The Cognitive Structure of Students in Understanding Mathematical Concepts

Suharto Suharto, Wahyu Widada
Mathematics Education
University of Bengkulu
Bengkulu, Indonesia
suharto9813@gmail.com, w.widada@unib.ac.id

***Abstract*—The mathematics was a difficult subject for students to study in senior high school. The purpose of this study was to determine the role of cognitive structure of students in understanding mathematical concepts. The Samples were randomly selected as many as 140 students from the whole students of senior high school at the Kota Bengkulu. There are two the latent variable. It was the ability to understanding math concepts and cognitive structure. There are two and seven indicator variables, respectively. The research instrument was a test of the ability to understanding mathematical concepts and tests of cognitive structures. The data from the two tests were analyzed using Confirmatory Factor Analysis (CFA). The results, the suitability of the whole model was good (i.e. model fit). Conclusion, there is a positive direct effect of cognitive structures on the ability to understanding mathematical concepts.**

Keywords*—cognitive structure; understanding mathematical concepts

I. INTRODUCTION

Problem solving ability is one of the competencies students Mathematics is one of the required subjects in senior high school. The results of the study show that mathematics is difficult for students [1]. The teacher was managing mathematics learning through a structuralistic and mechanistic approach. Teachers are more active in delivering material orally than students [2,3]. Students tend to be passive and wait for the teacher's command [4]. He does not have initiative and is less creative. The consequence is that students have misconceptions [5]. All of this leads to students having low concepts comprehension abilities [1,6]. Even though, students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge [7]. Students must to be communicate their thinking and reasoning both orally and in writing [8]. So that, a student to be mathematically proficient.

Students can understand mathematical concepts correctly, when they are able to do abstractions, idealizations and generalizations [3]. Therefore, it needs to be designed two sets of lesson materials that were identical except for the use abstract and contextualised examples. This enabled a precise exploration of the relative benefits of abstraction and contextualisation for learning advanced mathematics [9].

Concepts are abstract ideas that are used to classify examples and not examples. Restrictions from a concept are definitions. The formation of the definitions is "concept name" is "genus proximum" which is a "specific differentiator". Therefore, to understanding the concept students must be able to make sense of a concept [3]. In mathematics, trying to master processes and symbolism might create enough pressure on limited working memory capacity. Students cannot overcome all concepts (understandings), procedures, symbolism and applications at the same time [10].

Understanding mathematical concepts is a problem for students. When students must determine the proximal genus and specific differentiator [11]. In addition, how to distinguish between understanding actions and processes and when connecting "good understanding" of mathematical situations (concepts, theories, problems) with action sequences overcoming specific obstacles to this situation [12]. According to Hirschfeld-cotton [8], increased understanding occurs through probing questions that cause students to reflect on their learning and reevaluate their reasons. This occurs when students are expected to write more than one concept for a mathematical journal. By making students aware of their understanding through oral and written communication, students realize that true understanding does not originate from the completion of homework, but from evaluating and evaluating their own ideas and those of others.

In learning mathematics, students need to think actively. There are two student mindsets namely predicative and functional which are often referred to as cognitive structures [13]. According to her, predictive thinking is in terms of relations and judgments; functional thinking and available actions and achievable effects. Depending on the way of orientation in the world, the type of sources for getting insight are not the same.

The students must maximise the use of his/her cognitive structure by focusing on concepts and methods that work, discarding earlier intermediate stages that no longer have value [14]. According Schwank, the term functional thinking for motoric thinking which includes only such motoric actions useful for productions [13]. In contrary, the thinking that doesn't care so much on dynamics but on static structures and the embedded complex relationships, it is predicative thinking. Therefore, the teacher must be a good learning agent [15].

Powerful agent in learning with understanding is by going through mathematical constructions knowledge – thinking about thinking [14].

To set this in context, we need to focus on the cognitive structure developed by students calculating with numbers and manipulating symbols. The mathematical thinking uses one of the most powerful and natural constructions of the human mind—the ability to use symbols to switch between concepts and processes [16]. The link between the cognitive structure and modelling to improve mathematics education [17]. Therefore, increasing the mathematical abilities of high school students is influenced by their cognitive structure. Widada found that in term of the cognitive structure of learners that was reviewed based on the abstraction ability in the formation of mathematical concepts, 74% of students had the constructive functional element and 26% of students whose constructive elements were predictive [1]. Thus, most cognitive structures of learners in performing abstraction is functional.

The results of research Weerd and Verhoef, students that are good in modelling have a rich cognitive structure [17]. Students that are placed in a high class have a rich cognitive unit. When a student is good in modelling and is placed in a high class, there is compression. These students have a rich cognitive unit as well as a rich cognitive structure. The cognitive structure can be developed to a rich cognitive structure by repetition. Repetition can be done in several ways, and can be compressed. As stated by Tall, there are various methods of compression of knowledge in mathematics, including: representing information visually (a picture is worth a thousand words); using symbols to represent information compactly; if a process is too long to fit in the focus of attention, practice can make it routine so that it no longer requires much conscious thought [14].

In the process of learning mathematics, students practice various forms of thinking activities aimed to substantially contribute to the development of their different cognitive structures [18]. Furthermore, we can read the results of study of Widada [19], that was long-term study of the cognitive structure of students in understanding mathematics. Also, the results of his research strengthen and complement students' cognitive development (Extended Level Triad ++) theories in learning mathematics [3].

Increasing the ability to understanding mathematical concepts is better if the teacher manages learning through a realistic approach. The average mathematical ability of students taught in the classroom applies a higher realistic mathematics learning approach compared to those taught by applying conventional learning [1,20-22]. Because, cognitive structures help students learn how to learn [23]. Thus, cognitive structure is closely related to students' ability to understanding mathematical concepts [1]. Without effective cognitive structures, students often drop out of school mentally as early as third grade [23].

Teachers can use everyday lessons to help students develop more effective cognitive structures to learn how to learn so they can make sense of what is taught. The basic cognitive structures compare bits of data to process information for meaning [23].

Based on the above quotation, cognitive structure influences the ability to understanding concepts. There is a form of cognitive structure of students that is predicative and functional [13]. While the ability to understanding concepts can be measured based on seven indicators. The indicator is (1) Restate a concept; (2) classify objects according to certain properties in accordance with the concept; (3) provide examples and non-examples of the concept; (4) presents the concept in different forms of mathematical representation; (5) develop a condition necessary or sufficient condition of a concept; (6) use, utilize and choose specific procedures; and (7) apply the concept to algorithm to problem solving [24]. Thus, we measure the contribution of cognitive structures to the ability to understanding concepts based on these indicators.

II. METHOD

The type of research is a survey. The population is the whole students of senior high school at the Kota Bengkulu. Samples were randomly selected as many as 140 students. There are two the latent variable. It was the ability to understanding math concepts and cognitive structure. There are two and seven indicator variables, respectively. The research instrument was a test of the ability to understanding mathematical concepts and tests of cognitive structures. The data from the two tests were analyzed using Confirmatory Factor Analysis (CFA).

III. RESULTS AND DISCUSSIONS

The data of the test were the ability to understanding mathematical concepts (= con) and cognitive structures (= cog) as latent variables. As for each indicator variable is the predicative (X1) and functional (X2) for cognitive structure and restate a concept (X3); classify objects according to certain properties in accordance with the concept (X4); provide examples and non-examples of the concept (X5); presents the concept in different forms of mathematical representation (X6); develop a condition that is sufficient or sufficient condition of a concept (X7); use, utilize and choose specific procedures (X8); and apply the concept to algorithm to problem solving (X9), for variable concept comprehension ability. Data were analyzed using Lisrel 9.1. The results of data analysis are presented in Table 1.

TABLE I. GOODNESS OF FIT (GOF)

Statistics	**Calculation Results**	**Criteria (fit)**	**Description**
Minimum fit function chi-square	44.64 (P = 0.013)	p > 0.05	Not good
RMSEA	0.067	< 0.08	Fit
RMR	0.020	≤ 0,10	Fit
Standarized RMR	0.053	≤ 0,10	Fit
GFI	0.94	≥ 0,90	Fit
AGFI	0.89	**0.80≤ AGFI < 0,9**	Fit
NFI	0.94	≥ 0,90	Fit
NNFI	0.96	≥ 0,90	Fit
CFI	0.97	≥ 0,90	Fit
IFI	0.97	≥ 0,90	Fit
RFI	0.92	≥ 0,90	Fit
PNFI	0.68	≥ 0,00	Fit

Based on the results of data analysis, it was found that all statistical tests met the criteria of fit. Thus the theoretical model corresponds to empirical data (= fit). Based on Table 1, there are 11 sizes of GOF that show good compatibility, and only one is not good. So it can be concluded that the suitability of the whole model is good (model fit).

CFA results, then contained in the path diagram as can be seen in Figure 1 (Basic Model Standardized Solution).

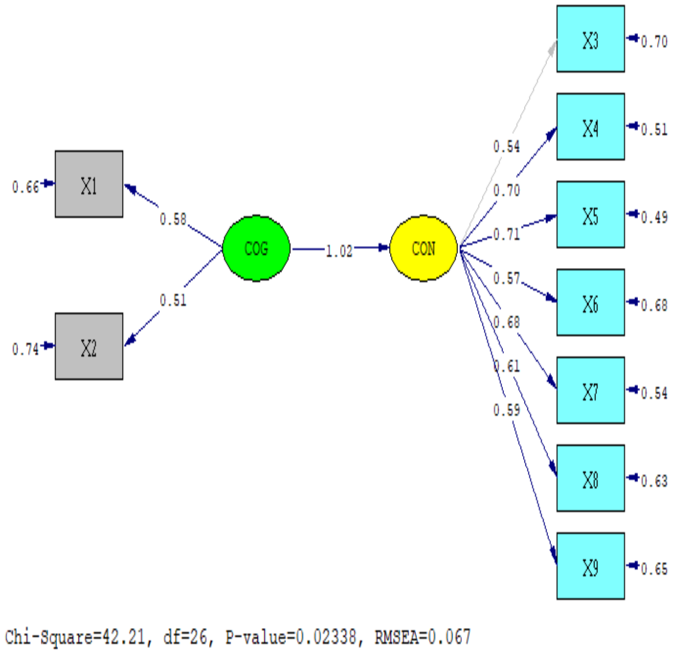

Fig. 1. Basic model standarized solution.

Furthermore, Lisrel 9.1 also produces the Basic Model T-Values path diagram. The diagram is Figure 2.

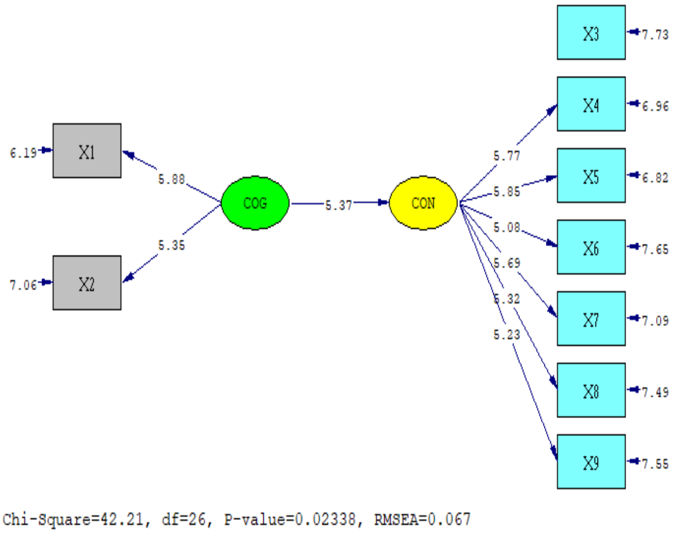

Fig. 2. Basic model T-values.

Based on the path diagram of Figure 1 and Figure 2, summarized the validity and reliability of two latent variables, namely the cognitive structure and the ability to understanding mathematical concepts. These summaries are Table 2 and Table 3.

TABLE II. VALIDITY AND RELIABILITY OF COGNITIVE STRUCTURE

Indicator	**SLF ≥ 0.50**	***Standard Errors***	***t-value***	**Declaration**	**Reliability**	
					CR≥0.70	***VE≥0.50***
X1	0.58	0.66	5.88	Good validity	0.46	0.30
X2	0.52	0.74	5.35	Good validity		

Based on Table 2, this shows that there are two observed variables (i.e., X1, and X2) for latent variables (cognitive structures) that have passed the validity test. This is because it meets the requirements, namely the factor loading value ≥ 0.50 and t-value ≥ 1.96. However, construct reliability (CR) is 0.46 <0.70, indicating that the reliability test of the cognitive structure variables is still lacking in consistency.

TABLE III. VALIDITY AND RELIABILITY OF THE UNDERSTANDING MATHEMATICS CONCEPTS

Indicator	***SLF≥ 0.50***	***Standard Errors***	***t-value > 1.96***	**Declaration**	**Reliability**	
					CR ≥ **0.70**	VE ≥ **0.50**
X4	0.54	0.71	**	Good validity	0.86	0.48
X5	0.56	0.69	5.77	Good validity		
X6	0.58	0.66	5.85	Good validity		
X7	0.87	0.24	5.08	Good validity		
X8	0.84	0.3	5.69	Good validity		
X9	0.84	0.29	5.32	Good validity		
X10	0.52	0.73	5.23	Good validity		

To explain the indicator variables for understanding the ability of mathematical concepts, see Table 3. The table confirms the validity of the seven observed variables (X3 - X9) on the variable ability to understanding mathematical concepts. The seven observed variables are valid. The seven manifest variables are restating a concept (X3); classify objects according to certain properties in accordance with the concept (X4); provide examples and non-examples of the concept (X5);

presents the concept in different forms of mathematical representation (X6); develop a condition that is sufficient condition of a concept (X7); use, utilize and choose specific procedures (X8); and apply the concept to algorithm to problem solving (X9). This is in accordance with the provision that the loading factor value is ≥ 0.50 and t-value ≥ 1.96. For reliability, the construct reliability value (CR) is 0.86 ≥ 0.70, indicating that the reliability test of the mathematical ability variable produces good values. Thus, mathematical abilities have good consistency.

To determine the significance of the positive direct effect of cognitive structure variables on the ability to understanding mathematical concepts, consider Figure 2. The path diagram shows that t-value = 5.37> 1.96. This means that with a 95% confidence level, the test states Ho is rejected and Ha is accepted. Ha in this study is that cognitive structures have a positive direct effect on the ability to understanding concepts. The results of this study support the conclusions of Weerd and Verhoef that the cognitive structure can be developed to a rich cognitive structure by repetition [17]. Repetition is in the development of modeling. Tall, state that to set this in context, we need to focus on the cognitive structure developed by students calculating with numbers and manipulating symbols [16]. The mathematical thinking uses one of the most powerful and natural constructions of the human mind—the ability to use symbols to switch between concepts and processes.

According to Catarreiraa et al. that are knowledge is organized from small elements we call concepts [25]. Each concept in mind equates to a relatively stable structure, the cognitive structure, with the elements interrelated. The elements forming these structures follow a functional correspondence with the neuronal circuits of the human brain and a mental correspondence with the representations in the form of diagrams. Prior knowledge can be represented by these structures. Learning corresponds to the modification of the cognitive structure, by assimilation and accommodation. Thus, the cognitive structure has a positive direct effect on understanding mathematical concepts.

IV. Conclusion

The CFA test produces conclusions as follows: there is a positive direct effect of cognitive structures on the ability to understanding mathematical concepts. The data of the two construct variables show, there are 11 sizes of GOF that show good compatibility, and only one is not good. Finally, concluding that the suitability of the whole model is good (model fit).

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