratio, variance, and first order coefficients cannot match with actual data, either. In actual data, the price-divident ratio is 3.36, larger than 3.29 (95%) and 3.32 (97.5%); the variane is 0.45, larger than 0.28 (95%) and 0.3(97.5%). The first order autocorrelation coefficient of price-divident ratio is 0.87, larger than 0.82 (97.5%). In terms of volatility of risk-free rate, the upper bound of interval 2.12 is closer than 1.59 of Bansal, Kiku, Yaron model (2012) to the actual value 2.86, so this paper better reflects the actual historical risk-free rate.

Although the model proposed in this paper, long run ris model with risk factors introduced, can explain the equity risk premium puzzle, it must be evaluated in terms of predictability of consumption and stock return. To compare with Bansal, Kiku, and Yaron (2012), the data set and time period used are the same, so the number of statistics are also the same. On the aspect of predictability, the method used is similar to Bansal, Kiku, and Yaron (2012): using vector autoregression (VAR) to match estimates and evaluate the predictability of growth rate. Table 3 lists consumption growth rate and predictability based on VAR. It is evident that the first-order VAR model of consumption growth rate, price-divident ratio, and risk-free rate implies that consumption growth rate is predictable. Comparing to the multivariate regression model of Bansal, Kiku, and Yaron (2012), the predictability in this paper reduces from 21% of 1 year to 12% of 5 years, while that in the Bansal, Kiku, and Yaron model increases from 27% to 31%. Therefore, in terms of predictability, the model in this paper matches the acutal data better.

Table 2. Comparision with BKY(2012)

Moment	Data	Bansal, Kiku, and Yaron (2012)						The Proposed Model					
	Estimate	Median	2.5%	5.0%	95%	97.5%	Pop	Median	2.5%	5.0%	95%	97.5%	Pop
$E[\Delta g]$	1.93	1.80	0.72	0.92	2.73	2.93	1.79	1.82	0.13	0.42	2.91	3.15	1.76
$\sigma(\Delta g)$	2.16	2.47	1.52	1.64	3.60	3.84	2.83	3.04	1.65	1.83	4.81	5.20	3.87
$AC1(\Delta g)$	0.45	0.39	0.15	0.19	0.57	0.60	0.45	0.42	0.18	0.21	0.61	0.62	0.45
$AC2(\Delta g)$	0.16	0.15	-0.13	-0.10	0.37	0.41	0.19	0.22	-0.06	-0.03	0.38	0.42	0.21
$AC3(\Delta g)$	-0.10	0.09	-0.17	-0.13	0.31	0.35	0.15	0.11	-0.12	-0.09	0.32	0.38	0.16
$AC4(\Delta g)$	-0.24	0.05	-0.21	-0.17	0.28	0.32	0.10	0.09	-0.23	-0.18	0.32	0.31	0.15
$AC5(\Delta g)$	-0.02	0.03	-0.23	-0.19	0.24	0.28	0.08	0.05	-0.17	-0.18	0.24	0.35	0.08
$E[\Delta g_d]$	1.15	1.84	-2.79	-1.96	5.64	6.52	1.45	1.82	-4.39	-3.23	6.40	7.01	1.54
$\sigma(\Delta g_d)$	11.05	14.11	8.53	9.19	20.02	21.15	15.83	15.88	8.74	9.57	23.24	24.53	18.9
$AC1(\Delta g_d)$	0.21	0.27	0.03	0.07	0.45	0.48	0.27	0.26	0.05	0.06	0.45	0.48	0.29
$Corr(\Delta g_d, \Delta g)$	0.55	0.46	0.22	0.26	0.62	0.65	0.46	0.48	0.23	0.27	0.63	0.66	0.47
$E[R]$	7.66	8.12	3.59	4.38	13.62	14.96	8.75	7.21	1.89	2.68	11.31	13.41	7.68
$\sigma(R)$	20.28	20.44	12.45	13.62	30.15	31.90	23.37	21.45	12.89	13.99	33.87	36.21	25.69
$E[p-g_d]$	3.36	3.14	2.79	2.85	3.29	3.32	3.07	3.13	2.62	2.79	3.39	3.62	3.01
$\sigma(p-g_d)$	0.45	0.18	0.11	0.11	0.28	0.30	0.26	0.23	0.13	0.14	0.38	0.41	0.37
$AC1(p-g_d)$	0.87	0.62	0.33	0.38	0.79	0.82	0.79	0.66	0.36	0.42	0.83	0.84	0.81
$E[R^f]$	0.57	1.24	0.10	0.31	1.78	1.86	1.05	1.03	-0.85	-0.41	1.80	1.91	0.87
$\sigma(R^f)$	2.86	0.94	0.54	0.59	1.46	1.59	1.22	1.21	0.63	0.65	1.92	2.23	2.12

Table 3. VAR-implied predictability of consumption growth

	Data	Bansal, Kiku, and Yaron (2012)						The Proposed Model					
	Estimate	Median	2.5%	5.0%	95%	97.5%	Pop	Median	2.5%	5.0%	95%	97.5%	Pop
1 year	0.23	0.32	0.10	0.13	0.50	0.54	0.27	0.27	0.12	0.15	0.49	0.60	0.21
2 years	0.17	0.34	0.10	0.13	0.57	0.61	0.30	0.30	0.11	0.15	0.61	0.64	0.15
5 years	0.15	0.31	0.08	0.12	0.55	0.60	0.31	0.35	0.13	0.17	0.51	0.48	0.12
10 years	0.13	0.21	0.05	0.07	0.45	0.50	0.27	0.19	0.11	0.10	0.59	0.49	0.11
15 years	0.11	0.15	0.03	0.05	0.35	0.41	0.22	0.18	0.08	0.10	0.43	0.39	0.14
20 years	0.09	0.11	0.02	0.04	0.28	0.33	0.17	0.15	0.08	0.05	0.22	0.34	0.13

6. Future Work

In this paper, the Long run risk model and risk model with disaster are combined, and the explicit solution for equity risk premium is devised theoretically. The proposed model can explain equity risk premium, better than the traditional long run risk model. But when analyzing the correlation of stock return and consumption, we assumed that they follow the relationship of second order function, which is not in line with the actual situation. Thus, we need further analysis of the correlation between stock return and consumption through real-world data to obtain a more accurate assessment model. And we will continue to study the fusion structure of the long-term risk model, the catastrophic risk model and the habit formation model to build an integrated analysis model.

References

[1]. Mehra, Rajnish and Prescott, Edward C. The Equity Premium: A Puzzle. Journal of Financial Economics 15, 145-161.

[2]. Kurz, M. (1968). Optimal economic growth and wealth effects. International Economic Review, 9(3), 348-357.

[3]. Santos, T., & Veronesi, P. (2010). Habit formation, the cross section of stock returns and the cash-flow risk puzzle. Journal of Financial Economics, 98(2), 385-413.

[4]. Kaltenbrunner, G., & Lochstoer, L. A. (2010). Long-run risk through consumption smoothing. Review of Financial Studies, 23(8), 3190-3224.

[5]. Hansen, Peter L, Heaton, et al. Consumption Strikes Back ?: Measuring Long-Run Risk[J]. Nan Li, 2008, 116(2):260-302.

[6]. Mulligan, C. B. (2004). What do aggregate consumption euler equations say about the capital-income tax burden?. American Economic Review, 94(2), 166-170.

[7]. Bakshi, G. S., & Chen, Z. (1996). Inflation, asset prices, and the term structure of interest rates in monetary economics. Review of Financial Studies, 9(1), 241-275.

[8]. Bansal, R., & Yaron, A. (2004). Risks for the long run: a potential resolution of asset pricing puzzles. Journal of Finance, 59(4), 1481-1509.

[9]. Barro, Robert J. (2006). Rare Disasters and Asset Markets in the Twentieth Century. The Quarterly Journal of Economics 123(3):823-866.

[10]. Bansal, R., Dittmar, R. F., & Lundblad, C. T. (2005). Consumption, dividends, and the cross section of equity returns. Journal of Finance, 60(4), 1639-1672.

[11]. Stock, J. H., & Watson, M. W. (1988). Variable trends in economic time series. Journal of Economic Perspectives, 2(3), 147-174.

[12]. Tédongap, R. (2015). Consumption volatility and the cross-section of stock returns. Social Science Electronic Publishing, 19(1), 203–209.

[13]. Bansal, R., Kiku, D., & Yaron, A. (2010). Long run risks, the macroeconomy, and asset prices. American Economic Review, 100(2), 542-546.

[14]. Epstein, L. G., & Zin, S. E. (1991). Substitution, risk aversion, and the temporal behaviour of consumption and ass. Journal of Political Economy, 99(2), 263-86.

[15]. Ravi Bansal, Dana Kiku, & Amir Yaron. (2009). An empirical evaluation of the long-run risks model for asset prices. Nber Working Papers, 1, 183-221.