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Vibration Characteristics Analysis of Orthogonal Stiffened Thin Plate with the Viscoelastic Damping Topology Distribution

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***Abstract*—Dynamic characteristic of stiffened thin plate structure is superior to that of conventional thin plate structure, but it continues to suffer from the problem of less damping and large vibration sound radiation. The vibration noise of thin plate structure can be effectively reduced by damping treatment. In this paper, the dynamic model and eigenvalue equation of orthogonal thin plate structure with damping are established. Aiming at the rectangular reinforced thin plate with simply supported boundary, the influence of damping thickness and damping distribution on the natural frequency is studied. The relationship between damping thickness, damping distribution, position of excitation point and vibration response characteristics is obtained by vibration harmonic response analysis, and the topological structure of damping layer distribution is analyzed and calculated.**

Keywords—orthogonal stiffened thin plate; damping treatment; topological structure; vibration harmonic response analysis

I. INTRODUCTION

Orthogonal stiffened structure is a commonly used form of a thin plate structure, which is widely used in engineering because of its superior stiffness characteristics compared with conventional flat plate structure. Conventional metal thin plate structure usually has less damping and larger vibration acoustic radiation, which directly affects the dynamic characteristics and vibration acoustic radiation of machine and equipment. The vibration noise of thin plate structure can be effectively reduced by damping treatment. Some important progress has been made in modeling the dynamics of damping composite structures, analyzing the characteristics of viscoelastic damping layer structures and optimizing the layout of damping layer structures[1][2][3].However, there are few researches on damping treatment and vibration characteristics of stiffened plate. Therefore, this paper takes damping stiffened plate as the research object and studies the influence of damping parameters and distribution on the natural frequency of stiffened thin plate structure. On this basis, harmonic response analysis is carried out to analyze the vibration response characteristics of damping stiffened plate. Furthermore, the damping topology of stiffened plate structure is designed and analyzed so as to provide a basis for reducing vibration and noise of thin plate structure through damping treatment.

II. DYNAMIC MODEL OF DAMPING STIFFENED THIN PLATE

The governing differential equation for the forced vibration of a orthogonal anisotropic damping Plate is given by

$$D\nabla^4 w(x,y,t) + C_d \frac{\partial w(x,y,t)}{\partial t} + \rho_d h \frac{\partial^2 w(x,y,t)}{\partial t^2} = f(x,y,t) \tag{1}$$

Where

$$D\nabla^4 = D_x \frac{\partial^4}{\partial x^4} + D_y \frac{\partial^2}{\partial y^4} + 2\overline{D}_{xy} \frac{\partial^4}{\partial x^2 \partial y^2} \tag{2}$$

D_x and D_y are respectively the flexural rigidity in X and Y direction. $\overline{D}_{xy} = D_0 + 2D_{xy}$,where D_0 is a parameter related to the Poisson ratio of the plate, that is $D_0 = \mu E h^3/12(1-\mu^2)$. D_{xy} is the torsional stiffness of the reinforced composite plate. $w(x,y,t)$, C_d, ρ_d and h are respectively the flexural displacement, the mass density and the thickness of the plate. $f(x,y,t)$ is the applied load on the plate.

If the composite thin plate of orthogonal plate is obtained on the basis of isotropic uniform plate by adding longitudinal and transverse reinforcement, and the differential operator of equation (2) can be obtained by referring to [1], it can be expressed as:

$$\begin{aligned} D\nabla^4 = {} & \left(D + \frac{E}{b}\sum_{i=1}^{s} I_i Y_{ni}^2\right)\frac{\partial^4}{\partial x^4} + \left(D + \frac{E}{a}\sum_{j=1}^{r} I_j Y_{mj}^2\right)\frac{\partial^2}{\partial y^4} \\ & + 2\left(D_0 + \left((1-\mu)D + \frac{G}{2}\frac{\sum_{i=1}^{s} J_i (Y'_{ni})^2}{\int_0^b (Y'_n)^2 dy}\right) + \frac{G}{2}\frac{\sum_{i=1}^{s} J_j (X'_{mj})^2}{\int_0^a (X'_m)^2 dx}\right)\frac{\partial^4}{\partial x^2 \partial y^2} \end{aligned} \tag{3}$$

The equivalent density of the stiffened plate is as follows.

$$\rho_d = \rho + \rho_z + \frac{1}{b}\sum_{i=1}^{s} \gamma_i Y_{ni}^2 + \frac{1}{a}\sum_{i=1}^{r} \gamma_j X_{mj}^2 \tag{4}$$

E and D are the elastic modulus and stiffness of the plate without stiffener respectively. $G = E/2(1+\mu)$ is the shear modulus of the structure. I_i and s are respectively the moment of inertia and the number of bars of the ith stiffener in the x direction. I_j

and *r* are the moment of inertia and the number of bars of the *j*th stiffener in the y direction respectively. J_i and J_j are the polar moment of inertia of the *i*th and *j*th reinforcing bars respectively. X_m and Y_n are the vibration mode functions of *x* and *y* direction bars respectively. *a* and *b* are the lengths of rectangular plates in the x and y directions respectively. ρ_d, ρ and ρ_z are respectively the average equivalent surface density of reinforced plates, the surface density of polished plates and the equivalent surface density of damping materials. (See Figure I)

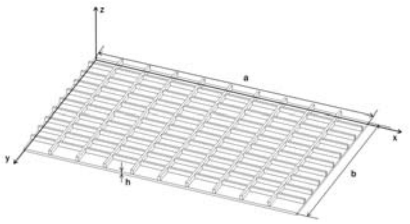


FIGURE I. REINFORCED RECTANGULAR THIN PLATE STRUCTURE

The eigenvalue equation of formula (1) is

$$\mathrm{D}\nabla^4 W_{mn}(x, y) = \rho h \omega_{mn}^2 W_{mn}(x, y) \tag{5}$$

In the formula, ω_{mn} is the (m, n) order modal frequency, and $W_{mn}(x,y)$ is the modal vibration mode.For the simple supported orthogonal anisotropic plate, the vibration mode function and the modal frequency are as follows (6) and (7) respectively.

$$W_{mn}(x, y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{6}$$

$$\omega_{mm}^2 = \frac{\pi^4}{\rho_d h}\left(\mathrm{D} + \frac{E}{\mathrm{b}}\sum_{i=1}^{s} I_i Y_{ni}^2\right)\left(\frac{m}{a}\right)^4 + \left(\mathrm{D} + \frac{E}{\mathrm{a}}\sum_{j=1}^{r} I_j Y_{mj}^2\right)\left(\frac{n}{b}\right)^4 + 2\left(D_0 + \left((1-\mu)D + \frac{G}{2}\frac{\sum_{i=1}^{s} J_i (Y'_{ni})^2}{\int_0^b (Y'_n)^2 dy}\right) + \frac{G}{2}\frac{\sum_{i=1}^{s} J_j (X'_{mj})^2}{\int_0^a (X'_m)^2 dx}\right)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 \tag{7}$$

The modal mode and frequency of the system can be obtained by using formula (6) and (7). For a given excitation source, the vibration response can be obtained by solving the formula (1). The vibration acceleration level is calculated by formula (8).

$$L_a = 20\lg\frac{a}{a_0} \tag{8}$$

where $a_0 = 10^{-6} m/s^2$.

III. THE INFLUENCE OF DAMPING PARAMETERS ON THE INHERENT CHARACTERISTICS OF STIFFENED PLATE

A. Damping Stiffened Thin Plate Structure

For stiffened thin plate structure, the damping layer will affect the quality and stiffness of the plate structure, thus affecting its inherent characteristics. In this section, on the basis of isotropic uniform plates, nine stiffened plates are added in the direction of length and width. The rectangular plates are divided into 100 small rectangular plates of equal parts, and damping materials are pasted on the surfaces of 100 small rectangular plates to form damping stiffened plates. The material parameters of damping stiffened plates are given in Table I.

TABLE I. MATERIAL PARAMETER TABLE OF DAMPING REINFORCED PLATE STRUCTURE

Materials	E/MPa	μ	$\rho/kg\cdot m^{-3}$	η
Rectangular Plate	2.06×10^5	0.3	7850	0.02
Reinforced ribs	2.06×10^5	0.3	7850	0.02
Damping Materials	3.5×10^3	0.49	1500	0.4

B. Effect of Damping on Natural Frequency of Stiffened Thin Plate

Under the simple supported boundary condition, the reinforced thin plate with a thickness of 0mm (no damping material is added), 1mm, 2mm, 3mm, etc are respectively pasted into the small rectangular block separated by the reinforcing ribs. Through numerical simulation and modal analysis, the first six natural frequencies of the rectangular plate (denoted as $f_1 - f_6$) can be obtained, as shown in Table II.

TABLE II. VARIATION OF NATURAL FREQUENCY OF THE FIRST 6 ORDER OF THE THICKNESS DAMPING REINFORCED PLATE OF DIFFERENT DAMPING LAYER

Damping Layer Thickness/mm	f_1/Hz	f_2/Hz	f_3/Hz	f_4/Hz	f_5/Hz	f_6/Hz
0	184.52	369.64	605.72	719.08	726.05	1007.6
1	182.25	367.57	597.41	716.29	719.52	997.58
2	180.91	364.71	592.75	710.28	714.37	990.39
3	180.02	362.63	589.33	705.52	710.96	985.55
4	179.62	361.44	587.35	702.2	709.53	983.39
5	179.78	361.25	586.96	700.49	710.26	984.1
6	180.57	362.17	588.32	700.55	713.37	987.98
7	182.09	364.36	591.74	702.63	719.21	995.45
8	184.31	367.81	597.19	706.76	727.7	1006.4
9	187.28	372.58	604.77	713.02	738.93	1020.9
10	190.9	378.5	614.17	721.17	752.46	1038.3

It can be seen from the data in Table II that, with the change of the thickness of damping layer, the natural frequencies of the first 6 orders of the damping stiffened plate have the same trend of change. Here, the first natural frequencies and the second natural frequencies of the damping stiffened plate are plotted as curves, as shown in Figure II and Figure III.

It can be concluded from the synthesis of Figure II, Figure III and Table II that the natural frequency of the first 6 orders of the damping stiffened plate shows a trend of decreasing first and then increasing with the change of the thickness of the damping layer. In other words, there is a damping layer thickness with the lowest natural frequency. After that, with the increase of damping layer thickness, the natural frequency of damping stiffened thin plate increases further.

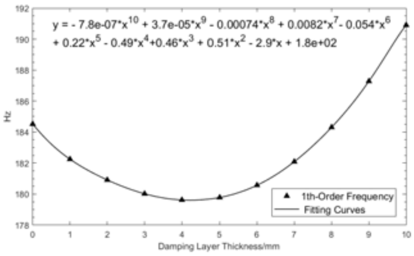


FIGURE II. THE FIRST ORDER NATURAL FREQUENCY CURVE OF DAMPING STIFFENED PLATE WITH DIFFERENT DAMPING LAYER THICKNESS

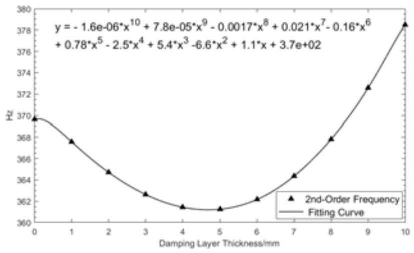


FIGURE III. THE SECOND ORDER NATURAL FREQUENCY CURVE OF DAMPING STIFFENED PLATE WITH DIFFERENT DAMPING LAYER THICKNESS

C. *Influence of Damping Layer Position Distribution*

The influence of damping layer on the inherent properties of stiffened sheet is different with the distribution of damping layer. For the model of transverse and longitudinal stiffened plate (s,r) = (9,9), damping plates are pasted in part areas of the 100 small rectangular blocks divided by the stiffened plate. Its damping layout is shown in the green part shown in Figure IV, in which 16 damping slices are pasted in the upper three graphs and 36 damping slices are pasted in the lower three graphs.

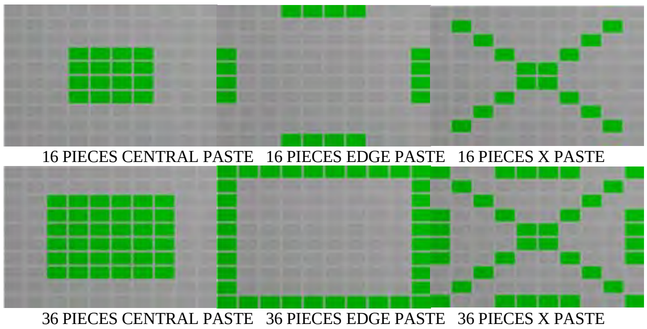


FIGURE IV. SCHEME DIAGRAM OF DISTRIBUTION POSITION OF DAMPING MATERIALS

The first 6 natural frequencies of damping stiffened plate shown in Figure IV are calculated by simulation, and the results are shown in Table III.

TABLE III. THE FIRST 6 NATURAL FREQUENCIES OF DAMPING STIFFENED PLATE WITH DIFFERENT DAMPING LAYER DISTRIBUTIONS

Damping layer distribution position	f_1 / *Hz*	f_2 / *Hz*	f_3 / *Hz*	f_4 / *Hz*	f_5 / *Hz*	f_6 / *Hz*
16 pieces damping in the center	181.11	368.43	599.11	717.36	725.4	1005.4
16 pieces damping at the edge	183.96	371.17	602.9	722.01	727.11	1007.4
16 pieces damping along the X-type	182.38	368.59	599.25	718.12	720.22	1001.2
36 pieces damping in the center	179.51	363.51	591.66	711.16	716.99	997.92
36 pieces damping at the edge	184.61	371.77	603.07	722.01	726.89	1006.1
16 pieces damping along the X-type	182.56	368.94	599.23	717.04	721.58	1002.2

It can be seen from Table III that, under the condition that the number of damping layers remains unchanged, the position distribution of damping layer has a certain impact on the natural frequency of reinforced thin plate. When damping material is distributed at the edge of the structure, the natural frequency of each order is relatively higher, and the central concentrated damping natural frequency is lower. Different position of damping layer will influence the stiffness change of plate structure and the natural frequency of plate structure to some extent.

IV. Harmonic Response Analysis of Damping Stiffened Thin Plates

The stiffened plate shown in Figure V adopts the boundary condition of simply supported four sides. The excitation force is 10N, the frequency is 50Hz-1100Hz, and the excitation point is the middle part of the bottom of the rectangular plate. The influence of the thickness of the damping layer and the distribution position of the damping material on the vibration response of the stiffened plate is studied by simulation calculation. Figure V shows the distribution of one of the damped stiffened plates. The restraint and force of the other damped stiffened plates are the same.

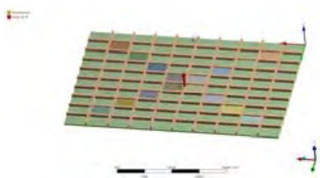


FIGURE V. CONSTRAINTS AND FORCE DIAGRAMS OF HARMONIC RESPONSE ANALYSIS MODEL FOR DAMPED STIFFENED PLATES

A. *Effect of Thickness of Damping Layer on Vibration Response of Plate*

When the thickness of the damping layer is 0 mm, 1 mm, 2 mm, 3 mm, 4 mm, 5 mm, 6 mm, 7 mm, 8 mm, 9 mm and 10 mm respectively, the RMS value of acceleration and its spectrum in Z direction (direction of force) are obtained by harmonic response analysis, as shown in Table IV and Figure VI-VIII.

TABLE IV. RMS ACCELERATION VALUE OF DAMPED STIFFENED PLATES WITH DIFFERENT THICKNESSES OF DAMPING LAYERS

Damping Layer Thickness (mm)	0	1	2	3	4
Root mean square value (dB)	123.93	123.35	123.22	123.02	122.80
Damping Layer Thickness (mm)	6	7	8	9	10
Root mean square value (dB)	122.29	122.05	121.77	121.46	120.85

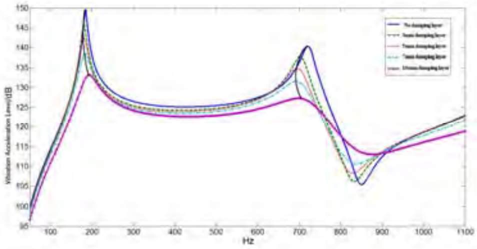


FIGURE VI. ACCELERATION HARMONIC RESPONSE CURVES OF DAMPED STIFFENED PLATES WITH DIFFERENT THICKNESSES

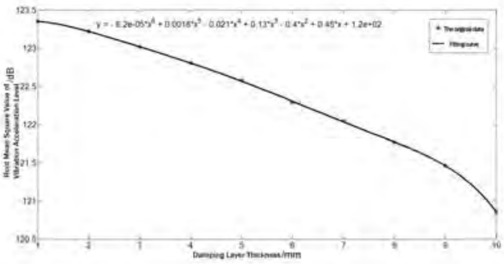


FIGURE VII. RMS CURVE OF VIBRATION ACCELERATION LEVEL OF DAMPED STIFFENED PLATE WITH DIFFERENT THICKNESS OF DAMPING LAYER

B. *Influence of Excitation Point Position on Plate Vibration Response*

Analyze the influence of different excitation points on the vibration of damped stiffened panels, under the same constraints, excitation force and frequency, the influence of excitation points on the vibration response of damped stiffened panels is studied by pasting viscoelastic dampers with a thickness of 5 mm on the surface of stiffened panels, as shown in Figure VIII.

From Figure VIII, it can be seen that different locations of excitation points have obvious influence on the amplitude of the damped stiffened plate. The maximum amplitude of vibration occurs at the middle of the structure, far from the constraint boundary of the excitation. At the same time, the vibration response of stiffened plate near the corner of the structure is larger than that near the edge of the structure. The possible reason is that there are more dampers near the edge, so the energy absorbed by the edge is slightly larger.

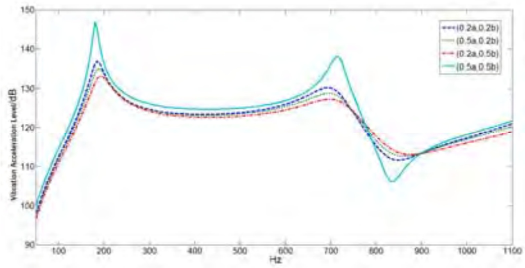


FIGURE VIII. ANALYSIS CURVE OF ACCELERATION HARMONIC RESPONSE OF DAMPED STIFFENED PLATES WITH DIFFERENT EXCITATION POSITIONS

C. *Effect of Damping Distribution Position on Vibration Response of Plate*

Investigate the influence of the distribution of the damping layer on the vibration response of the stiffened plate, the 16 damped plates shown in Figure IV are arranged according to the middle, edge and X-shape. The constraints, excitation force, frequency and excitation point position of harmonic response analysis are the same as before. The simulation results are shown in Table V.

TABLE V. PEAK POINT AND RMS VALUE OF ACCELERATION HARMONIC RESPONSE ANALYSIS OF STIFFENED PLATES WITH DIFFERENT DAMPING LAYER DISTRIBUTIONS

Distribution of Damping Layer	***16 pieces damping in the center***	***16 pieces damping at the edge***	***16 pieces damping along the X-type***
The first peak amplitude (dB)	147.51	148.95	148.14
The second peak amplitude(dB)	138.93	139.61	138.11
Root mean square value(dB)	123.02	123.80	123.42

From Table V, it can be seen that the amplitudes of the two peak points of the edge arrangement damping are the largest. For the first peak value, the amplitude of the X-shaped arrangement damping is next, and the middle arrangement damping is the smallest. For the second peak point, the amplitude of the middle arrangement damping is next, and that of the X-shape arrangement damping is the smallest. From the root mean square value of vibration, the amplitude of the middle arrangement damping is the smallest, the edge arrangement damping is the largest, and the X arrangement is between the two. Figure IX and Figure x are vibration response curves of undamped stiffened plates and modal shapes of corresponding two peak points.

The lowest group of the first peak value of the vibration response is the damped stiffened plate with damping material distributed in the middle, and this group of dampers just stick to the peak position of the vibration mode at the first peak frequency of the vibration response. Because the first peak of the vibration response is larger than the second peak, the root mean square value of the overall vibration acceleration level of the stiffened plate with distributed damping material in the middle is the smallest.

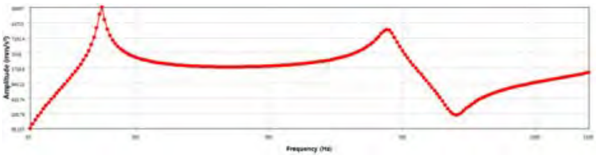


FIGURE IX. ANALYSIS CURVE OF VIBRATION HARMONIC RESPONSE OF UNDAMPED STIFFENED PLATES

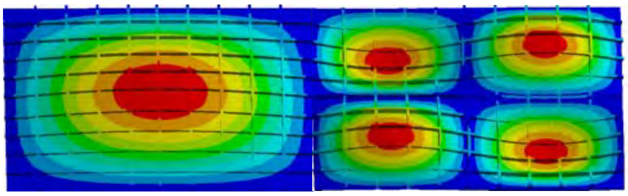
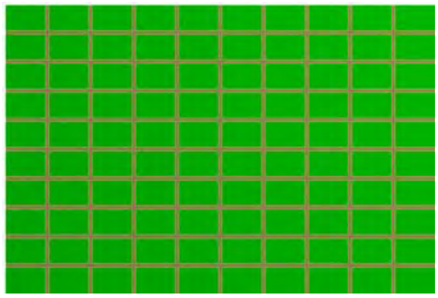


FIGURE X. FIRST-ORDER AND SECOND-ORDER MODE SHAPES CORRESPONDING TO PEAK FREQUENCIES AT PEAK VIBRATION POINTS

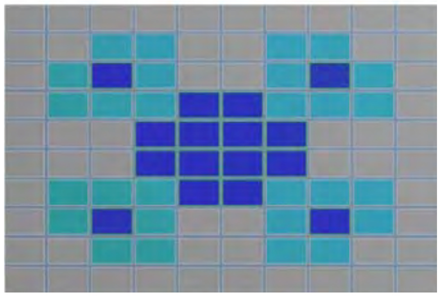
The lowest group of the first peak value of the vibration response is the damped stiffened plate with damping material distributed in the middle, and this group of dampers just stick to the peak position of the vibration mode at the first peak frequency of the vibration response. Because the first peak of the vibration response is larger than the second peak, the root mean square value of the overall vibration acceleration level of the stiffened plate with distributed damping material in the middle is the smallest. For the case of X-shaped distributed damping materials, because the damping covers the position of the first peak and the second peak corresponding to the modal frequencies, it has obvious vibration suppression effect on the first and second peaks, especially the X-shaped distribution is the only one covering the second peak modal frequencies in the three groups. For the mode position, the vibration of the second peak frequency is also effectively controlled. For the case where the damping material is distributed on the edge, the acceleration level at the two peak frequencies is the largest due to the staggered mode shapes corresponding to the two peak frequencies of the vibration response. It can be seen from the above that the bonding position of the damping material is closely related to the natural frequency and mode shape of the plate structure on the one hand, and to the excitation frequency and position on the other hand. Only when the modal shape corresponding to the peak frequency of the vibration response is bonded with the damping material, the better vibration suppression effect can be achieved.

V. TOPOLOGICAL OPTIMIZATION ANALYSIS OF DAMPING LAYER

Based on the analysis of the influence of the distribution position of the damping layer on the vibration of the stiffened plate structure, the topological structure of the distribution form of the damping layer is designed and optimized, as shown in Figure X. Among them, figure A shows the distribution of damping layer before optimization, and 100 damping materials (green part) with thickness of 3 mm are evenly pasted in the gap between stiffeners of stiffened panels. Figure B is the optimized topological distribution of the damping layer. There are 44 dampers. Among them, 16 dampers (dark blue position) with 10 mm thickness are pasted according to the larger peak values of the two modes corresponding to the peak frequencies of the stiffened plates, and the location of the lower modes with larger peak values is pasted preferentially. The remaining 28 damper plates (light blue part) with a thickness of 5 mm are pasted on the positions where the peak values of first and second modes are smaller.



A UNIFORM DISTRIBUTION FORM OF DAMPING LAYER BEFORE OPTIMIZATION



B TOPOLOGICAL DISTRIBUTION OF OPTIMIZED DAMPING LAYER

FIGURE XI. DISTRIBUTION DIAGRAM OF DAMPING LAYER TOPOLOGICAL STRUCTURE BEFORE AND AFTER OPTIMIZATION

The damping materials used before and after the optimal distribution are the same, and the constraints and loads are the same. The comparison of vibration response before and after optimization of damping layer distribution is shown in Figure XII, and the root mean square values of peak vibration and vibration response before and after optimization are shown in Table VI.

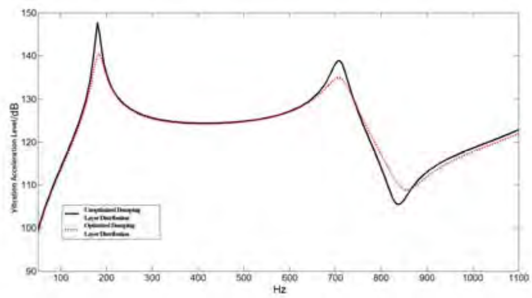


FIGURE XII. CONTRAST CURVE OF VIBRATION RESPONSE BEFORE AND AFTER OPTIMIZING DAMPING LAYER DISTRIBUTION

As can be seen from Figure XI and Table VI, compared with the uniform distribution of the damping layer before optimization, the effect of the topological distribution of the damping material pasted at the peak positions of the two modal modes contributing to the vibration peak is obvious. The first peak amplitude and the second peak amplitude are both lower than the uniform distribution. From the whole RMS value of vibration acceleration level, the optimized value is also lower than that before optimization.

TABLE VI. PEAK VALUE AND ROOT MEAN SQUARE VALUE OF ACCELERATION HARMONIC RESPONSE ANALYSIS OF STIFFENED PLATE BEFORE AND AFTER OPTIMIZATION OF DAMPING LAYER DISTRIBUTION

Distribution of Damping Layer	*No Optimal Distribution Form*	*Optimal distribution form*
The first peak amplitude (dB)	147.69	140.39
The second peak amplitude(dB)	138.79	134.97
Root mean square value(dB)	122.53	123.22

VI. CONCLUSION

(1) The thickness of the damping layer affects the natural frequency of the stiffened thin plate. There exists a natural frequency corresponding to the minimum thickness of the damping layer within a certain thickness range of the damping layer. When the number and thickness of the damping plates are fixed, the location and distribution of the damping layer have relatively small influence on the natural frequency of the stiffened thin plate.

(2) The thickness of the damping layer affects the energy consumption of vibration. Under the same excitation force, the vibration response decreases with the increase of the thickness of the damping layer. The position distribution of the damper mainly affects the peak vibration, but has little effect on the total vibration level of the plate.

(3) Damping topology design based on modal and peak value of vibration response is an effective method to reduce the vibration of stiffened plate structure. It can maximize the effect of damping and energy dissipation of stiffened thin plate structure and control the vibration of plate structure.

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