

Application of Markowitz's Portfolio Theory in Obtaining the Best Portfolio in the Stock Market

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Abstract—The article mainly explores the mean-variance model proposed by Markowitz, and then applies it to the concept of contemporary securities investment. Investors can find the historical average weekly interest rate and covariance matrix of each stock in the A-share market, use Markowitz's investment concept model, and then use mathematics such as Matlab to calculate the collection of portfolio. After the study we find that Markowitz proposed portfolio concept has a specific using value in the domestic A-share market, but in practice it also has some shortcomings.

Keywords—Markowitz mean-variance model; Mean; Variance; Covariance; Optimal portfolio

I. INTRODUCTION

The portfolio concept explores investors' portfolios of diversified securities and diversified investments [1]. Its goal is to enable investors with portfolio investment capabilities to maximize their profitability and minimize their risks through their scientific combination [2]. The portfolio concept that Western investment used was first proposed by the famous American economic scholar Markowitz in his book "Securities Portfolio Selection", and then had gradually developed over the past 50 years and finally became a main theory in western investment concept system [3].

In 1952, Markowitz proposed the concept of investment portfolio [4]. Through the "Securities Portfolio Selection" article [5], we can find that Markowitz used the link between interest rates and risks of all capitals as a basis to further explore how to use the optimal portfolio in an uncertain economic system, and then obtain the fund separation law and laid the foundation for the creation of asset pricing theory. The idea of capital integration mean-variance put forward by Markowitz not only laid the foundation for the formation of the concept of contemporary capital combination, but also became the cornerstone of the entire modern financial theory, and was widely used in the financial industry in economically developed countries and regions. This theory is used to quantitatively determine the best investment portfolio and help people form a reasonable and reliable investment philosophy, which will help to stabilize the financial market [6]. Although the establishment of the model still had some deficiencies, the quantitative analysis of the investment interest rate and risk has been carried out for the first time, and a mathematical model has been constructed so that investors had the earliest reference basis for decision making.

II. THE ESTABLISHMENT OF A MODEL

A. The assumption of the Markowitz's model

There are no fees and taxes in the transaction process. There are no barriers to entering or exiting the market. Assets can be divided.

All investors are price acceptors who meet the risk aversion assumption.

The utility function has a gradually decreasing marginal feature, and the investment objective is consistent of the utility maximization.

The expected rate of return on all securities investments is expressed in a normal distribution; Investors use the volatility of the expected rate of return to assess the risk of the portfolio.

Investors only rely on the risk and profitability of the securities portfolio to make investment decisions. Its utility function only represents the relationship between risk and profitability [7].

B. Mean-variance model

1) Expected yield of securities portfolio

We use average rate of return to measure the return of a single stock. We use the weighted average of the expected yield of each stock to represent the yield of the portfolio:

$$R_p = \sum_{i=1}^n w_i \cdot r_i \quad (1)$$

where R_p represents the overall profitability of the portfolio, r_i represents the yield of the i -th stock, w_i represents the investment weight of the i -th stock.

2) Securities portfolio risk

We use the portfolio's variance to represent the portfolio's risk.

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot \sigma_{ij} \quad (2)$$

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j \quad (3)$$

where σ_p^2 represents the variance of the portfolio of securities, w_i, w_j represent the proportion of i-th, j-th investment in securities portfolio. σ_{ij} represents the covariance between i-th, j-th investment securities. ρ_{ij} represents the correlation coefficient between i-th, j-th investment securities. σ_i, σ_j represent the standard deviation of the i-th and j-th securities.

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} - 1 < \rho_{ij} < 1 \quad (4)$$

The proof is as follows:

First of all, we discuss the situation of having only A and B assets.

Variance Formula:

$$\sigma^2(ax + by) = a^2\sigma^2(x) + b^2\sigma^2(y) + 2ab\text{Cov}(x, y) \quad (5)$$

Portfolio yield:

$$R_p = w_a \cdot r_a + w_b \cdot r_b \quad (6)$$

Where w_i represents the proportion of capital i-th in the portfolio, and r_i represents the rate of return of capital i-th.

Thus, we can get it as follows:

$$\begin{aligned} \sigma^2(R_p) &= \sigma(w_a \cdot r_a + w_b \cdot r_b)^2 \\ &= w_a^2\sigma^2(r_a) + w_b^2\sigma^2(r_b) + 2\text{Cov}(w_a r_a, w_b r_b) \\ &= w_a^2\sigma^2(r_a) + w_b^2\sigma^2(r_b) + 2w_a w_b \text{Cov}(r_a, r_b) \\ &= w_a^2\sigma^2(r_a) + w_b^2\sigma^2(r_b) + 2w_a w_b \rho_{ab} \sigma_a \sigma_b \end{aligned} \quad (7)$$

If there are n kinds of assets:

$$D(R_p) = E [R_p - E(R_p)]^2 \quad (8)$$

$$\sigma^2(R_p) = E[\sum_{i=1}^n w_i r_i - E(\sum_{i=1}^n w_i r_i)]^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot \sigma_{ij} \quad (9)$$

From formula (9), it can be found that the degree of risk in the securities portfolio is related to the correlation between two or more securities. There is a relationship between investment weights and standard deviations for various securities. It is beneficial to share risks when adopting securities portfolios that have little connection with each other.

C. Markowitz effective boundary model

The effective boundaries for finding investments are through investigating different investment portfolios and discovering the best portfolio plans [8].

1) The contents of the effective boundary model

According to Markowitz's book "Selective Portfolio Theory," the $\sigma_p - \mu_p$ chart can be used to find the best portfolio's required boundary curves.

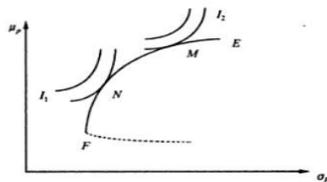


Fig. 1. Effective Boundary Model of Portfolio

The abscissa axis σ_p represents the risk of the portfolio, the ordinate axis μ_p represents the expected rate of return that portfolio has.

All the best investment portfolio points are unified on the border EF line. In the corresponding case, all points in the effective boundary are not valid.

From a perspective of investing, adopting the points on the effective boundary EF depends on the preferences of investors.

The two graphs I_1, I_2 represent the indifferent curves of different investor preferences. When investor 1 adopts a N-point portfolio, he can obtain the most effective investment portfolio under his investment preferences. Investor 2 is a proactive investor who wants to take on greater risks in order to get high returns.

III. DATA AND METHODOLOGY

A. Selection of stock samples

This article selected 5 representative stocks in the A-share market before December 31, 2010. By selecting CSMAR China Stock Market Trading Database, 601919 (Zhongyuanhaikong), 600519 (Kweichow Moutai), 600100 (Tongfang co), 000002 (Vanke A), 002092 (Zhongtai Chemistry), we have considered the overall industry representativeness of all stocks, transaction volatility, regional representation, and financial status of listed companies [9]. We have adopted these five typical stocks.

B. Time limit of sample data and choice of stock yield types

The article used the time period from January 1, 2009 to December 31, 2010 as the time required for each security. During this period, there were a total of 503 trading days. In this sample time, China's A-share market has both long-term upswings and downturns for a period of time. This has a strong sense of representativeness in the case study of data samples.

In different time periods, the commonly used profitability usually has an annual interest rate, monthly interest rate, weekly interest rate, and daily interest rate. Since the focus of the text is on the risks of investing in securities portfolios, the use of interest rates for investments in medium and long-term time periods takes into account risks, so the article uses weekly interest rates. During the period of article exploration, a total of 102 weeks of interest was used.

C. Sample data collection

From the CSMAR China Stock Market Transaction Database, the article selected the week-long stock dividend yield information for a total of 102 weeks of cash reinvestment from 2009 to 2010.

D. Sample data description

Calculate the respective weekly returns of the five stocks within the sample time limit.

$$r_i = \frac{\sum_{t=1}^N r_{it}}{N} \quad (10)$$

where r_i represents the weekly average profitability yield of the i -th stock, N represents the number of weeks in the sample time period, and N is equal to 102.

Calculate the standard deviation of the return rate of sample stocks within the sample time limit

$$\sigma_i = \sqrt{\frac{\sum_{i=1}^N (r_i - r_{it})^2}{N}} \quad (11)$$

Where σ_i is the standard deviation of the i -th stock.

Calculate the covariance between the sample stock returns.

$$\sigma_{ij} = E \{ [r_i - E(r_i)] [r_j - E(r_j)] \} \quad (12)$$

Where σ_{ij} is the covariance between stock returns.

The five sample stocks in the Shanghai-Shenzhen A-share market have their average weekly interest rate during the sample time period, and the final calculation results for the standard deviation and sample covariance are shown in Table 1 and Table 2.

TABLE I. STANDARD DEVIATION OF STOCK AVERAGE WEEKLY RATE OF RETURN

Security Weekly Trading price	Zhong Yuan haikong 601919	Kweichow Moutai 600519	Tongfang co 600100	Vanke A 000002	Zhongtai Chemistry 002092
Average weekly rate of return	0.003743307	0.006428505	0.011858881	0.004918396	0.014405248
Standard deviation	0.065722917	0.043054087	0.063604862	0.059534587	0.076499149
Source of Original Data: CSMAR China Stock Market Trading Database					

TABLE II. COVARIANCE MATRIX

	Zhong yuan haikong 601919	Kweichow Moutai 600519	Tongfang co 600100	Vanke A 000002	Zhongtai Chemistry 002092
Zhongyuan haikong 601919	0.0044	0.0006	0.0015	0.0012	0.0021
Kweichow Moutai 600519	0.0006	0.0019	0.0005	0	0.0007
Tongfang co 600100	0.0015	0.0005	0.0041	0.0012	0.0005
Vanke A 000002	0.0012	0	0.0012	0.0036	0.0005
Zhongtai Chemistry 002092	0.0021	0.0007	0.0005	0.0005	0.0059

Substituting the above data into the Markowitz portfolio model

$$\sigma_P^2 = \sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot \sigma_{ij} \quad (13)$$

$$R_P = \sum_{i=1}^n w_i \cdot r_i \quad (14)$$

limitation factor: $\sum_{i=1}^n w_i \geq 0$

Use Matlab tool modeling to achieve the results.

A. When the expected return 1 is set to 0.008, the following Table 3 results are obtained:

The weighted investment right of the most suitable combination of five stocks 1 is: Zhongyuanhaikong $w_1 = 0.0045$ Kweichow Moutai $w_2 = 0.4883$, Tongfang co $w_3 = 0.1596$, Vanke A $w_4 = 0.2167$, Zhongtai Chemistry, $w_5 = 0.1309$, The minimum variance is $\sigma_P^2 = 0.0011$.

B. When the expected return 2 is set to 0.011, the following Table 4 results are obtained:

The weighted investment right of the most suitable combination of five stocks 2 is: Zhongyuanhaikong $w_1 = 0.0002$ Kweichow Moutai $w_2 = 0.1579$ Tongfang co $w_3 = 0.8419$ Vanke A $w_4 = 0.0002$ Zhongtai Chemistry $w_5 = 0$ The minimum variance is $\sigma_P^2 = 0.0031$.

TABLE III. RESULTS A

	601919	600519	600100	000002	002092
Expected yield of securities portfolio 1	0.008				
Best Portfolio 1	0.0045	0.4883	0.1596	0.2167	0.1309
Standard deviation of securities portfolio 1	0.0336				
Variance of securities portfolio 1	0.0011				

TABLE IV. RESULTS B

	601919	600519	600100	000002	002092
Expected yield of securities portfolio 2	0.011				
Best Portfolio 2 (Sellable)	-0.188	0.3995	0.3385	0.1435	0.3066
Best Portfolio 2 (Not for sale)	0	0.1579	0.8419	0.0002	0
Standard deviation of securities portfolio 2	0.0554				
Variance of securities portfolio 2	0.0031				

IV. RESULT

Investors can use the mean-variance model proposed by Markowitz to obtain the existence of effective portfolios when they understand the historical average interest rate of each stock and the covariance matrix. Based on the above calculations, when the portfolio's interest rate is lower than 0.008, it can bear a very small investment risk: 0.0336. At this time, the investor spreads his own capital among the 5 stocks: Zhongyuanhaikong $w_1 = 0.0045$, Kweichow Moutai $w_2 = 0.4883$ Tongfang co $w_3 = 0.1596$ Vanke

A $w_4=0.2167$ Zhongtai Chemistry $w_5=0.1309$. When the investor hopes to get a profit rate of 0.011, he must also bear the risk of 0.0554. At this time, the investor spreads the funds in three stocks: Kweichow Moutai $w_2=0.1579$ Tongfang $w_3=0.8419$ Vanke $w_4=0.0002$. This result shows that investment portfolios with a high degree of diversification can obtain very few returns, and high risk corresponds to a portfolio with a high interest rate [10]. Using Markowitz's investment portfolio, an optimized portfolio structure can be achieved. Effective combination proposed by Markowitz is significantly stronger than random combinations, allowing investors to take small risks and gain as much as possible, which can be assessed using coefficient of variation. If Markowitz's effective combination has a coefficient of variation that exceeds the random weights, it means that under the established risk, the profitability that Markowitz proposed for the effective combination exceeds that of the random weights [11].

V. CONCLUSION

We can use the Markowitz capital portfolio concept in the domestic A-share market to find that its overall profitability exceeds the portfolio currently used by the securities market. The result proves that there is still some value in applying the capital combination idea proposed by Markowitz to the domestic stock market. It is precisely because of this that the concept of capital combination is of major practical significance for establishing a rational investment atmosphere and promoting the flourish of the domestic stock market. However, Markowitz's concept of capital composition also has some defects. There are transaction costs in actual operations, and transaction costs are particularly evident in the case of buying a great number of stocks in small quantities. However, this cost is missing from the theory proposed by Markowitz [12].

The asset portfolio theory proposed by Markowitz assumes that the interest rate is normally distributed, and the variance is the proper measure of its risk. However, many current case studies have raised doubts about the idea that securities are expected to meet the normal distribution of interest rates.

Under current technical conditions, it is very difficult for professionals first to obtain a set of investment portfolios through complex computer programs, and then to respond quickly to the rapidly changing securities market. In addition, when introducing a lot of data, it cannot ensure its accuracy

[13]. It can be found that although Markowitz's portfolio model has strict logic, it lacks practicality in the real environment. Investors cannot fully rely on this theory to implement investment, and they need to make appropriate changes and corrections based on actual conditions and their own experiences in order to avoid risks in investment and thus maximize investment diversification

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