

# The Simulation of Oligopoly Information Equilibria with Agents' Capacity Constraints

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**Abstract**—The game model of the oligopoly under the linear demand function and the non-linear functions of agents costs is considered, taking into account the capacity constraints of the agents. The simulation of optimal strategies for unlimited agents is carried out. The Pareto-efficient information equilibria of the game are found under various reflective assumptions of the agents.

**Index Terms**—stackelberg oligopoly, reflexion, capacity constraint

## I. INTRODUCTION

The game-theoretic oligopoly models, widely represented in the modern literature [1]–[13], are based on the search for Nash equilibria [14] without taking into account the constraints associated with the limits of the agents technological capacity. In this case, the equilibrium model is the vector of actions, that is, the sales volumes of the vendors (agents), simultaneously optimizing their criteria. The equilibrium is determined by solving the system of reaction equations or agents best responses on selected counterparty actions. In the case of exceeding the Nash equilibrium actions over the limits of agents technological capacity, they, as limited, choose smaller actions. As a result, a free market volume is formed, that is, the equilibrium of supply and demand is violated, therefore, the rest, unlimited agents, must choose actions that restore equilibrium.

The model of agent's action choice under capacity constraints is a dynamic problem, since these constraints are the results of agents investments in previous periods [15], [16]. The dynamics of the problem is manifested in the adaptation of capacities to the dynamics of market equilibrium [17], [18]. The result of the capacity constraints is manifested in the emergence of excessive demand, and the equilibrium depends on the strategy of the most competitive agent [19]. The capacity limitations were considered in models with linear [20]–[22] and quadratic [23], [24] functions of agents costs in the symmetric Cournot oligopoly. The asymmetric capacity-limited oligopoly was investigated in the presence of the Stackelberg leader [12], [25]. The Pareto-effective solution was obtained [26] for the problem of unlimited agents actions choice for oligopoly market.

However, the problem of finding the equilibrium under capacity constraints in the case of agents' reflexive behavior, leading to the various levels Stackelberg leaders appearance, is not solved and it is important.

## II. METHODOLOGY

The following model of oligopoly market agents choice is considered: the agent chooses the non-negative and not exceeding the capacity limit action, maximizing his utility function (profit), taking into account the market demand inverse function, which determines the price dependence on the total supply of all agents. The following hypotheses are accepted: 1) the inverse demand function is linear; 2) the functions of agents costs are power; 3) the utility functions of agents are continuous, continuously differentiable and concave over a certain interval into which the capacity limit enters; 4) the capacity limit of the agent in a certain  $t$ -th period is proportional to the profit in the  $(t - 1)$ -th period.

Under these assumptions, the model for choosing the action of the  $i$ -th agent in the  $t$ -th period under the capacity constraint has the form

$$\begin{aligned} Q_{it}^* &= \arg \max_{0 < Q_{it} \leq \bar{Q}_{it}} \Pi_{it}(Q_t, Q_{it}) = \\ &= \arg \max_{0 < Q_{it} \leq \bar{Q}_{it}} \{ (a + bQ_t) Q_{it} - C_{Fi} - B_i Q_{it}^{\beta_i} \}, \quad (1) \\ \bar{Q}_{it} &= f_i(\Pi_{it-1}), \quad i \in N, \end{aligned}$$

where  $Q_i, \Pi_i$  are the sales volume and profit of the  $i$ -th agent;  $N$  is the set of agents;  $n$  is the number of agents;  $P(Q) = a + bQ$ ,  $a > 0$ ,  $b < 0$  is the inverse demand function;  $Q = \sum_{i \in N} Q_i$  is the total market volume;  $C_{Fi} > 0$ ,  $B_i > 0$ ,  $\beta_i \in (0, 2)$  are coefficients of agents' cost functions, that have the form  $C_i(Q_i) = C_{Fi} + B_i Q_i^{\beta_i}$ ;  $a > 0$ ,  $b < 0$  are coefficients of the inverse demand function;  $\bar{Q}_{it}$  is the limit of the production capacity of the  $i$ -th agent;  $Q_i^N$ ,  $i \in N$  is the Nash equilibrium vector of agents' actions;  $f_i(\cdot)$  is nondecreasing function. Further, the index " $t$ " is omitted; it is assumed that problem (1) is considered in the  $t$ -th period.

It was proved [26] that the equilibria of oligopoly market agents are calculated by the formula

$$Q_i^N = \max \{ 0, \min \{ Q_i^*, \bar{Q}_i \} \}, \quad i \in N, \quad (2)$$

where the values  $Q_i^*$  are determined from the solution of system

$$\begin{aligned} a + bQ^* + bQ_i^* \left( 1 + b \sum_{j \in N} Q_j^* \right) - \\ - B_i \beta_i Q_i^{*\beta_i-1} = 0, \quad i \in N, \quad (3) \end{aligned}$$

where  $Q'_{jQ_i}$  is the conjectured variation, that is, the expected change in the supply of the  $j$ -th agent in response to a single increase in the supply of the  $i$ -th agent, determined based on the  $i$ -agent's leadership level.

The sum  $Q^N = \sum_{i \in N} Q_i^N$  of the equilibrium agents' actions (2), taking into account the capacity limitations, does not exceed the volume of market demand at an equilibrium price corresponding to  $Q^* = \sum_{i \in N} Q_i^*$ , therefore, a free market volume  $q$  is formed:

$$q = \sum_{i_2 \in N_2} \Delta Q_{i_2}, \quad \Delta Q_{i_2} = \max \{Q_{i_2}^*, 0\} - \bar{Q}_{i_2} > 0, \quad (4)$$

$$i_2 \in N_2,$$

where  $N_2 = \{i_2 \in N\}$  is the subset of agents for which the capacity constraints are effective (hereinafter "limited" agents). It has been shown [26] that agents for which capacity constraints are not effective (hereinafter "unlimited" agents) forming a subset  $N_1 = \{i_1 \in N\}$ , can implement two types of strategies: 1) increasing the supply by some amount  $0 < q_{i_1} < \bar{Q}_{i_1}$ ,  $i_1 \in N_1$ ,  $\sum_{i_1 \in N_1} q_{i_1} \leq q$ ; 2) the constancy of equilibrium supply  $q_{i_1} = 0$ ,  $i_1 \in N_1$ . It was proved [26] that in the case of full awareness, when the common knowledge for all agents are vectors  $Q_i^N$ ,  $i \in N$  and  $\bar{Q}_i$ ,  $i \in N$ , for agents having  $\beta_{i_1} \in (0, 1]$ , Pareto-effective is the strategy  $q_{i_1} = 0$ ,  $i_1 \in N_1$ .

The empirical study of country-regionplace Russian Federation telecommunications market [27] allowed to obtain vectors  $Q_i^*$ ,  $i \in N$ , expressing information equilibria for various reflective assumptions of agents about environing strategies, including strategies of follower agents and Stackelberg leaders. However, the influence of capacity constraints was not investigated. The purpose of this article is to form vectors  $\bar{Q}_i$ ,  $i \in N$  and  $Q_i^N$ ,  $i \in N$ , as well as to investigate the Pareto effectiveness of strategies  $q_{i_1} = 0 \vee q_{i_1} > 0$ ,  $i_1 \in N_1$ .

### III. RESULTS AND DISCUSSION

Based on information about Russian telecommunications market companies (MTS [28], Megafon [29], VimpelCom [30]) for 2003–2016, regression models for the functions of technological capacity limits are formed. The analysis of the dynamic series [31], [32] of voice traffic (indicated by the "V" index), depending on the agents' profits for the previous period, shows (Fig. 1) the nonlinear trend. The analysis of the dynamic series of Internet traffic (denoted by the "I" index) shows (Fig. 2) the non-linear (exponential) trend, depending on the dynamics of voice traffic. Therefore, regressions of voice traffic limits capacity are formed as power functions depending on profits, and regression of Internet traffic – as exponential functions depending on voice traffic capacity limits:

$$\bar{Q}_i^V(\Pi_i^V) = A_i^V (\Pi_i^V)^{\alpha_i^V}, \quad \bar{Q}_i^I(\bar{Q}_i^V) = A_i^I e^{\alpha_i^I \bar{Q}_i^V}, \quad (5)$$

$$i = 1, 2, 3,$$

where  $\bar{Q}_i^V$ ,  $\bar{Q}_i^I$  are maximum volumes of voice services and Internet traffic of the  $i$ -th agent for the corresponding period

(billion minutes, billion Mb);  $A_i^V$ ,  $A_i^I$ ,  $\alpha_i^V$ ,  $\alpha_i^I$  are coefficients of regression;  $\Pi_i^V$  is profit of voice traffic (billion rubles<sup>1</sup>). The following indexation of market agents is introduced: 1 – "MTS", 2 – "MegaFon", 3 – "VimpelCom". The results of estimating the regression coefficients (5) by the method of least squares in the Excel spreadsheet and the statistical characteristics of the regressions are given in Table I. High values of the determination coefficient and the Fisher criterion, calculated at a significance level of 0.05, indicate the high explanatory characteristics of regression models, their adequacy and statistical significance.

Further, the most important information equilibria, calculated in [27] for the Russian telecommunications, are analyzed: the Cournot equilibrium (denoted by  $\theta = 1$ ), the equilibrium with the first level Stackelberg leader (MTS), denoted by  $\theta = 2$ , the equilibrium with the two second level Stackelberg leaders (MTS, MegaFon) and the first level Stackelberg leader (VimpelCom), denoted by  $\theta = 3$ . The latter case most accurately (deviations in the agents' traffic less than 10%) corresponds to the current state of the market.

In Table II–IV the parameters of Pareto effective agents' strategies for voice traffic are shown, in Table V–VII the parameters for Internet traffic are shown. The profit deviations  $\Delta_i = \Pi_i(Q_i^N) - \Pi_i(Q_i^*)$ ,  $i \in N$  are calculated in the tables; these values characterize efficiency of Pareto effective strategies in comparison with the strategies without taking into account capacity limitations.

In the case of implementing the strategy  $q_{i_1} > 0$ ,  $i_1 \in N_1$ , two possible situations are considered:

1) the unawareness of all unlimited agents except one ( $\eta$ -th) about the existence of a free market volume – in this case  $q_\eta \leq q$ , that is, the  $\eta$ -th agent, if the technological capacity limit allows it, occupies the entire free market volume, consequently  $Q_\eta^N = Q_\eta^* + q \quad \forall Q_\eta^N \leq \bar{Q}_\eta$ ,  $Q_{i_2}^N = \bar{Q}_{i_2}$ ,  $i_2 \in N_2$ ;

2) the awareness of all agents about the existence of a free market niche – in this case  $q_{i_1} = \frac{q}{N_1}$ ,  $i_1 \in N_1$ , that is, it is accepted that all unlimited agents divide the free market volume equally, that is,  $Q_{i_1}^N = Q_{i_1}^* + \frac{q}{N_1} \quad \forall Q_{i_1}^N \leq \bar{Q}_{i_1}$ ,  $i_1 \in N$ ,  $Q_{i_2}^N = \bar{Q}_{i_2}$ ,  $i_2 \in N_2$ . In the case of the implementation of the strategy  $q_{i_1} = 0$ ,  $i_1 \in N_1$ , it was assumed that  $Q_{i_2}^N = \bar{Q}_{i_2}$ ,  $i_2 \in N_2$ ,  $Q_{i_1}^N = Q_{i_1}^*$ ,  $i_1 \in N$ , that is, the equilibrium supply of the limited agents is equal to the limit of their capacity; equilibrium supply of the unlimited agents is equal to the optimum without taking into account the capacity limitations.

The analysis of Table II–VII results shows that for both voice traffic and Internet traffic, the strategy  $q_{i_1} = 0$ ,  $i_1 \in N_1$  dominates the strategy  $q_{i_1} > 0$ ,  $i_1 \in N_1$ , that is

$$\Delta \Pi_i|_{q_{i_1}=0} > \Delta \Pi_i|_{q_{i_1}>0}, \quad i \in N. \quad (6)$$

It should be noted that (6) is also performed for limited agents.

The formula (6) follows from the fact that the sensitivity of the agents' profit to the change in the equilibrium price, corresponding to the strategy  $q_{i_1} > 0$ ,  $i_1 \in N_1$ , is higher

<sup>1</sup>The average course of the Russian ruble against the US dollar in 2018 was 60.

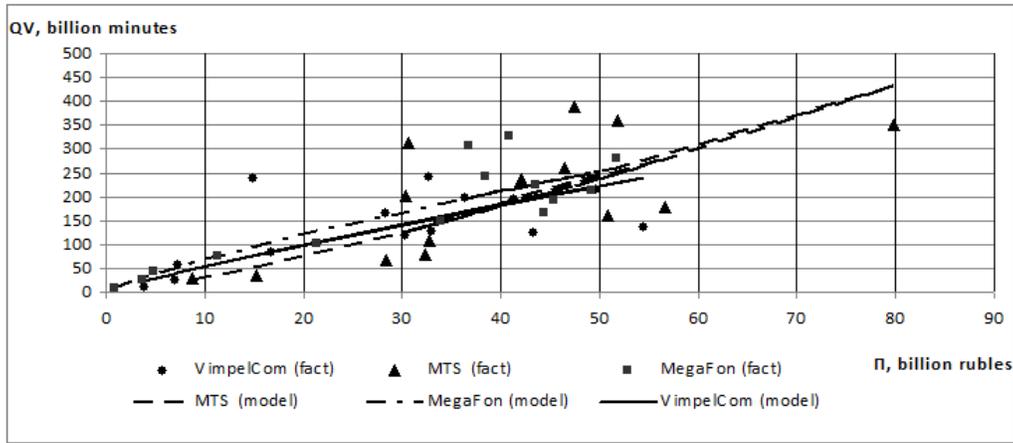


Figure 1. Dependence of agents' technological capacity of voice services on profits

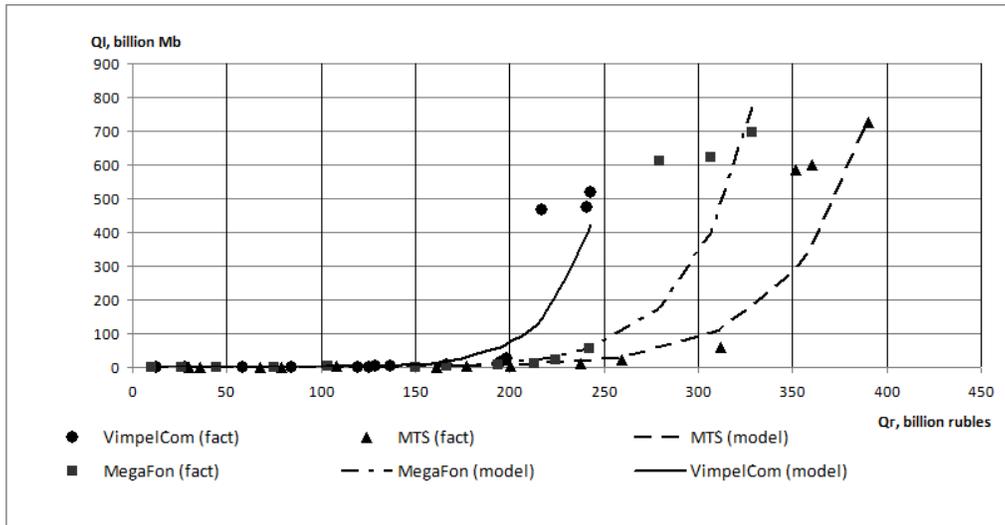


Figure 2. Dependence of agents' technological capacity of Internet traffic on voice capacities

Table I  
COEFFICIENTS OF REGRESSION MODELS (5),  $F_{table} = 3.81$

Regression models	$A_i^V, A_i^I, i = 1, 2, 3$	$\alpha_i^V, \alpha_i^I, i = 1, 2, 3$	$R^2$	$F_{fact}$
$\bar{Q}_1^V(\Pi_1^V)$	1.667	1.269	0.71	13.47
$\bar{Q}_2^V(\Pi_2^V)$	11.02	0.801	0.95	104.50
$\bar{Q}_3^V(\Pi_3^V)$	7.078	0.883	0.71	13.47
$\bar{Q}_1^I(Q_1^I)$	0.0579	0.024	0.95	104.50
$\bar{Q}_2^I(Q_2^I)$	0.0347	0.030	0.94	86.17
$\bar{Q}_3^I(Q_3^I)$	0.0199	0.041	0.94	86.17

than the profit sensitivity to the change in supply under this strategy, since from (1) follows

$$\begin{aligned} \Pi'_{iP(Q)}(Q, Q_i) &= Q_i, \\ \Pi'_{iQ_i}(Q, Q_i) &= P(Q) - B_i \beta_i Q_i^{*\beta_i - 1}, \\ Q_i &\gg P(Q) - B_i \beta_i Q_i^{*\beta_i - 1}, \quad i \in N. \end{aligned} \quad (7)$$

Therefore, the conclusion about the Pareto efficiency of the

strategy  $q_{i_1} = 0, i_1 \in N_1$ , obtained by modeling, is consistent with the theory of oligopoly.

The information equilibria parameters (Table IV, VII) calculated for the case  $\theta = 3$  have a practical importance, since 1) it predicts the possibility of a significant reduction in the supply volume of agent 3 (VimpelCom) under the same costs and capacity trends that were observed in 2003–2016; 2) it

Table II  
EFFECTIVE STRATEGIES,  $\theta = 1$ , FOR VOICE SERVICES,  $p_{q>0} = 0.87, p_{q=0} = 0.97$

Agent	Q*, billion minutes	$\bar{Q}$ , billion minutes	$q_{i_1} > 0$				$q_{i_1} = 0$	
			unawareness		awareness		$Q^N$ , billion minutes	$\Delta$ , billion rubles
			$Q^N$ , billion minutes	$\Delta$ , billion rubles	$Q^N$ , billion minutes	$\Delta$ , billion rubles		
MTS	458.3	939.1	559.1	46.8	508.7	23.1	458.3	42.5
MegaFon	403.5	541.5	403.5	0.0	453.9	20.5	403.5	37.4
VimpelCom	111.3	10.5	10.5	-2.4	10.5	-2.4	10.5	-1.4
Sum	973.0	1491.0	973.0	44.4	973.0	41.3	872.2	78.5

Table III  
EFFECTIVE STRATEGIES,  $\theta = 2$ , FOR VOICE SERVICES,  $p_{q>0} = 0.91, p_{q=0} = 0.98$

Agent	Q*, billion minutes	$\bar{Q}$ , billion minutes	$q_{i_1} > 0$				$q_{i_1} = 0$	
			unawareness		awareness		$Q^N$ , billion minutes	$\Delta$ , billion rubles
			$Q^N$ , billion minutes	$\Delta$ , billion rubles	$Q^N$ , billion minutes	$\Delta$ , billion rubles		
MTS	331.9	616.7	403.7	33.9	367.8	16.8	331.9	21.9
MegaFon	493.3	726.9	493.3	0.0	529.2	16.6	493.3	32.6
VimpelCom	102.5	30.7	30.7	-2.1	30.7	-2.1	30.7	-0.1
Sum	927.7	1374.4	927.7	31.8	927.7	31.2	855.9	54.4

Table IV  
EFFECTIVE STRATEGIES,  $\theta = 3$ , FOR VOICE SERVICES,  $p_{q>0} = 0.88, p_{q=0} = 1.0$

Agent	Q*, billion minutes	$\bar{Q}$ , billion minutes	$q_{i_1} > 0$				$q_{i_1} = 0$	
			unawareness		awareness		$Q^N$ , billion minutes	$\Delta$ , billion rubles
			$Q^N$ , billion minutes	$\Delta$ , billion rubles	$Q^N$ , billion minutes	$\Delta$ , billion rubles		
MTS	415.1	803.2	552.0	63.6	483.5	31.3	415.1	52.3
MegaFon	326.2	442.6	326.2	0.0	394.6	27.4	326.2	41.1
VimpelCom	225.7	88.8	88.8	-1.1	88.8	-1.1	88.8	10.1
Sum	966.9	1334.6	966.9	62.5	966.9	57.7	830.0	103.5

Table V  
EFFECTIVE STRATEGIES,  $\theta = 1$ , FOR INTERNET TRAFFIC,  $p_{q>0} = 0.36, p_{q=0} = 0.68$

Agent	Q*, billion minutes	$\bar{Q}$ , billion minutes	$q_{i_1} > 0$				$q_{i_1} = 0$	
			unawareness		awareness		$Q^N$ , billion minutes	$\Delta$ , billion rubles
			$Q^N$ , billion minutes	$\Delta$ , billion rubles	$Q^N$ , billion minutes	$\Delta$ , billion rubles		
MTS	441.1	3826.9	878.6	145.7	659.8	72.3	441.1	143.2
MegaFon	448.1	7637.2	448.1	0.0	666.9	73.3	448.1	145.5
VimpelCom	439.4	1.9	1.9	-120.7	1.9	-120.7	1.9	-120.0
Sum	1328	11466	1328	25.1	1328	24.9	891.1	168.6

shows the optimal strategies of all agents in response to such actions of the agent 3.

Also, for comparison with the results of [27], the regression coefficients of the linear agents' costs functions of the form

Table VI  
EFFECTIVE STRATEGIES,  $\theta = 2$ , FOR INTERNET TRAFFIC,  $p_{q>0} = 0.23$ ,  $p_{q=0} = 1.01$

Agent	Q*, billion minutes	$\bar{Q}$ , billion minutes	$q_{i_1} > 0$				$q_{i_1} = 0$	
			unawareness		awareness		$Q^N$ , billion minutes	$\Delta$ , billion rubles
			$Q^N$ , billion minutes	$\Delta$ , billion rubles	$Q^N$ , billion minutes	$\Delta$ , billion rubles		
MTS	973.9	179.4	179.4	-163.4	179.4	-163.4	179.4	-24.1
MegaFon	266.2	118181	1313	225.2	1313	225.2	266.2	206.8
VimpelCom	253.7	1.3	1.3	-29.9	1.3	-29.9	1.3	-28.8
Sum	1493.9	118362	1493	32.0	1493	32.0	446.9	153.9

Table VII  
EFFECTIVE STRATEGIES,  $\theta = 3$ , FOR INTERNET TRAFFIC,  $p_{q>0} = 0.15$ ,  $p_{q=0} = 0.38$

Agent	Q*, billion minutes	$\bar{Q}$ , billion minutes	$q_{i_1} > 0$				$q_{i_1} = 0$	
			unawareness		awareness		$Q^N$ , billion minutes	$\Delta$ , billion rubles
			$Q^N$ , billion minutes	$\Delta$ , billion rubles	$Q^N$ , billion minutes	$\Delta$ , billion rubles		
MTS	533.0	1344.6	844.0	40.5	688.5	20.0	533.0	123.0
MegaFon	551.0	723.3	551.0	0.0	706.4	20.7	551.0	127.1
VimpelCom	519.0	208.1	208.1	-32.6	208.1	-32.6	208.1	15.4
Sum	1603.1	2276.0	1603	7.9	1603	8.1	1292	265.5

$C_i(Q_i) = d_i + c_i Q_i$ ,  $c_i, d_i > 0$ ,  $c_i < a$ ,  $i \in N$  were obtained. The regression models coefficients of the agents' costs functions, calculated by the least-squares deviation method in the Excel table processor, are given in Table VIII. The statistical significance criteria of regressions show the adequacy and high explanatory characteristics of models. The linear form of cost functions makes it possible to simplify the calculation of equilibria.

Table VIII  
COEFFICIENTS OF AGENTS' LINEAR COSTS FUNCTIONS AND CRITERIA FOR ESTIMATING REGRESSION ACCURACY

Coefficient	Agent 1	Agent 2	Agent 2
c, thousand rubles per minute	0.0005	0.0003	0.0006
d, billion rubles	20.42	86.11	49.61
$R^2$	0.98	0.93	0.84
$F_{fact}$	582.32	55.17	31.92
$F_{table}$	2.46	4.95	3.73

#### IV. CONCLUSION

The model of the oligopoly equilibrium under nonlinear agents' costs functions is considered, taking into account the limitations on the technological capacity. It is shown that the influence of capacity constraints leads to the emergence of a free market volume equal to the sum of the deviations of equilibrium supplies from the limits of agents' capacities. The consequence of this is the game for the additional market volume between unlimited agents.

The simulation conducted for Russian telecoms agents shows the power trends in the limits of voice traffic depending on the profit and exponential trends of the limits of Internet traffic, depending on the dynamics of voice traffic. The last correlation is technologically consistent with telecommunications, since the technical base of Internet data transmission services is equipment used for voice traffic.

Such game strategies of unlimited agents, as an increase in the supply of services and the preservation of the supply of services at an unchanged level, are analyzed. The simulation confirmed that the strategy of invariability of the supply of services is Pareto effective in comparison with another possible strategy. The found parameters of Nash information equilibria under capacity constraints can be used for programs of Russian telecommunications development.

#### REFERENCES

- [1] U. S. Karmarkar and K. Rajaram, "Aggregate production planning for process industries under oligopolistic competition," *European Journal of Operational Research*, vol. 223, no. 3, pp. 680–689, 2012.
- [2] A. Ledvina and R. Sigar, "Oligopoly games under asymmetric costs and an application to energy production," *Mathematics and Financial Economics*, vol. 6, no. 4, pp. 261–293, 2012.
- [3] S. Currarini and M. A. Marini, "Sequential play and cartel stability in tournot oligopoly," *Applied Mathematical Sciences*, vol. 7, no. 4, pp. 197–200, 2013.
- [4] A. Vasin, "Game-theoretic study of electricity market mechanisms," *Procedia Computer Science*, vol. 31, pp. 124–132, 2014.
- [5] F. Sun, B. Liu, F. Hou, H. Zhou, L. Gui, and J. Chen, "Cournot equilibrium in the mobile virtual network operator oriented oligopoly offloading market," in *2016 IEEE International Conference on Communications (ICC)*. ICC, may 2016.

- [6] A. K. Naimzada and L. Sbragia, "Oligopoly games with nonlinear demand and cost functions: Two boundedly rational adjustment processes," *Chaos, Solitons & Fractals*, vol. 29, no. 3, pp. 707–722, 2006.
- [7] S. Askar and K. Alnowibet, "Nonlinear oligopolistic game with isoelastic demand function: Rationality and local monopolistic approximation," *Chaos, Solitons & Fractals*, vol. 84, pp. 15–22, 2016.
- [8] A. Naimzada and F. Tramontana, "Two different routes to complex dynamics in a heterogeneous triopoly game," *Journal of Difference Equations and Applications*, vol. 21, no. 7, pp. 553–563, 2015.
- [9] F. Cavalli, A. Naimzada, and F. Tramontana, "Nonlinear dynamics and global analysis of a heterogeneous cournot duopoly with a local monopolistic approach versus a gradient rule with endogenous reactivity," *Communications in Nonlinear Science and Numerical Simulation*, vol. 23, no. 1–3, pp. 245–262, 2015.
- [10] R. Colacicco, "Ten years of general oligopolistic equilibrium: A survey," *Journal of Economic Surveys*, vol. 29, no. 5, pp. 965–992, 2015.
- [11] S. Jrgensen and G. Zaccour, "A survey of game-theoretic models of cooperative advertising," *European Journal of Operational Research*, vol. 237, no. 1, pp. 1–14, 2014.
- [12] H. Ino and T. Matsumura, "Welfare-improving effect of a small number of followers in a stackelberg model," *B.E. Journal of Theoretical Economics*, vol. 16, no. 1, pp. 243–265, 2016.
- [13] Y. Peng, Q. Lu, and Y. Xiao, "A dynamic stackelberg duopoly model with different strategies," *Chaos, Solitons and Fractals*, vol. 85, pp. 128–134, 2016.
- [14] J. Nash, "Non-cooperative games," *Annals of Mathematics*, vol. 54, pp. 286–295, 1951.
- [15] A. Spence, "Michael entry, capacity, investment and oligopolistic pricing," *Bell Journal of Economics*, vol. 8, pp. 534–544, 1977.
- [16] A. Dixit, "The role of investment in entry-deterrence," *The Economic Journal*, vol. 90, no. 357, pp. 95–106, mar 1980.
- [17] J. P. Benoit and V. Krishna, "Dynamic duopoly: Prices and quantities," *The Review of Economic Studies*, vol. 54, pp. 23–36, 1987.
- [18] H. Dixon, "Strategic investment with consistent conjectures," *Oxford Economic Papers*, vol. 38, no. supp, pp. 111–128, nov 1986.
- [19] D. Kreps and J. Sheinkman, "Cournot recommitment and bertrand competition yield cournot outcomes," *Bell Journal of Economics*, vol. 14, pp. 326–337, 1983.
- [30] "VimpelCom." [Online]. Available: <http://moskva.beeline.ru/about/about-beeline/disclosure/annual-reports/>
- [20] E. V. Ponkina, A. S. Manicheva, and P. V. Komarov, "A model of a dispersed market with barriers to entry in conditions of limited production capacity," *Izvestiya Altayskogo gosudarstvennogo universiteta*, vol. 1, no. 77, pp. 97–100, 2013.
- [21] L. A. dos Santos Vieira, S. F. Mayerle, C. L. dos Santos Vieira, and M. M. M. Luna, "Industrial capacity planning in oligopolistic markets," *Optimization and Engineering*, vol. 17, no. 4, pp. 761–780, mar 2016.
- [22] P. Mouche and F. Quartieri, "Cournot equilibrium uniqueness in case of concave industry revenue: A simple proof," *Economics Bulletin*, vol. 35, no. 2, pp. 1299–1305, 2015.
- [23] N. I. Aizenberg, V. I. Zorkaltsev, and M. A. Kiseleva, "Models of imperfect competition applied to the analysis of the electric power market of Siberia," *Zhurnal Novoy ekonomicheskoy assotsiatsii*, vol. 2, no. 18, pp. 62–88, 2013.
- [24] H. Hecking and T. Panke, "The global markets for coking coal and iron ore – complementary goods, integrated mining companies," *Energy Economics*, vol. 52, pp. 26–38, 2015.
- [25] M. Ya. Postan and I. V. Saveleva, "A method for finding an equilibrium solution for port operators in a competitive environment of the oligopoly type," *Tekhnologicheskii audit i rezervy proizvodstva*, vol. 2, no. 18, pp. 58–63, 2014.
- [26] M. I. Geras'kin and A. G. Chkhartishvili, "Analysis of game-theoretic models of an oligopoly market under constraints on the capacity and competitiveness of agents," *Automation and Remote Control*, vol. 78, no. 11, pp. 2025–2038, 2017.
- [27] M. I. Geras'kin, "Modeling reflexion in the non-linear model of the stackelberg three-agent oligopoly for the russian telecommunication market," *Automation and Remote Control*, vol. 79, no. 5, pp. 841–859, may 2018.
- [28] "MTS." [Online]. Available: [http://www.company.mts.ru/comp/ir/control/data/annual\\_reports/](http://www.company.mts.ru/comp/ir/control/data/annual_reports/)
- [29] "MegaFon." [Online]. Available: [http://corp.megafon.ru/investoram/shareholder/year\\_report/](http://corp.megafon.ru/investoram/shareholder/year_report/)
- [31] S. Sidorov, A. Faizliev, and V. Balash, "Measuring long-range correlations in news flow intensity time series," *International Journal of Modern Physics C*, vol. 28, no. 08, p. 1750103, 2017.
- [32] —, "Scale invariance of news flow intensity time series," *Nonlinear Phenomena in Complex Systems*, vol. 19, no. 4, pp. 368–377, 2016.