

The effect of social proximity on degree distribution of Scale-free network

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Abstract: With limited resources in knowledge seeking, firms are more likely to connect with influential firms in adjacent areas according to preferential attachment mechanism of BA model, and thus geographical proximity plays an important role in shaping typology of a scale-free network. However, prior researches, adopting BA model, cannot explain why there are more and more nonlocal ties constructed via social relationships of local firms in industrial clusters. This paper proposed a new model by changing the rule of preferential attachment to detect the effect of social proximity on the typology of scale-free network through simulation. The results indicate that the distance of in-degree between core and peripheral nodes would decrease with the expanding of searching range and number of connections affecting by social proximity. Though the degree distribution still follows power-laws, peripheral nodes are less depended on core actors in accessing to external knowledge in case that social proximity rather than geographical proximity becomes the main factor in actors' knowledge seeking strategy.

Key words: Social Proximity; Scale-free Network; BA Model; Degree Distribution

1 Introduction

Scale-free network has special typology that it obeys power-laws degree distribution, i.e. the probability $p(k)$ that a node has degree k can describe as $p(k) \sim k^{-\gamma}$ (Barabási et al., 1999; Barabasi & Albert, 1999; Albert et al., 1999). In recent years, a large number of researchers argued that many knowledge networks in real world follow power-laws in which core actors are more favored by other nodes in knowledge exchanging or learning (Lin & Li, 2010), and that structure is efficient in knowledge diffusion (Tang et al., 2010). Constrained by resources for knowledge seeking and partner sustaining, firms are more likely to search and connect with external organization that nearby their location (Dettmann et al., 2015). Thus, geographical proximity enhances the probability of constructing ties between local firms (Kabo et al., 2014) and induces agglomeration. In that case, local core actors such as leading firms or flagships are more attractive to others and that makes them become driving force of the generation of a local scale-free network.

However, with the widely application of advanced communication devices and intelligent technologies, accessing to nonlocal knowledge resources becomes more convenient in nowadays (Labrianidis, 2011; Ter Wal & Boschma, 2011). Some scholars pointed out that the role of geographical proximity in motivating knowledge ties' forming can be replaced, partially or even completely, by other types of proximities (Singh, 2005; Breschi & Lissoni, 2009), such as social proximity (Sternitzke et al., 2008; Sharmeen et al., 2014; Müller & Stewart, 2016) which is associated with personal relationships or past collaborations.

Though social relationships, nonlocal actors would be attracted into a local knowledge network and enhance the chance for normal local actors to be chosen as external knowledge sources by new comers (Fuentes & Dutrénit, 2016). In that case, the increasing nonlocal ties will change the typology of local scale-free network, e.g. former dense local network turns into a sparse one, via affecting the connecting mechanism of preferential attachment. Nevertheless, how and to what extent that social proximity affects scale-free network in real word still remains unknown.

More precisely, though BA model provided a way to exhibit the special typological characters of scale-free network, it failed in mimicking well the network structural properties of a scale-free network which influenced by social proximity. Little attention has been put on the connection mechanism of nodes based on social relationships and its effect on structure property of the entire network. Motivated by these considerations, the objective of this paper is to provide a revised BA model to detect, through simulation, the role of social proximity in changing degree distribution of a scale-free network.

The remainder of this paper is organized as follows. In Section 2, a simulation model which revised the preferential attachment mechanism is introduced. Then, the steps of simulation and results have been discussed in Section 3. In the last Section, we summarize our findings, contributions as well as limitations.

2 Simulation Model

BA model describes the forming of a scale-free network with two critical mechanisms, i.e. growth and preferential attachment (Barabási et al., 1999). The two mechanisms can be briefly described as follows.

(1) Growth: assuming that there are m_0 nodes in the original network and a new node will add into the network and connected with m ($m \leq m_0$) existing nodes in every time-step.

(2) Preferential attachment: The probability Π of any node connects to node i is proportional to the degree k_i of node i , i.e. $\Pi(k_i) = k_i / \sum_j k_j$. After t time-steps, a network with $N=t+m_0$ nodes and mt links will be generated. The degree distribution of this network follows power-law.

Though BA model has been widely adopted in theoretical researches, the mechanism of preferential attachment cannot comprehensively depict how firms choose their external knowledge resources. In real world, there are two critical factors determining firms' preference in searching external knowledge sources. The first one is the geographical location of firms and the second is the social relationships that a firm has. Normally, there are limited resources can be invested into knowledge seeking and partner sustaining (Freel, 2003). Thus, the knowledge searching range would be easily restricted in specific geographical space. In such region, external organization that already has many knowledge links may be more attractive since through which a broader knowledge basis will be available.

However, when considering nonlocal knowledge ties constructed by social relationships, new actors entering into the network have larger scope in knowledge seeking (Crescenzi et al., 2016). They may construct knowledge connections either based on geographical proximity or on social proximity and that may change the mechanism of preferential attachment. Therefore, we propose a modified BA model to describe the connecting decision of new nodes motivated by different proximities. The steps of network generation based on our model have been described as follows.

(1) Growth. We start with an ego network of a local flagship firm or leading company and there are m_0 nodes in all but m_0-1 nodes only have links with a core actor. New actors involves in the local knowledge network in two stages.

In stage one, new node entering in the existing network can search the entire network and construct m_1 ($m_1 \leq m_0$) links with nodes either nearby located or has close social relationships. By doing so, firm added in stage one is not limited by geographical space since it can contact with anyone. Larger parameter m_1 indicates greater social proximity. After t_1 time-steps, a network with $N_1 = m_0 + t_1$ nodes and $2(m_0-1) + 2m_1t_1$ links has been generated.

In stage two, with the growing of the network after t_1 time-steps, new actors cannot search the entire network and thus have smaller searching range. To better use their resource for knowledge acquiring, each of new nodes will focus on n_1 ($n_1 \leq N_1$) most closed, in geography, actors and from which build m_2 ($m_2 < m_1 \leq m_0$) links. The reason that new actors in this stage have smaller links m_2 is that it is more difficult to find appropriate partners within a limited searching scope. There will be N_2 new nodes come into the existing network after t_2 time-steps.

(2) Preferential attachment. New actors will prior choose nodes with larger in-degree in their searching range, i.e. $\Pi(k_i) = k_i / \sum_j k_j$ where Π is the probability that a new node generate ties with node i . After t ($t = t_1 + t_2$) time-steps, a network which has $N = N_1 + N_2 = m_0 + t_1 + t_2$ nodes and $2(m_0-1) + 2m_1t_1 + 2m_2t_2$ links would be formed.

3 Results and Discussion

Based on the model mentioned above, we conduct a simulation to detect the effect of social proximity on the typology of knowledge network using MATLAB6.5. We first detect the degree distribution of network generated based on our model. The simulation conducts as follows.

In stage one, there are 300 nodes has been added into the original network and each of them builds knowledge ties with 4 existing actors after searching the entire network. Thus, we have $N_1=300$, $m_1=4$ in stage one.

While in stage two, the network will continuously expand which significantly narrow down new comers' searching range. We assume that there are 1200 new comers in this stage and each of them just has only one knowledge connection, choosing from 20 most geographically closer nodes, in the network. By doing so, we

have $N_2=1200$ and $m_2=1$ in stage two. In all, a network with over 1500 nodes will be generated based on our new model.

The degree distribution has been shown in Figure 1 in which the horizontal axis represents the logarithm of the number of node connections and the vertical axis reflects the logarithm of the number of node connections probability. We also draw a distribution curve of BA model based on a network with same size but $m=3$. Figure 1 compares node degree distribution of our new model and BA model.

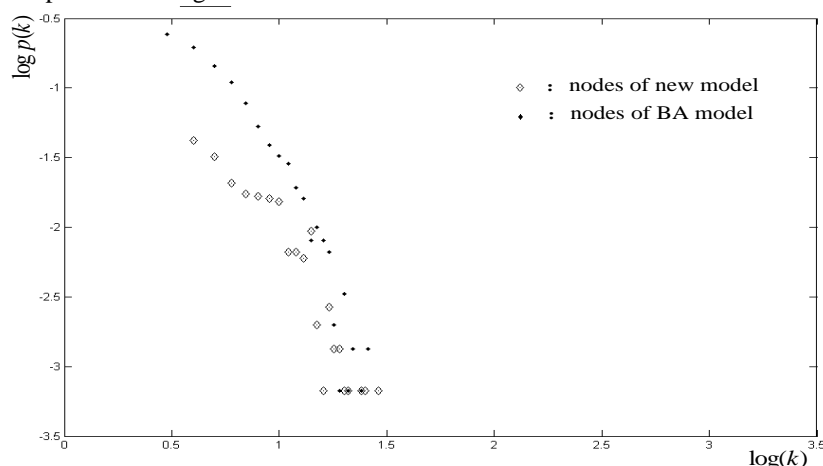


Figure 1 Degree distribution of new model and BA model

As is shown in Figure 1, the node degree distribution of our new model still follows power-laws but has smaller difference in degree between core actors, with maximum in-degree, and peripheral nodes, with the minimum in-degree, in the network compare with BA model. Through simulation we find out that the degree distribution of BA model is the same as our new model when nodes in the second stage can search the entire network and has the same links as the nodes in the stage one. We also notice that though the degree distribution obeys power-laws, the distinction of links between core actors (with a comparable larger number of connections) and normal nodes in the network would change with different number of N_1 and m_1 .

From the adjacent matrix which records the links of the final network, we confirm that nodes with wider searching scope have construct more links, while nodes added in stage two attracts fewer ties. Since in our model, nodes in stage one have larger searching scope which enable them to connect more external knowledge resources, local or nonlocal, via social relationships. These nodes are likely to grow up as core actors which occupy large proportion of knowledge channels.

However, nodes in stage two have relatively limited searching area and thus they are easily attracted by core actors already existed in the network and difficult to be connected by others. In other words, nodes in stage two are more likely to be trapped in specific geographical space and thus have weak influence in the knowledge network.

Above findings reflect the evolution of some types of knowledge networks in real world. For example, in an industrial cluster, the knowledge network often starts with a star-shape network in which a leading company connects with a group of cooperators. Outsiders entering into the local network in the early stage have broad eyesight in knowledge searching and thus enlarge the chance of these actors to be connected by late comers. Nevertheless, when the original local network grows up as a huge one, for instance, many industrial clusters have several hundred or even over one thousand firms, new nodes coming into this network will connect nodes with widely local knowledge channels so as to make best use of their limited resource in knowledge searching and that make them soon be embedded into the local network. That strategy will enhance the influence of local leading firm for one side, and make new adding nodes become less attractive to later comers since they will more depend on geographical proximity rather than social proximity in constructing knowledge links.

Thus, social proximity may help firms broaden their knowledge seeking space and has more in-degree. Otherwise, geographical proximity may convenient firms' knowledge acquirement but also raises the risk of becoming periphery actors in a local knowledge network. To further see through the findings presented above, we detect the effect of three parameters, i.e. relative proportions N_1/N_2 , m_1/m_2 as well as the search range of nodes in stage two, on the degree distribution.

Figure 2 reflects the change of the maximum in-degree, the average value of 100 times of simulation, with the increasing N_1/N_2 . The results indicate that the maximum in-degree of core actors have less links when there are fewer nodes added in stage one can search entire network. However, with the expanding of the number of nodes entering into the network in the first stage, core actors can construct more connections with other nodes. Thus, the ratio N_1/N_2 will increase the distance of core actors and periphery nodes in terms of links they have. Combing with the finding that the average path length of the final network, the average distance between all pairs of its nodes, is over 2, we conclude that a large number of nodes cannot reach each other in the absence of core actors.

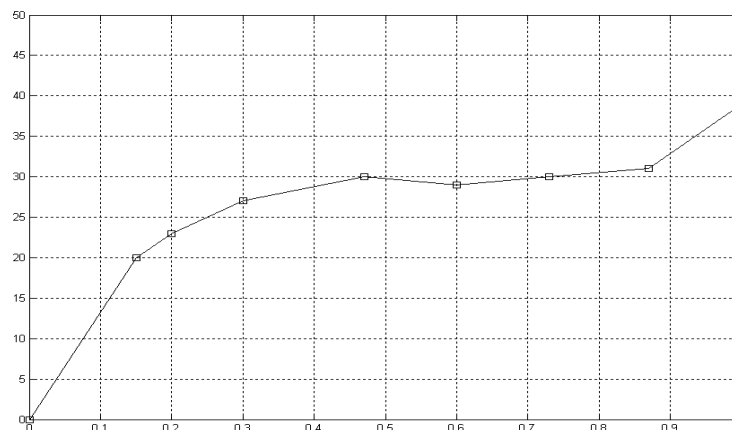


Figure 2 The relationship between N_1/N_2 and the maximum links

Next, we enlarge the searching range of nodes added in stage two. In our model, the searching range represents the breadth of the social network that a firm can reach. Thus, with a wider knowledge seeking scope, new comers in stage two may less depend on geographical proximity but more likely to favor nodes that have intimate social relationships with them.

To test the assumption, we expand the searching range, from 20 actors to 200, of nodes in the second stage and increase the average links (now $m_2=4$) in a network with 1500 nodes ($N_1=300$, $N_2=1200$). Figure 3 reveals the relationship between in-degree and the corresponding probability.

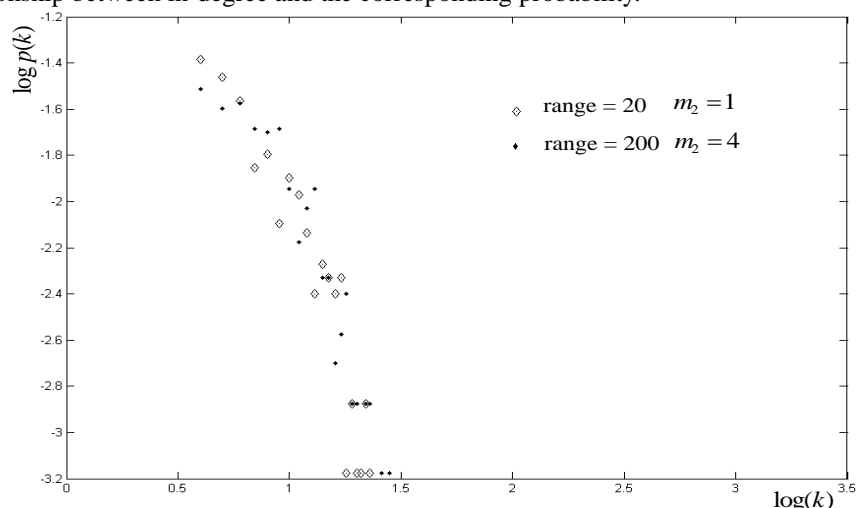


Figure 3 The effect of searching range and average links on degree distribution

From Figure 3, we find out that core actors have more links while the number of periphery nodes with minimum links decreased. The degree distribution becomes more “uniform”, and that means normal nodes has more chance to be connected by new comers in stage two. Since the searching range indicates the extent that a firm relies on social relationships in expanding its ego network, the shorten distance in terms of in-degree between core actors and peripheral nodes reflects that social proximity helps less important nodes

improve their position in local network and provides a way to free from the control of core actors in knowledge acquirement. In another angle, as the searching range enlarged, new comers are less affected by geographical proximity, but instead turn to nodes which have shorter social distance. Therefore, firms which are not deeply embedded in local network also can attract new ties via social relationships. This finding indicates that it is important for firms to be openness to external social networks so as to connect organizations in different locations and become influential in local knowledge networks. Our result also shows that social proximity has significant influence on the process that a local dense network turns into a sparse one. In real word, based on social relationships, ties from new comers can be shared by more actors rather than a few leading firms, and that reduces the centrality of the entire network and yields more separated knowledge groups.

4 Conclusions and Limitation

This paper detects the effect of social proximity on degree distribution of knowledge network by revising the connecting mechanism of BA model. Through simulation, we find out that the degree distribution still obeys power-laws when we change the preferential attachment mechanism, i.e. most of nodes have limited connections with other actors while few nodes have significant larger number of degree. Peripheral nodes with ties in a specific region would become more difficult in expanding the range of its ego network since the core actors have obviously advantage in attracting new ties.

However, if actors broaden their searching range via social relationships, such as former colleagues, friends (Ramírez-Pasillas, 2010), classmates, relatives as well as previous co-workers (Sharmeen & Arentze, 2014), peripheral nodes in the network will have more chance to be chosen as potential knowledge sources since local influential firms will has less advantage compare with the situation in which all new comers are trapped in specific space when seeking knowledge sources. If the role of geographical proximity could be replaced, partially in our model, by social proximity, new comers themselves will benefit from great extent of social proximity with external actors when they completely involve in a network and become a part of it. Therefore, the distance between peripheral nodes and core actors, in terms of in-degree, has been shortened. This finding is important to firms which endeavor to enhance their network position and get rid of the knowledge control of the core actors and local lock-in (Uzzi, 1996; Boschma, 2005).

In all, our research contributes to present literature in providing a model that can explain how social proximity affects the typology of knowledge network with power-laws degree distribution. Prior studies suggested that actors in a network with more ties have greater advantage in building new ties (Yang et al., 2015). Little attention has been put on how different proximities affect the constructing of knowledge ties during the growth process of a scale-free network. We modified the BA model and detect the role of social proximity in shaping the typology of a scale-free network by changing the proportion of nodes with globe information about the network and enlarging searching range of nodes that merely have local information.

Our model can be adopted to analyze the evolution of scale-free network in real world and solutions for some practical problems, such as why more and more nonlocal ties are involved in local knowledge network in industrial clusters (Fitjar & Rodríguez-Pose, 2011) and how to improve the status of less important firms in that network. To managers who endeavor to enhance their role in existing network, make a better use of their social network and construct nonlocal ties is another feasible channel besides being close to local leading firms.

There are also several limitations of our study. First, in this paper, we only detect the effect of two types of proximities. However, technological proximity is also an important factor affecting knowledge searching and acquirement and which has not been introduced in our model. Second, we assume that the social proximity may equally affect all actors' decision on knowledge acquirement which does not distinguish the role of various types of social relationships and their mutual effect.

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