

# *Mathematical Modeling of Seismic Vibrations of System: Tailings Dam and Soil Foundations*

Muzaev I.D.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Kharebov K.S.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Muzaev N.I.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Kozyrev E.N.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Makiev V.D.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Gabaraev A.F.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Morozov F.S.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

Chitishvili M.I.

Geophysical Institute - the Affiliate of Vladikavkaz Scientific  
Centre of the Russian Academy of Sciences  
Vladikavkaz, Russia  
cgi\_ras@mail.ru

**Abstract**—A mathematical model of the system of seismic vibrations, which consists of the flood dike of tailing dump, deposit material (tailings) and bottom ground layers, is developed. The model is represented as a contact boundary problem for the differential equation of shear-viscous lateral vibrations of the flood dike body with deposit materials and also for the differential equations of the shear-viscous lateral vibrations of the ground layers. These equations are interconnected through the boundary conditions on the contact surfaces. The boundary problem is solved analytically. Calculation formulas for computation of displacements, velocity and acceleration of the flood dike body during the propagation of incident to the system seismic wave in the ground layers and in the flood dike body are obtained.

**Keywords**—mathematical modeling; tailing dump; flood dike

## I. INTRODUCTION

Tailing dumps of mining waste (which are usually represented as a significant amount of toxic substances that pose a direct threat to the population) pose a particular potential hazard during an intensive influence of precipitation, landslides and strong seismic effects in mountainous regions [1, 2]. It assumes the creation of facilities for engineering protection of the territory and selection of areas with certain soil conditions

(relief, engineering-geological structure, type of soil layers, watering, etc.) in order to exclude various hazardous phenomena, including anthropogenic disasters [3, 4]. A detailed study of the problem allows assessing adequately possible environmental contamination of the territory [5, 6] under the conditions of a stress-strain state of rocks. On the basis of a deep study of the situation, it allows managing the risk by selecting the appropriate site for the future tailing dump and developing special measures to exclude any abnormal situations, up to the subsequent rehabilitation of the site. Actually, this is the basis of rational subsoil management [7]. Therewith the study of all elements of the “structure”, including the natural soil massif or mountain massif with an artificial dike (wall, etc.) in their natural occurrence is of particular importance [8-19].

## II. MATHEMATICAL MODEL OF THE SYSTEM

A quite complex engineering system consisting of the flood dike of tailing dump, deposit material (tailings) and bottom ground layers is considered below. Figures 1a and 1b show a schematic drawing and a design diagram of a system consisting of the flood dike of tailing dump, deposit material (tailings) and two bottom ground layers undergoing transverse vibrations caused by a transverse seismic wave falling on the lower layer.

The following contact boundary problem of mathematical physics [20-25] is represented by the mathematical model of vibrations of the entire system:

$$\rho B \frac{\partial^2 w}{\partial t^2} - G \frac{\partial}{\partial x} \left( B \frac{\partial w}{\partial x} \right) - G \beta^* \frac{\partial}{\partial x} \left( B \frac{\partial^2 w}{\partial x \partial t} \right) = 0, \quad 0 < x < L, \quad (1)$$

$$\rho_1 \frac{\partial^2 u_1}{\partial t^2} - G_1 \frac{\partial^2 u_1}{\partial x_1^2} - \beta^* G_1 \frac{\partial^3 u_1}{\partial x_1^2 \partial t} = 0, \quad 0 < x_1 < H, \quad (2)$$

$$\rho_2 \frac{\partial^2 u_2}{\partial t^2} - G_2 \frac{\partial^2 u_2}{\partial x_2^2} - \beta^* G_2 \frac{\partial^3 u_2}{\partial x_2^2 \partial t} = 0, \quad 0 < x_2 < \infty, \quad (3)$$

$$\left. \frac{\partial w}{\partial x} \right|_{x=0} = 0, \quad (4)$$

$$w(x, t)|_{x=L} = u_1(x_1, t)|_{x_1=0}, \quad G \left. \frac{\partial w}{\partial x} \right|_{x=L} = G_1 \left. \frac{\partial u_1}{\partial x_1} \right|_{x_1=0}, \quad (5)$$

$$u_1|_{x_1=H} = u_2|_{x_2=0}, \quad G_1 \left. \frac{\partial u_1}{\partial x_1} \right|_{x_1=H} = G_2 \left. \frac{\partial u_2}{\partial x_2} \right|_{x_2=0}, \quad (6)$$

where the following notations are used:  $t$  is time;  $x, x_1, x_2$  are vertical coordinates;  $w(x, t)$  are transverse shear displacements of the central body axis consisting of a dike and tails;  $B(x)$  - body thickness;  $\rho$  and  $G$  are density and shear modulus of the body material;  $\rho_1, G_1$  and  $\rho_2, G_2$  are densities and shear moduli in the upper and lower layers, respectively;  $\beta^*$  is the coefficient of viscous friction in accordance with the Kelvin - Foigt hypothesis;  $H$  is the thickness of the upper layer of soil;  $u_1$  and  $u_2$  are shear displacements of the layers of soil.

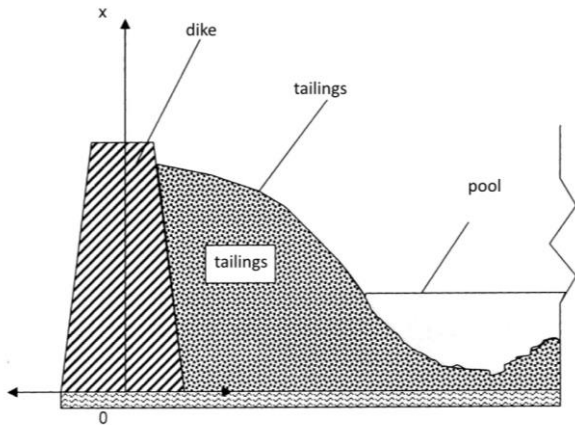


Fig. 1. a) A schematic drawing of the tailing dump.

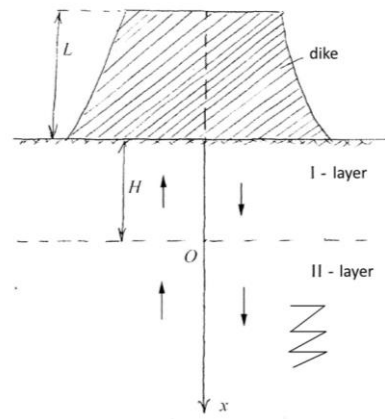


Fig. 2. b). An analytic model of seismic vibrations of the dike.

It is easy to see that the boundary problem (1) - (6) is written in different parallel coordinate systems. For the first system, the computing origin is taken on the crest of the dike ( $x=0$ ). For the second system, the computing origin is at the point of contact between the dike and the top layer of soil, and for the third system, the computing origin is at the place of contact of the soil layers. Such approach makes the solution of the set boundary problem considerably easier.

The body thickness (dike with tailings) is approximated by an exponential function of the following form:

$$B(x) = B_0 e^{Sx}, \quad S > 0, \quad (7)$$

where  $B_0$  is the body thickness on its crest (top); shape variable  $S$  is calculated as follows:

$$B(x)|_{x=L} = B(L) = B_0 e^{SL},$$

$$SL = \ln \frac{B(L)}{B_0},$$

$$S = \frac{1}{L} \ln \frac{B(L)}{B_0} \quad (8)$$

Density  $\rho$  and shear modulus of the body are calculated according to the weighted formulas:

$$\rho = \frac{\rho_d \cdot Q_d + \rho_t \cdot Q_t}{Q_d + Q_t}, \quad G = \frac{G_d \cdot Q_d + G_t \cdot Q_t}{Q_d + Q_t}, \quad (9)$$

where  $\rho_d, Q_d, G_d$  and  $\rho_t, Q_t, G_t$  are density, volume and shear modulus of the dike and tailings, respectively.

When approximating the thickness of a body  $B(x)$  according to dependence (7), differential equations (1) - (3) are reduced to the following types:

$$\frac{\partial^2 w}{\partial t^2} - C^2 \frac{\partial^2 w}{\partial x^2} - C^2 S \frac{\partial w}{\partial x} - \beta^* C^2 \frac{\partial^3 w}{\partial x^2 \partial t} - \beta C^2 S \frac{\partial^2 w}{\partial x \partial t} = 0, \quad 0 < x < L, \quad (10)$$

$$\frac{\partial^2 u_1}{\partial t^2} - a_1^2 \frac{\partial^2 u_1}{\partial x_1^2} - a_1^2 \beta^* \frac{\partial^3 u_1}{\partial x_1^2 \partial t} = 0, \quad 0 < x_1 < H_1, \quad (11)$$

$$\frac{\partial^2 u_2}{\partial t^2} - a_2^2 \frac{\partial^2 u_2}{\partial x_2^2} - a_2^2 \beta^* \frac{\partial^3 u_2}{\partial x_2^2 \partial t} = 0, \quad 0 < x_2 < \infty, \quad (12)$$

where

$$C = \sqrt{\frac{G}{\rho}}, \quad a_1 = \sqrt{\frac{G_1}{\rho_1}}, a_2 = \sqrt{\frac{G_2}{\rho_2}}. \quad (13)$$

During damped harmonic vibrations of the system, unknown functions  $w(x, t)$ ,  $u_1(x_1, t)$  and  $u_2(x_2, t)$  can be searched as follows:

$$w(x, t) = e^{(-\beta^* \frac{\omega^2}{2} + i\gamma\omega)t} v(x), \quad (14)$$

$$u_1(x_1, t) = e^{(-\beta^* \frac{\omega^2}{2} + i\gamma\omega)t} (A_1 e^{i\omega \frac{x_1}{a_1}} + B_1 e^{-i\omega \frac{x_1}{a_1}}), \quad (15)$$

$$u_2(x_2, t) = e^{(-\beta^* \frac{\omega^2}{2} + i\gamma\omega)t} (A_2 e^{i\omega \frac{x_2}{a_2}} + B_2 e^{-i\omega \frac{x_2}{a_2}}), \quad (16)$$

where  $v(x)$  is a function unknown in advance:

$$\gamma = \sqrt{1 - \beta^{*2} \frac{\omega^2}{4}},$$

where  $A_1, B_1, A_2$  and  $B_2$  are also constants unknown in advance. In this case,  $A_2$  is the amplitude of the seismic wave incident on the lower layer. It is easy to verify that functions (15) and (16) automatically satisfy differential equations (11) and (12), respectively.

As a result of the substitution of the expression (14) into the differential equation (10) for an unknown function  $v(x)$ , the following ordinary second-order differential equation is obtained:

$$\frac{d^2 v}{dx^2} + S \frac{dv}{dx} + \frac{\omega^2}{C^2} v = 0. \quad (17)$$

The general solution of equation (17) is the following:

$$v(x) = \begin{cases} e^{\alpha x} (C_1 \sin \beta x + C_2 \cos \beta x) & \text{at } \frac{\omega^2}{C^2} > \frac{S^2}{4}, \\ e^{\alpha x} (C_1 \operatorname{sh} \beta x + C_2 \operatorname{ch} \beta x) & \text{at } \frac{\omega^2}{C^2} < \frac{S^2}{4}, \\ e^{\alpha x} (C_1 x + C_2) & \text{at } \frac{\omega^2}{C^2} = \frac{S^2}{4}, \end{cases} \quad (18)$$

where

$$\alpha = -\frac{S}{2}, \quad \beta = \sqrt{\frac{\omega^2}{C^2} - \frac{S^2}{4}}. \quad (19)$$

Let's consider the first case when the following inequality takes place:

$$\frac{\omega^2}{C^2} > \frac{S^2}{4}. \quad (20)$$

$$v(x) = e^{\alpha x} (C_1 \sin \beta x + C_2 \cos \beta x).$$

The constants  $C_1, C_2, A_1, B_1, A_2, B_2$  are determined in the process of satisfying the boundary conditions (4) - (6). We obtain the following system of linear algebraic equations for them:

$$\begin{cases} \beta C_1 + \alpha C_2 = 0, \\ e^{\alpha L} (C_1 \sin \beta L + C_2 \cos \beta L) = A_1 + B_1, \\ G [C_1 \cos \beta L - C_2 \sin \beta L] + \alpha (C_1 \sin \beta L + C_2 \cos \beta L) e^{\alpha L} = G_1 \frac{i\omega}{a_1} (A_1 - B_1), \\ A_1 e^{i\omega \frac{H}{a_1}} + B_1 e^{-i\omega \frac{H}{a_1}} = A_2 + B_2, \\ G_1 \frac{i\omega}{a_1} (A_1 e^{i\omega \frac{H}{a_1}} - B_1 e^{-i\omega \frac{H}{a_1}}) = G_2 \frac{i\omega}{a_2} (A_2 - B_2). \end{cases} \quad (21)$$

In system (21), the number of equations is 5, and the number of unknown constants is 6. Therefore, the system is underdetermined.

Further, we will set the amplitude of the seismic wave incident on the lower layer  $A_2$ . The system (21) becomes a defined system of relatively constants  $C_1, C_2, A_1, B_1$  and  $B_2$ . A solution of the system (21) is obtained in the following form:

$$A_1 = 2A_2 \frac{r_2}{\Delta^*}, \quad B_1 = -2A_2 \frac{r_1}{\Delta^*}, \quad C_1 = -4A_2 \frac{\alpha}{\Delta} e^{-\alpha L} \frac{i\omega}{K_0 C \Delta^*}, \quad C_2 = 4A_2 \frac{\beta}{\Delta} e^{-\alpha L} \frac{i\omega}{K_0 C \Delta^*}, \quad (22)$$

where the following notations are used:

$$\Delta = \beta \cos \beta L - \alpha \sin \beta L, \quad \Delta^* = (1 + K_1) r_2 e^{i\omega \frac{H}{a_1}} - (1 - K_1) r_1 e^{-i\omega \frac{H}{a_1}} \quad (23)$$

$$r_1 = -S_1 \frac{\alpha}{\Delta} + S_2 \frac{\beta}{\Delta} - \frac{1}{K_0} \frac{i\omega}{C}, \quad r_2 = -S_1 \frac{\alpha}{\Delta} + S_2 \frac{\beta}{\Delta} + \frac{1}{K_0} \frac{i\omega}{C}, \quad (24)$$

$$S_1 = \beta \cos \beta L + \alpha \sin \beta L, \quad S_2 = -\beta \sin \beta L + \alpha \cos \beta L, \quad (25)$$

$$K_0 = \frac{G\alpha_1}{G_1 C} = \frac{\rho C}{\rho_1 \alpha_1}, \quad K_1 = \frac{G_1 \alpha_2}{G_2 \alpha_1} = \frac{\rho_1 \alpha_1}{\rho_2 \alpha_2}, \quad (26)$$

Substituting the values  $S_1$  and  $S_2$  from (25) to (24), we will have:

$$r_1 = -\frac{\omega^2 \sin \beta L}{C^2 \Delta} - \frac{1}{K_0} \frac{i\omega}{C}, \quad r_2 = -\frac{\omega^2 \sin \beta L}{C^2 \Delta} + \frac{1}{K_0} \frac{i\omega}{C}. \quad (27)$$

Substituting the values  $r_1$  and  $r_2$  from (27) to (23):

$$\Delta^* = (1 + K_1) \left( -\frac{\omega^2 \sin \beta L}{C^2 \Delta} + \frac{1}{K_0} \frac{i\omega}{C} \right) e^{i\omega \frac{H}{a_1}} - (1 - K_1) \left( -\frac{\omega^2 \sin \beta L}{C^2 \Delta} - \frac{1}{K_0} \frac{i\omega}{C} \right) e^{i\omega \frac{H}{a_1}}.$$

Using the Euler formula, the expression for  $\Delta \cdot \Delta^* K_0$  is simplified and leads to the following form:

$$K_0 \Delta \Delta^* = -2 \left[ K_1 \left( K_0 \frac{\omega^2}{C^2} \sin \beta L \cos \frac{\omega H}{\alpha_1} + \frac{\omega}{C} \Delta \sin \frac{\omega H}{\alpha_1} \right) + \left( K_0 \frac{\omega^2}{C^2} \sin \beta L \sin \frac{\omega H}{\alpha_1} - \frac{\omega}{C} \Delta \cos \frac{\omega H}{\alpha_1} \right) i \right] = \tilde{\Delta}. \quad (28)$$

Substituting the values  $C_1$  and  $C_2$  from (22) in equation (20) for  $v(x)$ , we will obtain:

$$v(x) = 2A_2 \frac{i\omega}{C} \frac{(\alpha \sin \beta x - \beta \cos \beta x)}{\operatorname{Re} \tilde{\Delta} + i \operatorname{Im} \tilde{\Delta}},$$

$$\operatorname{Re} \tilde{\Delta} = K_1 \left( K_0 \frac{\omega^2}{C^2} \sin \beta L \cos \frac{\omega H}{\alpha_1} + \frac{\omega}{C} \Delta \sin \frac{\omega H}{\alpha_1} \right), \quad (29)$$

$$\operatorname{Im} \tilde{\Delta} = K_0 \left( K_0 \frac{\omega^2}{C^2} \sin \beta L \sin \frac{\omega H}{\alpha_1} - \frac{\omega}{C} \Delta \cos \frac{\omega H}{\alpha_1} \right).$$

Initially unknown function  $w(x, t)$  is obtained in the following complex form:

$$w(x, t) = 2A_2 \frac{i\omega}{C} \frac{(\alpha \sin \beta x - \beta \cos \beta x)}{\operatorname{Re} \tilde{\Delta} + i \operatorname{Im} \tilde{\Delta}} e^{-\alpha(L-x)} e^{(-\beta^* \frac{\omega^2}{2} + i\gamma\omega)t}. \quad (30)$$

The real part of the expression (30) will be the solution of the stated boundary problem (1) - (7) provided  $\frac{\omega^2}{C^2} > \frac{S^2}{4}$ :

$$\operatorname{Re} w(x, t) = 2A_2 \frac{\omega}{C} \frac{(\alpha \sin \beta x - \beta \cos \beta x)}{(\operatorname{Re} \tilde{\Delta})^2 + (\operatorname{Im} \tilde{\Delta})^2} e^{-\alpha(L-x)} (\operatorname{Im} \tilde{\Delta} \cos \gamma \omega t - \operatorname{Re} \tilde{\Delta} \sin \gamma \omega t) e^{-\beta^* \frac{\omega^2}{2} t}. \quad (31)$$

For the amplitude of seismic vibrations of the dike body, the following expression is obtained:

$$y_1 = 2A_2 \frac{\omega}{C} \frac{|\alpha \sin \beta_1 x - \beta_1 \cos \beta_1 x|}{\sqrt{(\operatorname{Re} \tilde{\Delta}_1)^2 + (\operatorname{Im} \tilde{\Delta}_1)^2}} e^{-\alpha(L-x)} e^{-\beta \frac{\omega^2}{2} t}. \quad (32)$$

In the second case, when there is inequality:

$$\frac{\omega^2}{C^2} < \frac{S^2}{4}. \quad (33)$$

For the vibration amplitude of the dike body, the following expressions are obtained:

$$y_2 = 2A_2 \frac{\omega}{C} \frac{|\alpha \operatorname{sh} \beta_2 \operatorname{ch} \beta_2 x| e^{-\beta \frac{\omega^2}{2} t}}{\sqrt{(\operatorname{Re} \tilde{\Delta}_2)^2 + (\operatorname{Im} \tilde{\Delta}_2)^2}} e^{-\alpha(L-x)}, \quad (34)$$

$$\beta_2 = \sqrt{\frac{S^2}{4} - \frac{\omega^2}{C^2}},$$

$$\operatorname{Re} \tilde{\Delta}_2 = K_1 \left( K_0 \frac{\omega^2}{C^2} \operatorname{sh} \beta_2 L \cos \frac{\omega H}{\alpha_1} + \frac{\omega}{C} \Delta_2 \sin \frac{\omega H}{\alpha_1} \right), \quad (35)$$

$$\operatorname{Im} \tilde{\Delta}_2 = K_0 \frac{\omega^2}{C^2} \operatorname{sh} \beta_2 L \sin \frac{\omega H}{\alpha_1} - \frac{\omega}{C} \Delta_2 \cos \frac{\omega H}{\alpha_1},$$

$$\Delta_2 = \beta_2 \operatorname{ch} \beta_2 L - \alpha \operatorname{sh} \beta_2 L. \quad (36)$$

For the third case, when equality  $\frac{\omega^2}{C^2} = \frac{S^2}{4}$  is satisfied, the following formulas are obtained for calculation of the vibration amplitude of the dike body:

$$y_3 = 2A_2 \frac{\omega}{C} \frac{|\alpha x - 1|}{\sqrt{(\operatorname{Re} \tilde{\Delta}_3)^2 + (\operatorname{Im} \tilde{\Delta}_3)^2}} e^{-\alpha(L-x)} e^{-\beta \frac{\omega^2}{2} t}, \quad (37)$$

$$\operatorname{Re} \tilde{\Delta}_3 = K_1 \left( K_0 L \frac{\omega^2}{C^2} \cos \frac{\omega H}{\alpha_1} + \frac{\omega}{C} (1 - \alpha L) \sin \frac{\omega H}{\alpha_1} \right), \quad (38)$$

$$\operatorname{Im} \tilde{\Delta}_3 = K_0 L \frac{\omega^2}{C^2} \sin \frac{\omega H}{\alpha_1} - \frac{\omega}{C} (1 - \alpha L) \cos \frac{\omega H}{\alpha_1}. \quad (39)$$

Attenuation rate of a seismic wave is equal to  $\beta \frac{\omega^2}{2}$ .  $\beta^* \approx 0.02$  (sec) for the soil mass [25]. The circular frequency of the incident seismic wave,  $\omega = 2\pi\nu$  where  $\nu$  is the frequency in Hertz. The obtained formulas (34) - (38) show that high-quality seismic waves in the dike body do not spread practically. Thus, for example, at frequencies  $\nu = 6 - 10$  Hertz, the attenuation rate in the SI system varies within 14–40 sec<sup>-1</sup> and, therefore, the wave attenuates in a fraction of a second. Low-frequency seismic waves with negligible attenuation propagate in the dike body. Displacements of velocity and acceleration in the dike body can be calculated using the obtained formulas (34) - (38).

In the particular case when the dike height is zero,  $L = 0$ , the formulas obtained are considerably simplified and are reduced to the following form:

$$y = \frac{2A_2}{\sqrt{K_1^2 \sin^2 \frac{\omega H}{\alpha_1} + \cos^2 \frac{\omega H}{\alpha_1}}} e^{-\beta \frac{\omega^2}{2} t}. \quad (40)$$

This formula shows that in case of a low-frequency seismic wave outlet, a homogeneous soil medium on a day surface, its amplitude at the initial moment increases twice and then attenuates exponentially with an exponent  $\beta \frac{\omega^2}{2}$ .

Figures 3–6 show the dependencies of relative amplitude and relative acceleration of the dike crest on its height for various values of the frequency of seismic vibrations of the area of dike construction. The heights of the dike are measured up to 150 m on the axis of abscissa. Exactly the same dependencies

are obtained for the relative acceleration of the dike crest. According to the graphs, the given relative characteristics considerably depend on the height of the dike. For example, a dike with a height from 65 m to 110 m at a frequency of the incident seismic wave of 1 Hz seems less seismic resistant than a dike with a height from 110 m to 150 m.

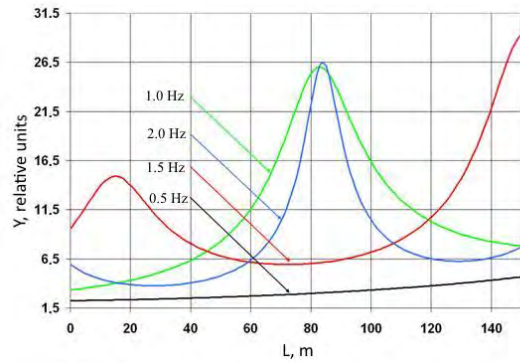


Fig. 3. Dependencies of relative displacement amplitudes and accelerations of the dike crest on its height for different frequencies of the incident wave (0.5 Hz – 2.0 Hz).

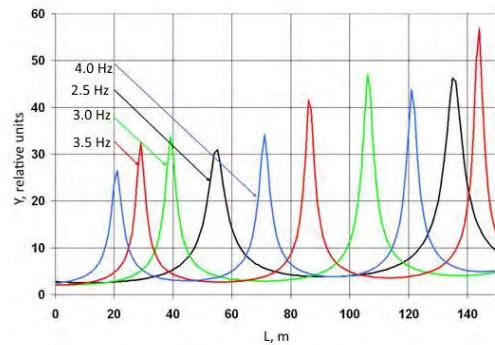


Fig. 4. Dependencies of relative displacement amplitudes and accelerations of the dike crest on its height for different frequencies of the incident wave (2.5 Hz – 4.0 Hz).

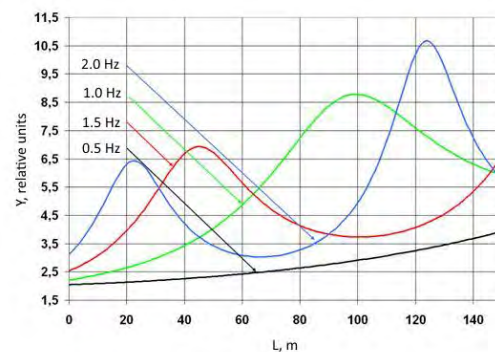


Fig. 5. Dependencies of relative displacement amplitudes and accelerations of the dike crest on its height for different frequencies of the incident wave (0.5 Hz – 2.0 Hz).

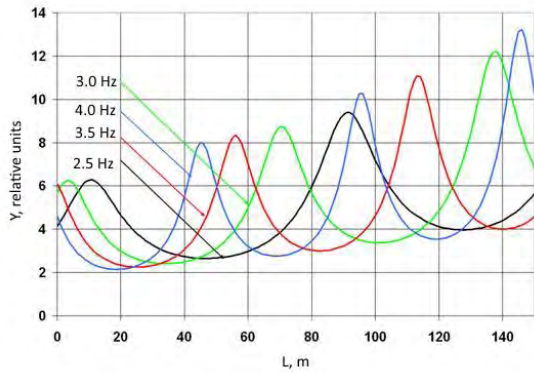


Fig. 6. Dependencies of relative displacement amplitudes and accelerations of the dike crest on its height for different frequencies of the incident wave (2.5 Hz – 4.0 Hz).

### III. CONCLUSIONS

1. A mathematical model of the system of seismic vibrations, which consists of the flood dike of tailing dump, deposit material (tailings) and bottom ground layers, is developed. The model is represented by a contact boundary problem for three differential equations of transverse shear vibrations of a dike body with tailings and two bottom ground layers with different physical and mechanical and power characteristics.

2. Calculation formulas for computation of relative displacement amplitudes, velocities and accelerations of the dike body depending on its height and the frequency of seismic vibrations of the area are obtained as a result of the stated boundary problem solution.

3. The obtained theoretical results allow choosing (at the design stage of the tailings) the overall dimensions of the bund wall of the storages so that acceleration and displacement of the dike body do not exceed the maximum permissible values during seismic events.

### References

[1] O.G.Burdzieva, "The mechanism of mine refuses influence on the environment," Seismic hazard and seismic risk management in Caucasus, Proc. of the III Caucasian International School-Seminar for Young Scientists, pp. 361-365, 2009.

[2] V.B.Zaalishvili, O.G.Beriev, O.G.Burdzieva, T.V.Zaks, and A.S.Kanukov, "Cancer morbidity of the population of Vladikavkaz and its relationship with various factors," Geology and Geophysics of the South of Russia, vol. 3, pp. 29-38, 2013.

[3] O.G.Burdzieva and V.B.Zaalishvili, "Investigation of tailings` impact to the environment under the conditions of high seismic hazard," Seismic hazard and seismic risk management in the Caucasus. Procs. of the III Caucasian International school-seminar for young scientists, pp. 355-360, 2009.

[4] O.G.Burdzieva, E.V.Shevchenko, and E.B.Ermishina, "The mechanism of anthropogenic disasters under the influence of mining production," Hazardous natural and anthropogenic geological processes in the mountain and foothill areas of the North Caucasus, pp. 157-161, 2010.

[5] O.G.Burdzieva, V.B.Zaalishvili, O.G.Beriev, T.V.Zaks, and A.S.Kanukov, "On the environmental aspects of modern urbanized territory during strong earthquakes," Earthquake engineering. Safety of buildings, vol. 3, pp. 62-67, 2012.

[6] V.B.Zaalishvili, S.A.Bekuzarova, and O.G.Burdzieva, "Assessment of technogenic pollution of the territory," Seismic hazard and seismic risk

management in the Caucasus. Procs. of the V Caucasian International school-seminar for young scientists, pp. 353-355, 2013.

[7] O.G. Burdzieva, "Rational subsoil management in the RNO-Alania region (problems and solutions)," Coal, vol. 1011, pp. 70-71, 2010.

[8] D.M.Banu and S. Mihai, "The probabilistic modeling of safety factor for mining tailings ponds," 16th International Multidisciplinary Scientific Geoconference, Sgem 2016: Science and Technologies in Geology, Exploration and Mining, Vol II, pp. 921-928, 2016.

[9] A.J. Breitenbach, "Seismic behavior on lined fill structures," Balkema Publishers, 1998 [International Conference on Tailings and Mine Waste, 5; 1998; Fort Collins, Colo].

[10] D. Chakraborty and D. Choudhury, "Pseudo-static and pseudo-dynamic stability analysis of tailings dam under seismic conditions," Proceedings of the National Academy of Sciences, India, Section a-Physical Sciences, vol. 83, pp. 63-71, 2013.

[11] W.S.Fei and Z.Y. Zhang, "Research of the stability of tailing dam under earthquake," Applied mechanics and materials, Stafa-Zurich: Trans Tech Publications Ltd, vol. 295-298, pp. 2820-2824, 2013.

[12] W.S.Fei and Y.Y. Zhou, "Analysis on the stability of tailing dam under earthquake," Marrickville: Orient Acad Forum, 2010.

[13] G.Ferrer, E. Saez, and C. Ledezma, "Numerical modeling of cracking pattern's influence on the dynamic response of thickened tailings disposals: a periodic approach," Earthquake Engineering and Engineering Vibration, vol. 17, 179-190, 2018.

[14] M.K. Ghose and P.K. Sen, "Investigation of soil engineering properties for safe design and construction of the iron ore tailing dam," Indian Journal of Engineering and Materials Sciences, vol. 8, pp. 318-326, 2011.

[15] J.X. Jin, C.G. Song, B. Liang, Y.J. Chen, and M.L. Su, "Dynamic characteristics of tailings reservoir under seismic load," Environmental Earth Sciences, vol. 77, p. 11, 2018.

[16] W.L. Liu, J.B. Xie, H.M. Cheng, and T.C. He, "Research on the dynamic response of Zhuziqing tailings dam", Progress in Civil Engineering, vol. 1-4, pp. 170-173, 2012.

[17] W.Schott, T. McGee, I. Richardson, and R. Stoll, "High-resolution seismic profiles on a mine tailings pond," Essex: Elsevier Applied Science Publishers, 1992.

[18] B.T. Xu and Y.Y. Wang, "Stability analysis of the Lingshan gold mine tailings dam under conditions of a raised dam height," Bulletin of Engineering Geology and the Environment, vol. 74, 151-161, 2015.

[19] Y.Y. Zhou, W.S. Fei, and W.L. Liu, "Analysis on the stability of tailing dam under seepage," Advances in Civil Engineering, vol. 1-6. Advanced Materials Research. Durnten-Zurich: Trans Tech Publications Ltd, 2011, p. 3234.

[20] I.I.Vorovich, V.A.Babeshko, and O.Pryakhina, "Dynamics of massive bodies and resonance phenomena in deformable media," Moscow: Nauchny Mir, 1999, p. 246.

[21] V.B.Zaalishvili, I.D.Muzaev, and Zh.D.Tuaeva, "Mathematical modeling of seismic vibrations of soil thickness at the base of the building," Procs. of the international scientific-practical conference "Hazardous natural and anthropogenic geological processes in the mountain and foothill territories of the North Caucasus," pp. 290-298, 2007.

[22] I.D.Muzaev, N.I.Muzaev, and K.S. Kharebov, "Statement and solution of the multilayer boundary problem of seismic vibrations of a high-rise structure," Seismic construction, safety of structures, vol. 1, pp. 22-25, 2014.

[23] M. Okomoto, "Earthquake resistance of engineering structures," Moscow, 1980, p. 342.

[24] E.E.Khachiyani, "The task of strengthening or weakening of the effect of seismic impact on the ground surface," Bulletin of the Research Institute of building constructions (TSNIISK) named after V.A. Kucherenko "Studies in the theory of structures", vol. XXVI, pp. 67-80, 2009.

[25] G.Bonnet and J.F. Heize "Non linear seismic response of a soft layer," 10-th European conference on Earthquake Engineering, Balkema, Rotterdam, pp. 361-364, 1995.