

# *Method of Calculation of Institutional Investors Stability under Conditions of Digital Economy*

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**Abstract**—The relevance of the study is caused by the necessity to find conditions and ways to assess the stability of institutional establishments in the Internet economy, as well as to develop the practical methods forecasting the duration of their functioning. The transformation to the information society and digitalization results in an increase of the number of people using the services of the digital economy, and provokes a large amount of processing data. The development of a practical methodology for forecasting the long-term performance of institutional establishments is also an important challenge. The authors of the article give the theory and the methodology of the approach to the problem of mathematical modeling of the dynamics of highly profitable investment projects (projects HYIP). The obtained results provide an algorithmic basis for the creation of software products. This will allow real-time evaluation of information flows when the interface format will be changed to digital platforms. We can apply the mathematical model developed in this research, first of all, to the long-term forecasting of the activities of such a large institutional investor as the Pension Fund.

**Keywords**—mathematical model, project, institutional investor, forecast

## I. INTRODUCTION

Institutional investors play a significant role in the life of a modern society. The development of information technology, the transition to digital platforms, leads to the need of real-time processing of big data sets. It is necessary to develop the appropriate algorithms for implementation of these. In addition, we need to create software to manage digital interactive communication.

Since their functioning covers the majority of the population, the stability and reliability of their work allowing long-term planning is the first priority.

The relevance of research and search solutions in this direction is conditioned by the fact that the methods of calculation based on mathematical modeling were not sufficiently used to assess and predict the behavior of such large-scale market participants.

The primary activity of the institutional investor consists in collecting money from the population of the country for the purpose of their further putting in various investment tools including the real estate, securities. Their work is done in a dynamic mode and key parameters have a stochastic character. The purpose of their work as well as of any commercial project

is, on the one hand, generation of profit, but on the other hand, their social value. The scope of work and influence of institutional investors on the general business climate is proved by the fact that from the last two decades of the XX century they became one of the largest participants in the most influential securities markets. First of all let us mention NYSE (New York Stock Exchange), Tokyo stock exchange, LSE - London Stock Exchange, SSE (The Shanghai Stock Exchange).

### A. Problem Statement

For development of the mathematical model, we will assume as a basis the formalisms which are applied when calculating dynamics of development of [1] projects belonging to the category HYIP. This kind of activity have existed for several decades, but wide spread of High Yield Investment Program, for which the term “HYIP projects” is used, is caused by such phenomenon as global coverage of the majority of the countries of the world by the high-speed Internet, with conceptual base Web 3.0. Their work has been recently based on online technologies with transactions of electronic currencies.

### B. Research Questions

It is necessary to highlight that actually the key moments of the analysis are numerous payments, population shares collected by the legal entity such as pension fund, insurance company acting as the holder of this money. This financial structure has a distinctive, essentially important advantage caused by the duration of life cycle. If we analyze the work of the institutional investor in more detail, we will see that in spite of the fact that depending on the direction of their activity, the extent of the business horizons can be various, but the difference from life cycles of the people depositing funds in them forms the basis for stability of such establishments. As people have an accumulation phase during labor activity of an individual, a consumption phase takes place during living on provision of pensions and the limited term of life. Duration of these periods thanks to the available extensive statistical data is well described by the relevant mathematical laws of distribution.

### C. Purpose of the Study

As the range of crypto currencies continuously extends and forks of Bitcoin arise already weekly, there is a problem of development of the general technique of modeling of HYIP

projects. Assessment of stability of HYIP projects and the forecast of duration of their functioning become the main goal of such work. Development of systems of electronic payments and also use of anonymous on-line payment service providers is the HYIP projects driver

## II. RESEARCH METHODS

The greatest benefit during the research is brought by mathematical modeling with application of computer calculations. It is caused by possibility of replication of results and also realization of the received algorithms as a part of expert systems and the automated program complexes for consultation of experts and the help in decision-making. At the stage of formation of conditions for modeling [2] of activity of the institutional investor, it is necessary to consider that such serious players of this segment of business as insurance companies work in the conditions determined by high dispersion of function of distribution of approach in terms of payments to the clients. It results in need to make investments in highly liquid assets. From the point of view of business, it narrows the range of their investment opportunities. Other participant, pension funds, according to the statistics of life expectancy on pension, have all set of investment tools such as exchange goods, actions, antiques, real estate. The expanded investment horizon is available to such participants of the institutional market that gives the grounds for an investment in low-liquid assets as the probability of need for their sale before the term is very small. The corresponding function of distribution of probability of duration of payments possesses small dispersion and long-term expected value. Thus, the extended horizon of planning [3] activity is available to this category of an institutional investor that allows one to diversify risks.

From the point of view of development of mathematical model [4], these differences do not play a great role as only parameters of functions change, the essence remains the same. The main thing is that savings and payments of citizens, small investors are accumulated by institutional investors and that allows one to form considerable resources and to put in various profitable projects.

## III. FINDINGS

### A. Analysis of Activity of High Yield

Define abbreviations and acronyms the first time they are. The analysis of activity of High Yield segment allows differentiating them according to the presence term in the market. These are Fast HYIP which conduct activity for a very short time but provide to the investor a very big profitability on the brink of rational risk reaching 50% per day. Such projects are most similar to gamblings. The other type, Long term HYIP, provides a small percent; conditions of payments contain a delay, as a rule, for a week.

For modeling of activity of [5] such projects, we will enter a number of formalisms:

$i\%$  (interest rate) - value of a rate of profitability per investment;

$k$  - number of the period of payments of interest per investment,

$p$  - investment sum;

$F(k)$  - number of participants of the project;

$S(k)$  - size of change of total portfolio of the project in terms of money;

$P(k)$  - sum of money at the disposal of organizers of the project during the period  $k$ ;

$K$  - number of the periods or life expectancy of the project.

Here we should note an important distinction from the standard notations – the matter is that  $i$  in this case does not reflect an annual interest rate but only that taken for the period of payments which can vary in a very wide range. To have an opportunity to compare conditions of investments, it is necessary to apply a formula of calculation of an effective rate, the argument is the known nominal:  $EFFECTIVE(i * 360 / \omega; 360 / \omega)$ , where  $\omega$  - frequency of payments.

Under this condition, the basis counted by the American NASD method is accepted for a year.

In the beginning we will consider the simplest case of linear dependence  $F(k)$  for an illustration when in every period the constant number of investors  $M$  is involved in the project activity. We have the following dependence:  $S(k) = M * p - M * p * (k - 1) * i\%$ .

From a condition  $S(k) > 0$  of profitability of the project, it follows that the ratio  $k < 1 + i^{-1}$  has to be completed, or, having taken the whole part we have: what gives the chance to calculate period of validity of HYIP during which organizers get profit? For definition,  $P_{max}$  - the maximum sum of money which can be earned during activity of such project, we will have an expression:  $P(k) = \sum_{t=1}^k S(t) = \sum_{t=1}^k (M * p - M * p * (t - 1) * i)$ .

Having counted the progression sum, we have:  $P(k) = M * p * \left[ k - i \frac{k * (k - 1)}{2} \right]$  and having substituted value  $K$ , we will finally receive:  $P_{max} = M * p * \text{entier}(1 + i^{-1}) / 2$ . Thus, in the case under consideration, the difference between Fast and Long term HYIP is definitely seen as reduction of size of the paid percent  $i\%$ , the greatest possible income of organizers of the project increases. For example with monthly payment  $i = 5\%$  (that corresponds to 79,6% per annum)  $P_{max} = 10,5 * M * p$ , it is clear that we cannot reduce  $i$  strongly, first because of decrease of the attractiveness of money investment; second because such business cannot exist long because of existence of signs of a financial pyramid.

The reviewed example has been strongly simplified. In reality the need for general technique of modeling HYIP dynamics is caused by the big risks accompanying entry into such investment programs. It is difficult for an unprepared investor to define the degree of their stability and also the activity term determined by a point of zeroing of the profit of organizers called a scam point. Calculation (figure 01) for the linear model [6] gives values:  $P(K)$  and  $S(K)$ , the linear model,

profit, inflow of money, the number of participants, the number of participants HYIP, progression,  $k$  (period of payments).

In some cases such approach allows one to carry out the analysis of work of HYIP. Besides, thanks to a simple formula, it is possible to estimate difference between Fast and Long term HYIP at once. Nevertheless, in reality linear model can be seldom applied in the long horizon of planning. For the solution of this task it is expedient to take other techniques of modeling as a basis. First of all it is a widespread Verhulst equation [7]. In this research the number of the investors involved in the project during initial stages of payments was calculated in two ways:

- by geometrical progression set using denominator  $q$  and  $q > 1$ ;
- by exact solution of Verhulst equation  $F(k) = \theta F_0 e^{\gamma k} / [\theta + F_0(e^{\gamma k} - 1)]$  with parameters:  $F_0$  - the initial number of participants,  $\gamma$ - equivalent  $q$ ,  $\theta$  - an indicator of the number of potential participants of the project.

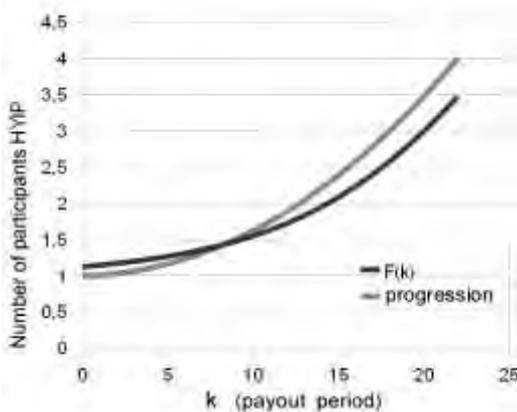


Fig. 1. The linear model.

Calculation (figure 02) of dynamics of change in the number of HYIP of investors executed by the computer has shown that with a considerable share of probability, at the initial stage, both approaches can be considered equivalent.

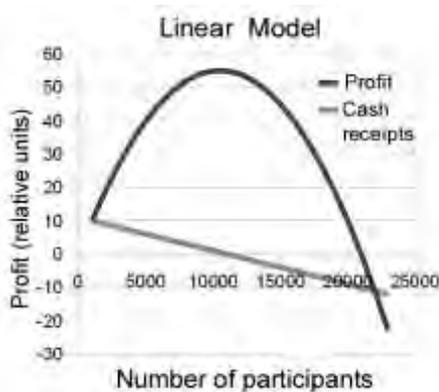


Fig. 2. Comparison of nonlinear methods.

It allows using summation of a geometrical progression when calculating that facilitates receiving analytical expressions. Then during the period  $k$ , money  $F_0 * p * q^{k-1}$  will be attracted. We will calculate how many percent during the period  $k$  we will have to pay to earlier attracted investors whose number for the  $k - 1$  periods will be equal to:  $F_0 \sum_{t=1}^{k-2} q^t$ . From a condition of profitability, it follows:

$$F_0 * P * q^{k-1} > i * p * F_0 \sum_{t=1}^{k-2} q^t \quad (1)$$

**B. Risk Degree Assessment on the Basis of Calculation of a Point of Profitability of HYIP Projects**

Entry into the risk projects relating to high yield segment online activity investor, as a rule, implies that the probability of loss of the capital is high. At the same time there is no clear technique of assessment of degree of risk. As universal [8] distribution of HYIP projects constantly increases, the tool in the form of mathematical model of their behavior will allow protecting investors from adoption of rash decisions. First of all, we need calculation of scam point that is time point when activity of the project stops and investors' funds are lost. Comparison of calculations for Verhulst equation [9] and geometrical progression with a good extent of approximation allowed receiving a condition (1) of profitability of the HYIP project. However, for the development of the recommendations useful to the investor, it is necessary to carry out the detailed analysis of this condition.

For this purpose, we will summarize geometrical progression in the right part of the condition (1) and we will receive the following ratio:  $i * p * M \sum_{t=1}^{k-2} q^t = i * p * M * \frac{q^{k-1}-1}{q-1}$ .

Having carried out cancellations, we will rewrite the profitability condition:

$$\frac{1}{q^{k-1}-1} > \frac{i}{q-q} - 1 \quad (2)$$

We will note that as inequalities  $q > 0$  and  $k > 1$  are satisfied, then the condition is true:  $q^{k-1} > 0$ . It follows from this that the left part of inequality (2) is positive. Then we will analyze the right part.

If inequality is satisfied:  $\frac{i}{q-1} - 1 < 0$  or, having rewritten the same in a form:  $i < q - 1$ , for this case we conclude that the condition of profitability is satisfied. It means that the gain of money (of new participants of the project) is more than the size of percent paid to investors and the sum of the money which is available in the HYIP project increases while the condition  $i < q - 1$  is satisfied.

However, the quantity of those interested in investment is limited and such analysis is true only at the initial stage. If  $i > q - 1$  then at great values of  $k$  the condition (2) is not satisfied because of the sharp increase of  $q^{k-1}$  in denominator of the left side. To obtain the result, we will solve inequality (2) in reference to  $k$ :  $q^{k-1} > i * \frac{q^{k-1}-1}{q-1}$  we will transform  $q^{k-1} > \frac{i}{q-1} q^{k-1} - \frac{i}{q-1}$ , from here it follows:  $q^{k-1} \left( \frac{i-q+1}{q-1} \right) < \frac{i}{q-1}$  that can be expressed as  $k - 1 < \log_q \left( \frac{i}{i-(q-1)} \right)$ .

Now it is possible to receive a final condition of profitability of the project:  $k < \frac{\ln \frac{i}{i-(q-1)}}{\ln q} + 1$ .

This expression allows calculating scam point from working conditions. [10] formulas received above give the chance to restore the general dynamics of work of the HYIP project according to the economic indicators. The matter is that in segments, average and high income payments, which, for example, are provided by Perfect Money, are carried out in real time. Though in the vast majority, such business has all signs of a financial pyramid and the considerable number of citizens suffered from the activity of real and virtual enterprises in fact constructed according to the Ponzi scheme there is a number of projects, including institutional ones which cornerstone is also the principle of a pyramid. Among the first ones, it is possible to name, for example, the pension fund, insurance institutes. It is necessary not only differentiate the available projects on the basis of potential fraud, but also to develop a clear technique of assessment of duration of their activity, calculation of financial performance and degree of risk of entry into such enterprises.

It allows one to receive necessary parameters of the model [11] and, respectively information on the current state and the degree of solvency of organizers. Such techniques first of all will be demanded by investors when planning the financial activity and also hedging of risks.

### C. Scope of Application of the Results

The received results allow applying this approach when modeling quite wide range of projects, including institutional. The mathematical model developed in this research can be applied first of all at long-term forecasting of activity of such a large institutional investor as the pension fund. As demographic processes have considerable lag effect, there is an opportunity to plan both the number of capable citizens, and birth rate for several decades ahead.

We will consider a technique of application of this model for the solution of a relevant problem of stability of pension system. In this case, it is necessary to unite data on demography, statistics of employment of the population and salary level. Besides, when forming mathematical model [12] of the institutional project, it is necessary to consider such major factors as a time log before retirement, in payments equal to  $\lambda$  of periods and nonlinear dependence  $F(k)$  of number of participants of the project where  $k$  - number of the period of payments stipulated by conditions of the percent of size  $i$ . For a start we will consider a technique of accounting of the characteristic  $\lambda$  as a casual parameter. Then, approximating growth of number of the participants having invested in the project during the period to number  $k$  we will receive that their number will be equal  $F_0 \sum_{i=1}^{k-2} q^i$ , where  $F_0$  - initial number of participants,  $q$  - progression denominator, and  $q > 1$ .

Further, during the period  $k$  payment should be made to investors; it is counted as the sum of the following row:  $ipF_0 + ipF_0q + ipF_0q^2 \dots + ipF_0q^{k-2-\lambda}$ , where  $p$  - the investment sum,  $\lambda$  - a time lag. To define a condition of profitability of the project, it is necessary to summarize this progression and to

compare with the amount of receipt during  $k$  of payments from investors.

As a result, we will receive a ratio:  $q^{k-1} \geq i * (q^{k-1-\lambda} - 1)/(q - 1)$ . It can be transformed as  $q^{k-1} \left[ i * \frac{q^{-\lambda}}{(q-1)} - 1 \right] \leq i(q - 1)^{-1}$ . It allows one to get the equation of profitability of the project:

$$k \leq 1 + \frac{\ln i - \ln [i/q^\lambda - (q-1)]}{\ln q} \quad (3)$$

We will note that, solving a ratio as an equality, we will receive value of a point of zeroing of the profit  $k^*$  called a scam point. We will carry out the analysis of inequality. As existence of a logarithm requires performance of a condition:  $i * q^{-\lambda} - (q - 1) > 0$ , that is  $\lambda < \frac{\ln i - \ln(q-1)}{\ln q}$  at violation of this inequality the financial pyramid disperses also when maintaining values  $q$  and  $i$  becomes always profitable. It means that if one pays profit to participants of a pyramid starting with the period  $\lambda$  more than  $\lambda^* = \frac{\ln i - \ln(q-1)}{\ln q}$  or, in other words, with a delay on *entier*( $\lambda^* + 1$ ) of the periods then while preserving conditions on growth of the number of investors and paid percent, the project brings constantly increasing income.

Using statistics on life expectancy of participants of the institutional project, it is possible to receive distribution function  $\Psi(\lambda)$  that reflects, for example, the pension fund duration of payments to pensioners. Then calculations for a formula (3) are carried out on the computer with the use of the device of probability theory.

To solve this problem, it is necessary to formalize the data of the main activity of the pension fund and to consider them as arguments of functions. For this purpose, the life of a modern individual can be simply divided into several phases:

- the period preceding the achievement of working age;
- the main period of employment, during which funds are accumulated;
- life expectancy at a pension, social or some other form of security.

At the same time, it should be noted that on the one hand, this approach is relevant only in the conditions of a post-industrial society, but at the same time it should be taken into account that it was at this phase that public institutions of pension provision emerged that fully correspond to the conditions of the developed mathematical model.

Data on demographics, disease statistics, and life expectancy form the Big Data arrays. This makes it possible to conduct statistical processing, restore coefficients in statistical laws and effectively predict the work of an institutional investor. In this case, due to large sample arrays, the distribution functions are determined with high accuracy. This minimizes risks. The analysis should be carried out with determining the level of readiness for the transition to work activities. Further, it is necessary to calculate the coefficient of demographic load that is equal to the ratio of the number of able-bodied people to the number of people who are not able to work among the population, which is covered by the relevant legislative acts

concerning pensions. The stability of such institution as a pension fund is determined by the balance of these indicators.

While processing statistical data, it is necessary to segment the source material by such characteristics as the socio-demographic landscape, regional differences (in such a large country as Russia there are both favorable regions and depressive ones). Next, the probability distribution function of the transition to the phase of the main work activity is calculated. For this purpose, the expressions obtained in biostatistics and common in actuarial mathematics are used:

$$P(x < X \leq z | X > x) = \frac{F_x(z) - F_x(x)}{1 - F_x(x)}$$

where  $x$  is the age,  $z$  is the average age of transition to work activity,  $F_x(x)$  is the survival function. We obtain the probability distribution of duration  $n$  of the third stage length from the analysis of the expression:

$${}_n p_x = \exp\left[-\int_x^{x+n} \mu(y) dy\right], \text{ where } f_x(x) = F'_x(x), \mu(x) = \frac{f_x(x)}{1 - F_x(x)}$$

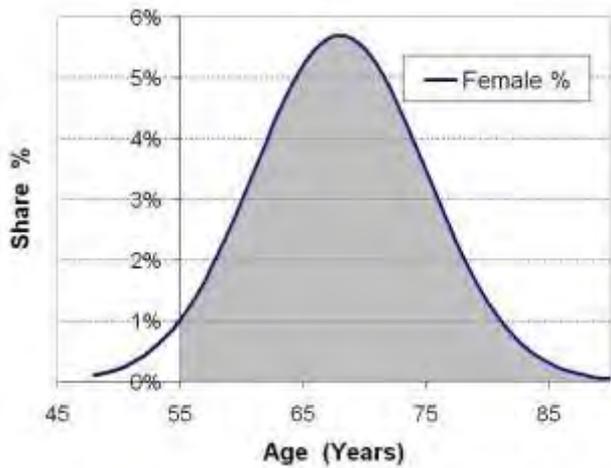


Fig. 3. Female age distribution.

As a result, we get the Euler-Poisson distribution parameters, and it allows constructing graphs on the computer, which visually interpret the ratio of the load on the pension fund. Since in Russia there are differences in the retirement age for men and women, as well as the various gender survival functions, the results for them are obtained separately.

In Figure 03 the curve represents the percentage of women in the third period, that is, above the legal age, from which pension payments are made.

Even from these graphs, it is clear that there is a significant difference in the conditions of citizens' provision depending on gender. Thus, the problem of determining institutional stability can be solved using the theory of probability.

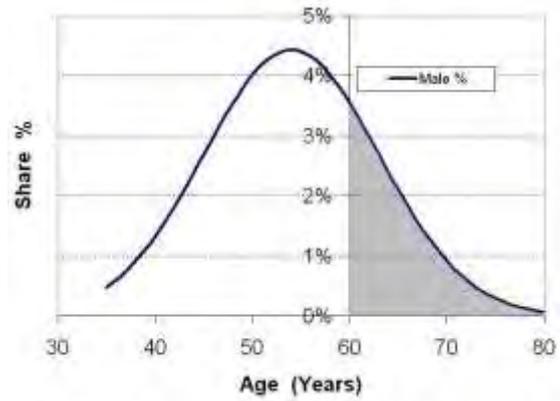


Fig. 4. Male age distribution.

#### IV. CONCLUSION

On the basis of the obtained data, it is possible to predict the condition of payments in social funds, financial stability not only of the pension, but also some other, for example, insurance funds.

The equations for insurance funds and organizations are used with that difference that not laws of distribution of time of survival, but functions describing probabilities of insured events are formed.

It should be noted that by results of inspection of The Sovereign Wealth Fund Institute (SWFI), the rating of the largest funds has been defined. It is remarkable that the pension fund belonging to the state of (Norway) is in the first place. The total capital of this pension fund amounts to about 900 billion USD. The scope of its activity is proved by the fact that the citizens of Norway invest through the fund in more than 9000 companies scattered in 75 leading states; at the same time the main part of the capital of the fund is invested in actions. Many countries which have endured a rapid growth actively develop institutional funds, so for example in People's Republic of China the national fund of social security which cumulative capital is estimated at the sum of more than 260 billion dollars is created. As SWFI carries out monitoring of direct transactions of sovereign funds, the sum of investments in state funds in the world is known; that exceeds 7 trillion USD. It is necessary to refer Linaburg-Maduell Transparency Index published in annual reports to advantages of information on public funds.

Other relevant direction of application of mathematical modeling of activity of institutional investors is the tendency to formation of network structures, primary use of information technologies, online operations. As these processes are correlated with increase in real income of the population in the developed countries of the world, the motivation to participate in programs of investment of savings of the population respectively increases. That in total with the increased financial literacy makes new demands to activity of institutional establishments.

And we respectively obtain the increase in possibilities of application of evidence-based techniques of forecasting of

stability of these establishments which activity has serious economic influence and big social value as it touches upon the interests of the vast majority of the population.

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