Observer Design for a Class of Systems with Mismatched Uncertainties via Variable Structure Control

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***Abstract*—A variable structure observer (VSO) design problem solved by VSC (variable structure control) theory for system together with mis-matched uncertainty is studied in this paper. In the sliding mode, if developed systems are with mis-matched uncertainties, then invariance condition cannot be satisfied. Hence, we endeavor to design a new VSO structure. Under this VSO, if matching condition is unsatisfied, then estimation error trajectories will be bounded uniformly, and these estimation error trajectories will asymptotically approach to zero if they satisfied matching condition. Finally, a set of simulation results for a helpful example is shown to illustrate the validity of this developed VSO way.**

Keywords—variable structure control; variable structure observer

I. Introduction

They all know that the closed-loop system trajectories under VSC law will be forced onto a sliding mode, and trajectories will be persisted in the sliding mode afterward [1]-[4]. System characteristic will be independent of matched uncertainty in sliding mode and will have invariance property. However, the system characteristic is not unchanging to the mismatched uncertainties if some mismatched uncertainties in developed system. Furthermore, when system achieved to the sliding mode, the order of the motion will be reduced. For these outstanding appearances, the VSC scheme is usually used to control for robust system and to deal with the existence of the model uncertainty.

Recently, the state-feedback approach has got a lot of interest to the stability problem for some types of uncertain system [3]-[4]. Nevertheless, practically speaking, part of uncertain system's components or uncertain system's states can be directly measured. Luenberger observer [5] can measure the states well while one knows the system dynamics. Based on other theory, for example, VSC theory, numerous investigators put forward the VSO design [6]-[10]. Since an additional non-linear dis-continuous term is presented to VSO, VSO is different from Luenberger observer. Because the dis-continuous term of VSO can be used to reject some inaccuracy between observer and system, and also can be used to reject some disturbances to system, while the perturbation to the system or the existence of plant uncertainties, Luenberg observer is not robust than VSO. Furthermore, control performance will usually be degraded by the presence of time delay, worse, instability of closed-loop system will usually be affected by time-delay, hence, stabilization problem of time-delay system is an interesting problem [3], [11] and [12]. However, in these papers, many applications of these results always restricted by these matching-conditions since their uncertainty has to satisfy matching condition. Hence, one is necessary to create a new VSO to treat these estimation problems for some time-delay systems with mis-matched uncertainties.

A new observer design problem via variable structure theory for time-delay systems with mis-matched uncertainties is studied in this document. It isn't easy and has seldom been studied for the time delay and uncertainty term which fulfill mismatched condition. Nevertheless, we endeavor to design a new VSO structure. Under this VSO, if matching condition is unfulfilled, then estimation error trajectories will be bounded uniformly, and these estimation error trajectories will asymptotically tend to zero if they fulfill matching condition. And a set of simulation results for a helpful example is shown to illustrate the validity of this developed VSO way.

II. System Descriptions

Let us consider some uncertain time delay dynamic systems together with mismatched uncertainty stated following

$$\dot{x}(t) = A_d x_d(t) + Ax(t) + Bu(t) + h(x, t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

$$x(t-d) = x_d(t) \text{ and } \theta(t) = x(t), \quad \text{for } -d \le t < 0 \tag{3}$$

here $u(t) \in R^m$, $y(t) \in R^k$ and $x(t) \in R^n$ indicate respectively control inputs, outputs, and state variables of the time delay mismatched uncertain dynamic system. Output matrix C can be assumed as $C = [I_k\ 0]$ without loss of generality. Matrices A, A_d, B and C are with suitable dimensions. Furthermore, function $\theta(t)$ indicates an initial function with continuous vector value. Symbol d represents a nonzero time delay term and the function $h(x, t) \in R^n$ can be considered as external perturbations to the system and the lumped perturbations in all of the plant. And the following assumption 1 and remark 1 will be helpful in the consecutive sections.

Assumption1:, Nominal system pair (A, C) is observable completely in the system (1), (2) and (3).

Remark 1: From Assumption 1, it is well-known that a matrix $K \in R^{nxk}$ can be found then the eigen-values of the following equation (4) locate at open left-half side.

$$\underline{A} = KC + A \tag{4}$$

In this paper, to represent the matched and mismatched uncertainties which existed in the developed system, the following equation (5) is decomposed as

$$h(x,t) = \begin{bmatrix} h_1(x,t) \\ h_2(x,t) \end{bmatrix} = Fh_m(x,t) + h_{um}(x,t) \tag{5}$$

where $h_m(x, t) \in R^k$ and $h_{um}(x, t) \in R^n$ are respectively the matched and mismatched parts of the developed system (1). Assuming that the matrix CF is nonsingular and matrix F is of advisable dimension. Furthermore, the mismatched parts $h_{um}(x, t)$ and matrix F are respectively decomposed as

$$h_{um} = \begin{bmatrix} 0 \\ h_2 - F_2 h_m \end{bmatrix}, F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{6}$$

And the following assumption 2 can be employed in succeeding sections.

Assumption2: A known non-negative constant β exists, and the matched part $h_m(x, t)$ satisfies the following condition.

$$\|h_m(x,t)\| \le \beta \tag{7}$$

III. The Proposed VSO Design

In this section, the important results of this work will be derived. The variable structure observer for system (1), (2) and (3), is proposed as the following equation (8).

$$\dot{\hat{x}}(t) = K[C\hat{x}(t) - y(t)] + A\hat{x}(t) + FL(t) + Bu(t) \tag{8}$$

where $\hat{x}(t)$ denotes the estimate of the system state $x(t)$, K and F are gain matrices, and the nonlinear term $L(t)$ will be took to guarantee the robust-ness contrary to the external disturbances to the system and the lumped uncertainties in the system. Moreover, the term $L(t)$ will be design as the following equation (9).

$$L(t) = -\frac{F^T C^T S}{\|F^T C^T S\|}\phi(t) \tag{9}$$

where $\phi(t) = \|(CF)^{-1} CA_d C^+\| \|y_d\| + \beta$. (10)

and $C^+ = C^T(CC^T)^{-1}$ indicates the generalized right inverse of C.

Let the following equation (11) denotes difference between the estimate and true states, then,

$$e(t) = -x(t) + \hat{x}(t) \tag{11}$$

And the following equation (12) is the designed switching surface $S(t)$

$$Ce(t) = S(t) = C\hat{x}(t) - y(t) \tag{12}$$

Deducing (1) from (8) generates the governing error systems as follows.

$$\dot{e}(t) = FL(t) - A_d x_d(t) - h(x,p,t) + (A+KC)e(t) \tag{13}$$

The Lemmas which shown in the follows must be built to guarantee the switching surface attraction and the sliding surface presence.

Lemma 1 [11]: If equation (14) is satisfied

$$S^T(t)\dot{S}(t) < 0 \tag{14}$$

Then the motion of the sliding surface is stable asymptotically.

Lemma2 [10]: A positive definite matrix $T \in R^{k \times k}$ is exactly exists, then

$$C\underline{A}C^+ = -T. \tag{15}$$

Now, we are going to establish a theorem as follows.

Theorem1: A class of time delay system with mismatched uncertainty stated in system (1), (2) and (3) is considered, and the proposed variable structure observer shown in (8). If one applied nonlinear term $L(t)$ defined in (9) to dynamical error system then attraction to sliding surface (12) and the sliding mode can be guaranteed.

Proof:

Choosing $V(t) = 0.5S^T(t)S(t)$ as the Lyapunov equation, then

$$\dot{V}(t) = S^T(t)\dot{S}(t) = S^T C\underline{A}e - S^T CA_d x_d + S^T CFL(t) - S^T Ch(x,p,t) \tag{16}$$

In this VSO design, it is going to use the un-measurable state e and x_d, that is,

$$e = C^{+}S \text{ and } x_d = C^{+}y_d.$$

Therefore, we have

$$\dot{V}(t) = S^{T}C\underline{A}C^{+}S - S^{T}CA_d C^{+}y_d + S^{T}CFL(t) - S^{T}Ch(x, p, t) \tag{17}$$

Then applying $\|AB\| \le \|A\|\|B\|$, (5), (7), (9), (10) and (15) to the equation above, we have

$$\dot{V}(t) \le -\lambda_{\min}(T)\|S\|^2 + \|S^{T}CF\|[(\|(CF)^{-1}CA_dC^{+}\|\|y_d\| + \|h_m\|] - S^{T}Ch_{um} - S^{T}CF\frac{F^{T}C^{T}S}{\|F^{T}C^{T}S\|}\phi(t) = -\lambda_{\min}(T)\|S\|^2 \tag{18}$$

where $\lambda_{\min}(T)$ indicates the positive minimum eigenvalue of T, one can see that $\dot{V}(t) < 0$. Then, attractiveness to sliding surface (12) and the sliding mode have been guaranteed.

IV. On the Subject of Mismatched Uncertainties

The effects of the mismatched uncertainties to the system (1), (2) and (3) will be shown in this section. Equations (19) is fulfilled on the sliding mode,

$$\dot{S}(t) = 0 \text{ and } S(t) = 0 \tag{19}$$

Therefore, the following equivalent control $L_{eq}(t)$ in the sliding mode may be got as

$$L_{eq}(t) = -(CF)^{-1}C[Ae - h(x, t) - A_d x_d] \tag{20}$$

Therefore, the error equivalent systems on the sliding surface will be

$$\dot{e}(t) = [I - F(CF)^{-1}C](Ae - A_d x_d) - h_{um}(x, p, t) \tag{21}$$

Since that the mismatched uncertainties, these systems are not with the invariant property.

If A_d, $e(t)$, A and x_d are separated respectively as follows,

$$A_d = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix},\ e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix},\ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ x_d = \begin{bmatrix} x_{d1} \\ x_{d2} \end{bmatrix} = \begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \end{bmatrix} \tag{22}$$

here $e_1(t) \in R^k$, $e_2(t) \in R^{n-k}$, and A_{ij}, A_{dij} are of advisable dimensions.

Thus (21) may be rewritten as

$$e_1(t) = 0 \tag{23}$$

$$\dot{e}_2(t) = Me_2 - (M_1x_{d1} + M_2x_{d2}) - (h_2 - F_2h_m) \tag{24}$$

where $M = A_{22} - F_2F_1^{-1}A_{12}$, $M_1 = A_{d21} - F_2F_1^{-1}A_{d11}$, $M_2 = A_{d22} - F_2F_1^{-1}A_{d12}$ (25)

To study the maximum error bound of $e_2(t)$, the Lyapunov function $V(e_2)$ can be chosen as follows.

$$V(e_2) = e_2^{T} P e_2 \tag{26}$$

Here, matrix $P \in R^{(n-k)\times(n-k)}$ is positive definite. More, assuming another matrix Q is positive definite, and

$$M^{T}P + PM = -Q \tag{27}$$

The derivative of $V(e_2)$ is shown as following

$$\dot{V}(e_2) \le -\lambda_{\min}(Q)\|e_2\|^2 + 2\|e_2\|\|P\|\,\|h_2 - F_2h_m + M_1x_{d1} + M_2x_{d2}\| \tag{28}$$

where $\lambda_{\min}(Q)$ indicates the mini-mum positive eigen-value of matrix Q.

Therefore, as in steady condition, the maximum error bound $\|e_2\|_{\max}$ can be got as

$$\|e_2\|_{\max} = \frac{2\|P\|\|h_2 - F_2h_m + M_1x_{d1} + M_2x_{d2}\|}{\lambda_{\min}(Q)}. \tag{29}$$

V. ILLUSTRATED EXAMPLE

The proposed VSO design approaches are going to apply to the following equation (30) in this section.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 3\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x_{d1}\\ x_{d2}\end{bmatrix}+\begin{bmatrix}0\\ -\sin x_1\end{bmatrix} \quad (30)$$

$$y=[1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad (31)$$

and the starting value $[x_{10}\ \ x_{20}]^T = [1\ \ -1]^T$ can be chosen arbitrarily. A gain matrix $K = [-1\ \ 0]^T$ exists, therefore, $\{-2, -2\}$ are the eigen-values of matrix $\underline{A} = KC + A$.

As well, the matched perturbation $h_m(x, t)$ will be zero if matrix F is designed as $F = [1\ \ F_2]^T$. Thus, the observer system homologous to (30) is given as

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix}=\begin{bmatrix}-2 & 3\\ 0 & -2\end{bmatrix}\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}-\begin{bmatrix}-1\\ 0\end{bmatrix}y+\begin{bmatrix}1\\ F_2\end{bmatrix}L(t) \quad (32)$$

Switching surface is derived as

$$S(t) = -y + \hat{x}_1$$

For this illustrated example, the following numerical values are chosen:

$$\beta = 2,\ d = 2|\sin(t)| \text{ and } F_2 = 2.$$

The time responses of system states $x_1(t)$ and $x_2(t)$ are performed in Figure I (a). Figure I (b) indicates the time reactions of estimated states $\hat{x}_1(t)$ and $\hat{x}_2(t)$. Figure I (c) indicates error states $e_1(t)$ and $e_2(t)$. One may see that all the estimated errors are stable asymptotically from these simulation results.

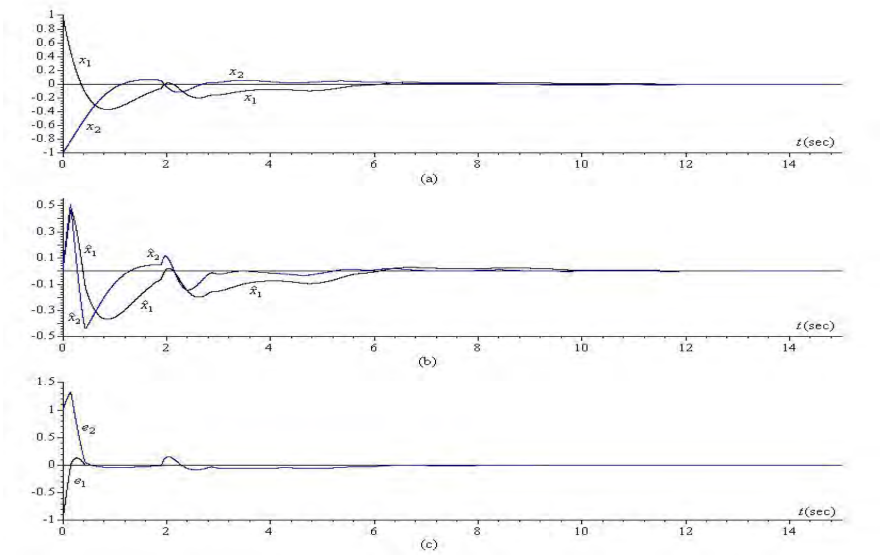

FIGURE I. TIME RESPONSES OF (A) SYSTEM STATES $x_1(t)$ AND $x_2(t)$. (B) ESTIMATED STATES $\hat{x}_1(t)$ AND $\hat{x}_2(t)$. (C) ERROR STATES $e_1(t)$ AND $e_2(t)$

VI. CONCLUSIONS

A new variable structure observer designing problem had been discussed for some time-delay systems which are with mis-matched uncertainty in the paper. We endeavor to design a new variable structure observer structure. Under this developed variable structure observer, if matching condition is unsatisfied, then estimation error trajectories will be bounded uniformly, and these estimation error trajectories will asymptotically tend to zero if they satisfied matching condition. At last, a set of simulation results for helpful numerical example are shown to illustrate validity to this developed variable structure observer way.

ACKNOWLEDGEMENTS

The authors appreciate Editor, Associated Editor and the incognito reviewers for these innovative recommendations to heighten this paper. Furthermore, the research was sustained by the Ministry of Science and Technology of Taiwan, under agreement MOST 105 – 2221 – E – 252 – 001 – and MOST 107– 2221 – E – 252 – 004 –.

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