

SIMFAC-A New Forecasting Method for Sporadic Time Series

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Abstract. This essay relates mainly to sporadic FC (forecasting) methods and error measures. The existing related FC methods of sporadic time series (STS), including the SES (Simple Exponential Smoothing), Croston's / SBA method and patented WSS method as well as two applicable error metrics APE and THEIL'S U are introduced briefly. Then the focus is laid on the analysis and presentation of a new forecasting yet unpublished method, SIMFAC (1), which is dedicated to STS and includes a new error metric, MEM (Matching Event Metric). For a more comprehensive comparison among methods, Cosine Similarity (CS) metric, will be introduced and applied in this essay.

Keywords: Sporadic data; Forecasting methods; Croston; SIMFAC; Error Metrics.

1. Introduction

The first method which is applied to STS forecasting is SES method, which was proposed in 1956 for forecasting data with no zeros and no clear trend or seasonality. But this method proved to be strongly biased-especially after a non-zero demand [1].

The first sporadic demand specific method was proposed by Croston [2], who applies the exponential smoothing method separately to estimate the inter-demand interval (IDI) and demand size. Croston's method is the most widely used approach that addressed the issues related to intermittent demand forecasting [3]. After that, many modified Croston's methods were also proposed.

Willemain et al. [4] proposed a patented method, called the WSS method, by using bootstrapping on observations of non-zero demand to forecast demand over a fixed lead time.

There are few substantial improvements in sporadic data forecasting except Croston's-like method and WSS methods. Sporadic demand series are difficult to forecast because they usually contain a (significant) proportion of zero values, with non-zero values randomly mixed.

2. Traditional FC Methods and Error Metrics

There are currently two main branches of forecasting methods for sporadic data, one is based on Croston's method; another is represented by patented WSS method.

2.1 Existing Forecasting Methods for Sporadic Data

2.1.1 SES Method

In SES, the forecast of demand in next period is defined as:

$$S_{t+1} = \alpha X_t + (1 - \alpha)S_t \quad (1)$$

where X_t is the observed value of both zero and nonzero demand, S_{t+1} is the smoothed average as well as the forecast for next period, α is the smoothing parameter, which can be adjusted between 0 and 1. Higher α will produce a forecast which is more responsive to recent changes in the data, whilst also being less robust to any errors that could occur.

2.1.2 Croston's Method and Modifications

In this method, Croston forecasted the inter-demand interval and demand size for the first time separately by SES.

Croston's method can be expressed as follows:

If $x_t = 1$, then

$$\begin{cases} Z_{t+1} = \alpha X_t + (1 - \alpha)Z_t \\ P_{t+1} = \beta q_t + (1 - \beta)P_t \\ q_t = 1 \end{cases} \quad (2)$$

If $x_t = 0$, then

$$\begin{cases} Z_{t+1} = Z_t \\ P_{t+1} = P_t \\ q_{t+1} = q_t + 1 \end{cases} \quad (3)$$

The mean demand per period is then

$$Y_{t+1} = \frac{Z_{t+1}}{P_{t+1}} \quad (4)$$

Syntetos and Boylan (2005) [5] recommended an adjustment of the Croston method, the SBA method (Syntetos-Boylan Approximation), a modified version of equation that is approximately unbiased, shown as formula (5), will also be tested in this thesis.

$$Y_{t+1} = \left(1 - \frac{\alpha}{2}\right) \frac{Z_{t+1}}{P_{t+1}} \quad (5)$$

Besides, there are lots of modifications based on the Croston's method, see Levén and Segerstedt (2004) [6], Teunter et al (2011) [7], Wallström and Segerstedt (2010) [8], Segerstedt and Levén (2014) [9] etc.

2.1.3 Patented WSS Method

WSS uses bootstrapping idea on historical observations of non-zero demand to forecast demand distribution over lead time. In order to avoid forecasting that only take the same values from the observation, Willemain et al introduced a so-called "jittering" process, which can produce more variation to the historical observation values. A two state Markov Chain model, corresponding to zero and non-zero demand observations, is adopted to simulate the autocorrelation in the demand.

2.2 Applicable FC Error Metrics for STS

On the one hand, it is about the "demand events", which includes the number of total positive demand events over forecast periods, which represents a big difference when compared with the standard time series, and the location or timing of the events on the time axis. No traditional forecasting methods have done this before. On the other hand, it is the demand size over forecast periods, which can be expressed in terms of the mean demand per forecast period or demand size of the demand events.

THEIL'S U method compares the forecasting quality between specified methods and the Naïve method; APE is one of the most intuitive and convenient metrics to measure the difference between non-zero values.

2.2.1 THEIL'S U

Naïve Method

The Naïve method is the simplest but not the worst of all forecasting methods. In this method, the forecast value of the next period is exactly just the historical actual value of the last period. Even now, the Naïve method is widely used as the benchmark method for measuring forecast accuracy because it requires little informational data. [10]

THEIL'S inequality coefficients (THEIL'S U)

This error measure was presented by THEIL in two different versions, the older version of the error metric is called U_1 , the other is called U_2 , in this essay THEIL'S U_2 is applied, shown as formula (6). However, both formulas are originally directed at standard time series, but for STS, if

the zero value can be turned into a value which is smaller than one but not zero, then the formula can also be applied.

$$U_2 = \sqrt{\frac{\sum_{t=2}^n (\frac{\hat{Y}_t - Y_t}{Y_{t-1}})^2}{\sum_{t=2}^n (\frac{Y_t - Y_{t-1}}{Y_{t-1}})^2}} \quad (6)$$

Table 1 interprets the index and the corresponding explanation of THEIL'S U.

Table 1. Interpretation of THEIL'S U

Wert	Implication
U=0	A perfect forecasting method was chosen.
0<U<1	Chosen method performs better than Naïve method.
U=1	Two FC methods are the same.
U>1	Naïve method performs better than chosen method.

2.2.2 APE (Absolute Percentage Error)

MAPE (Mean Absolute Percentage Error) cannot be applied in sporadic data although it is one of the most popular measures of the forecast accuracy. Moreover, MAPE is the average of absolute percentage errors (APE). Therefore, when it comes to measure only one non-zero value, such as the average of the forecasting over one year, APE is the most intuitive and convenient metric.

Before going into the APE, the errors can be defined by the formula (7):

$$e_t = X_t - F_t \quad \forall t \quad (7)$$

In this case, e_t denotes a forecast error (residual / deviation) at time t , X_t represents the actual value and F_t is the forecast values. The absolute percentage error APE then takes place as the formula (8):

$$APE = \frac{|e_t|}{X_t} = \frac{|X_t - F_t|}{X_t} \quad \forall t \quad (8)$$

3. SIMFAC (1) and CS

3.1 SIMFAC (1) for "Demand Events"

Up to now there is no solution reported, forecasting the number of demand events along a specified lead time. SIMFAC (1), developed by Spicher [12], offers a solution for this unsolved problem for the first time. SIMFAC (1) can be applied to time series of any (relevant) length and works without any pre-set theoretical assumptions, i.e. just using the given historical sporadic data.

Following is the specific steps of SIMFAC (1) heuristic:

Given a sporadic time series of length N ; $V(t)$ for $t=1$ to N ;

For some points in time t , $V(t) = 0$ and $V(t) > 0$ else. The function

$$z(t) = \begin{cases} 0, & \text{if } V(t) > 0 \\ 1, & \text{if } V(t) = 0 \end{cases} \quad (9)$$

identifies the value gaps in the original data.

The forecast period of length K has to be specified.

The algorithm:

Step 1: Accumulating the zeros along the time axis up to N resulting in a discrete cumulative function denoted $Z(t)$ for $t=1, \dots, N$:

$$Z(t) = \sum z(t) \text{ for } t=1 \text{ to } N \quad (10)$$

Step 2: Approximation of $Z(t)$ (e.g. linear, polynomial regression, ...); the approximation function is denoted $Z^*(t)$.

Step 3: Extrapolation of $Z^*(t)$ over the forecast period from $t=N+1$ to $t=N+K$.

Step 4: Rounding the values of $Z^*(t)$ down to Integers resulting in $ZI^*(t)$.

The result might look like $ZI^*(t) = \{18, 19, 19, 20, 21, 22, 22, 23, \dots\}$ for $t=N+1$ to $N+K$, covering the complete forecast period of length K.

Step 5: Transformation of $ZI^*(t)$ into a $\{0,1, \dots\}$ -sequence according to:

If $ZI^*(t-1) = ZI^*(t)$ then the corresponding function $ZI^{**}(t)=1$ else $ZI^{**}(t)=0$.

The corresponding function is $ZI^{**}(t) = \{0,1,0,0,0,1,1,0, \dots\}$ in line with example in step 4.

Step 6: The estimate of number and timing of events results from the Intersections of Integers and approximation function.

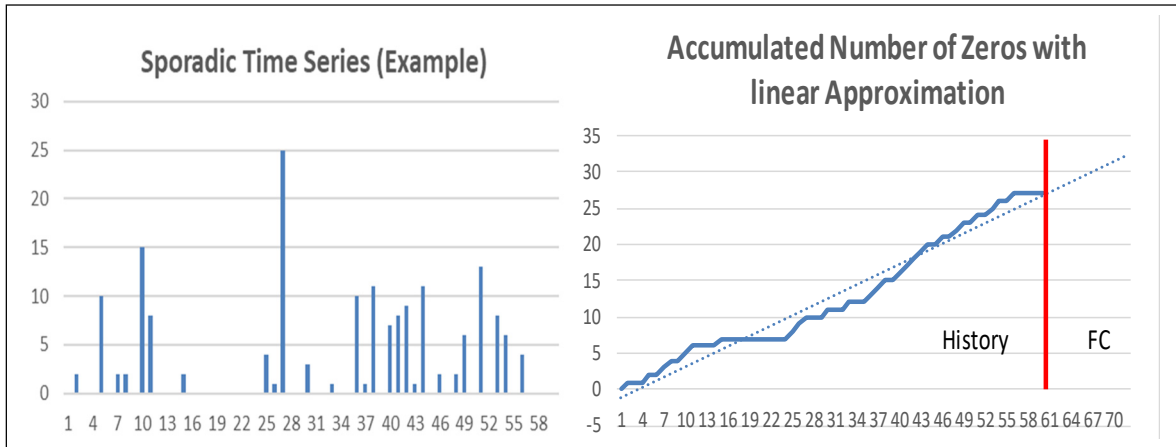


Fig. 1 Schematic of SIMFAC (1) method

All t values with $ZI^{**}(t)=1$ identify points on the time axis with estimated future demand > 0 . The sum

$$\sum ZI^{**}(t) \text{ for } t= N+1 \text{ to } N+K \quad (11)$$

specifies the forecasted number of “demand events” during the forecast periods.

The result consists of:

- The number of events during the forecast period;
- Their timely locations (months in this case).

3.2 Matching Event Metric (MEM)

The SIMFAC (1) approach, estimating the number of events, the timing and the related values require a new quality conception for comparing the accuracy of different time series and of different methods.

Let K be the length of the forecast period. Let E be the number of real events and E^* the forecasted number of events. Further, let S^* be the forecasted number of matching real events i.e. matching the real events in time. Then the Matching Error Metric MEM is defined as

$$MEM = |E - E^*| + (E - S^*) / K \quad (12)$$

with $(E - S^*) / K = 0$ for $E = 0$.

In order to explain this problem more clearly, an example is given below.

Forecast Period of length k with $k = N = 12$;

Forecasted Events: $E^* = 3$, in months e.g. $\{1, 7, 10\}$;

Real Events: $E = 5$, in period e.g. $\{2, 3, 5, 9, 12\}$;

No matches; Therefore $S^* = 0$.

$$MEM = |5 - 3| + (5 - 0) / 12 = 2.42$$

In case of $E^*=3$ with $\{3, 5, 8\}$; $S^*=2$; $MEM = 2 + 3/12 = 2.25$. In case of forecasting 5 events and 5 matches $\{2, 3, 5, 9, 12\}$, then $MEM = 0$, representing the perfect forecast.

The result of forecasting the number of events, the number of matches and the related MEM-Calculation can be seen from table 2. The smaller the MEM error, the better the forecasting quality of the number of events.

Table 2. Illustration of MEM

	In-Sample Number of Events Forecast				
	Month	Actual	Example 1	Example 2	Example 3
Forecast Period	1	-	X	-	-
	2	X	-	-	X
	3	X	-	X	X
	4	-	-	-	-
	5	X	-	X	X
	6	-	-	-	-
	7	-	X	-	-
	8	-	-	X	-
	9	X	-	-	X
	10	-	X	-	-
	11	-	-	-	-
	12	X	-	-	X
Event E		5	3	3	5
Matches S^*			0	2	5
MEM Error			2.42	2.25	0

Table 3. FC aspects and error metric for STS

FC Methods	Demand Values FC			Events FC	
	Mean	Distribution	Values	Num- ber	Tim- ing
Croston / SBA	X				
WSS	x	X			
Naïve	X		X	X	X
SIMFAC(1)	X		X	X	X
Error metric	APE & THEIL'S U			MEM	
	Cosine Similarity				

3.3 SIMFAC (1) for Event-related Values

The estimation of the future (demand) values is regarded highly speculative. The method of estimating the demand values follows in principal the method for event forecasting, but there are some modifications. Instead of accumulating the numbers of Zeros now accumulate the (demand) values as Step 1. In order to include the structure of (demand) values of history in Step 2 a stepwise approximation can be applied using either a polygon shape or stepwise (linear) regression. For demonstrating the principle, here a linear approximation is applied. Of course, also the value estimations depend on the shape of the approximation function(s).

In line to forecasting the number of events, now get a set of (demand) value forecasts for each forecasted event.

Let F_i^* represent all (demand) value forecasts based on histories with forecast period time index i ($i=1$ to k). F_i^* result from the extrapolation of the approximation functions – more precisely – here from their constant slope m .

$$F_i^* = y_i = m * x_i + b \quad (13)$$

representing value forecasts.

Matching the selected event forecast and F_i^* value forecasts will represent the compound result of SIMFAC (1). The result consists of ① estimated number of events, which means the number and

exact timing of demand events, in months e.g. {2, 3, 5, 9, 12}; and ② values per estimated events, which means the exact demand values of demand events, e.g. {0, 15, 23, 0, 7, 0, 0, 0, 54, 0, 0, 65}.

3.4 Cosine Similarity and Euclidean Distance

Cosine Similarity between two vectors (or two documents on the Vector Space) is a popular measure of similarity that calculates the cosine of the angle between them [13]. This metric is a measurement of orientation and not magnitude.

In positive space, the cosine offers the suitable property that it is 1.0 for identical vectors, maximally “similar” and 0.0 for orthogonal vectors, maximally “dissimilar”. [14]

If a and b represent two vectors, then the cosine similarity is calculated as follows:

$$S_{cos}(a, b) = \cos(\theta) = \frac{a \cdot b}{||a|| * ||b||} \quad (14)$$

where $a \cdot b = a^T * b$ denotes the scalar product and $||a|| = \sqrt{a^T * a}$ the norm of vector.

3.5 Summary

SIMFAC (1) method gives us a prediction information of demand events as well as the events-related demand size for the first time, which no one has done before. Naïve method, as a reference method, can provide us a basic standard for forecasting quality. Including Croston & SBA and WSS, there are totally five forecasting methods.

Seen from table 3, those methods can be compared together only in “Mean” through APE and THEIL'S U or some other error metrics since existing FC methods i.e. Croston & SBA and WSS provide no information about either total number or the timing of demand “events”. The demand “events”, which can be predicted only by Naïve method and SIMFA C (1), can be compared by the MEM error metric. The only error metric, which can measure “Demand Values” and “Demand Events” together, is CS. CS measures directly the similarity between vectors, which, from my point of view, is much closer to nature of sporadic data. Traditional error metrics, such as APE, measure the average demand size of a given forecast period, which is only one side of sporadic data. In practice, when considering costs such as inventory costs, then Cosine Similarity promises to be of value for practitioners. Moreover, Cosine Similarity represents a holistic view on forecast accuracy / -errors which opens a new door to sporadic data specific forecasting methods.

4. Forecasting Results Comparison

The tests described in this essay are based on 2 years history and 1-year forecast period. SIMFAC (1) allows selecting the length of history to be used. Here, we used 2 years as history.

4.1 FC comparison of the Average Demand Size

Example of 156/1# (name of one time series):

Time series in 2013 - {16,6,8,8,25,34,29,0,4,0,8,20};

Time series in 2014 - {0,8,4,8,0,0,8,0,14,16,24,12};

Actual time series in 2015- {8,4,0,12,4,12,8,0,0,8,0,4}.

Here, the average demand size of different forecasting methods, including Croston & SBA, WSS and SIMFA C (1) will be compared with Actuals under APE and THEIL'S U.

Table 4. Average comparison for data 156~1#

Data: 156~1#		Forecasting Methods					
FC Periods	Actuals	Croston	SBA	WSS		Naïve	SIMFAC(1)
				Interval	Frequency		
25	8	10.45	9.41	32~51.9	3	0	6
26	4	10.45	9.41	51.9~71.8	5	8	0
27	0	10.45	9.41	71.8~91.7	14	4	11
28	12	10.45	9.41	91.7~111.6	26	8	8
29	4	10.45	9.41	111.6~131.5	23	0	10
30	12	10.45	9.41	131.5~151.4	10	0	0
31	8	10.45	9.41	151.4~171.3	5	8	11
32	0	10.45	9.41	171.3~191.2	9	0	10
33	0	10.45	9.41	191.2~211.1	3	14	0
34	8	10.45	9.41	211.1~231	2	16	11
35	0	10.45	9.41			24	12
36	4	10.45	9.41			12	0
Average	5	10.45	9.41	9.91		7.83	6.58
156/1	APE	109%	88%	98%		57%	32%
	Theil's U	0.71	0.56	-		0.00	0.18

Table 5. Comparison under APE and Theil's U

TS	Metrics	Croston	SBA	WSS	Naïve	SIMFAC(1)
75/3#	APE	62.3%	66.1%	42.8%	0	148.1%
	Theil's U	0.896	0.905		1	1.481
108/3#	APE	110%	89.6%	97.6%	92.6%	16.7%
	Theil's U	0.857	0.849		1	1.164
157/3#	APE	37.6%	43.9%	23.3%	11.6%	24.4%
	Theil's U	0.639	0.667		1	0.773
164/1#	APE	60.3%	44.2%	42.9%	24.2%	12.9%
	Theil's U	0.360	0.341		1	0.277
...
Results	Intermittent Data: Naïve > SIMFAC(1) > WSS > Croston > SBA					
	Lumpy Data: SBA > WSS > Croston > Naïve > SIMFAC(1)					
	Smooth Data: SBA > WSS ≈ Naïve > Croston ≈ SIMFAC(1)					
	Erratic Data: WSS > Croston > SBA ≈ Naïve > SIMFAC(1)					

From table 4, we can see that SIMFAC (1) method performs best under the error metric APE. Among the left four methods, Naïve method is better than WSS, SBA and Croston's method, and WSS, SBA and Croston's method have almost the same forecasting accuracy.

However, under Theil's U the other four methods are better than Naïve method. The reason behind is that APE is simple and intuitive to apply although, it has not taken the existence of zeros into account. Since Theil's U measures the relative accuracy, which uses as reference forecast the last observed values recorded in the data series.

Based on the same idea, the left data are tested in different groups, results are shown as table 5.

From table 5, we can see that SBA method and WSS methods predict the average demand values with a better forecasting accuracy in general. The reason behind is that SBA method is based on huge statistical tests already and has its own data area with best average demand size accuracy. Naïve method predicts also well in intermittent data. While SIMFAC (1), as the only method which can predict both demand size and inter-demand intervals, has also not bad forecasting accuracy in intermittent data.

4.2 Comparison of FC Number and Timing of Events

The process is to compare the forecasting of number of events estimated by SIMFAC (1), Naïve method. And error metric is MEM.

All data are tested and the results are shown as table 6:

Table 6. Demand Events Comparison under MEM

Intermittent			Lumpy			Smooth			Erratic		
TS	Naïve	SIMFAC(1)	TS	Naïve	SIMFAC(1)	TS	Naïve	SIMFAC(1)	TS	Naïve	SIMFAC(1)
74/1#	1.08	1.17	76/1#	1.08	2.08	138/1#	0.33	2.17	139/1#	0.17	3.08
75/3#	0.17	1.17	81/3#	0.08	0.08	138/3#	4.08	4.00	139/3#	5.50	6.50
77/3#	0.08	0.08	83/1#	1.17	2.25	153/1#	0.17	1.33	140/1#	5.42	4.42
84/1#	0.08	0.08	85/3#	1.17	1.17	153/3#	2.33	1.33	140/3#	0.33	1.08
84/3#	2.08	1.08	86/1#	1.00	0.08	157/1#	1.08	1.25	141/1#	5.42	2.42
89/1#	1.17	2.17	95/3#	1.08	0.08	157/3#	5.00	4.08	141/3#	4.08	2.00
90/1#	1.17	0.17	96/1#	3.25	3.25				142/1#	4.17	4.08
92/1#	1.08	2.08	96/3#	1.08	1.08				142/3#	1.25	0.33
93/3#	0.08	0.08	98/1#	0.08	0.08				143/1#	2.42	1.25
97/1#	2.00	1.08	98/3#	1.17	1.17				143/3#	6.17	7.00
97/3#	1.17	1.08	100/1#	2.25	3.25				144/1#	1.00	2.25
104/1#	0.17	1.17	102/1#	2.08	3.00				144/3#	1.08	0.25
104/3#	1.08	0.17	102/3#	1.17	2.17				145/1#	4.42	3.42
107/1#	1.25	1.25	103/1#	2.17	2.25				145/3#	1.17	0.25
115/1#	0.17	1.25	103/3#	0.08	1.08				146/1#	4.42	2.33
115/3#	1.25	0.17	105/1#	5.42	3.42				146/3#	0.33	2.25
118/1#	1.33	2.17	106/1#	0.08	1.08				147/1#	3.00	3.08
118/3#	1.17	1.25	106/3#	2.08	0.08				147/3#	2.42	1.25
119/1#	1.17	1.17	108/3#	0.33	1.33				148/1#	0.08	1.42
119/3#	0.17	0.08	109/1#	2.33	3.33				148/3#	5.00	4.17
120/1#	0.17	0.17	109/3#	1.08	1.17				149/1#	1.15	2.25
...
Average	1.23	1.23		2.18	1.95		2.17	2.36		2.19	2.02
Results (MEM)			Intermittent Data: SIMFAC(1) = Naïve Method								
			Lumpy Data: SIMFAC(1) > Naïve Method								
			Smooth Data: SIMFAC(1) < Naïve Method								
			Erratic Data: SIMFAC(1) > Naïve Method								

Table 7. FC Comparison under CS

Intermittent			Lumpy			Smooth			Erratic		
TS	Naïve	SIMFAC(1)	TS	Naïve	SIMFAC(1)	TS	Naïve	SIMFAC(1)	TS	Naïve	SIMFAC(1)
107/1#	0.046	0.029	96/3#	0.196	0.654	138/1#	0.203	0.364	139/1#	0.172	0.652
115/3#	0.307	0.307	102/1#	0.000	0.530	138/3#	0.000	0.460	139/3#	0.157	0.518
118/1#	0.000	0.832	103/3#	0.316	0.258	153/1#	0.641	0.682	140/1#	0.573	0.453
118/3#	0.645	0.414	109/1#	0.129	0.105	153/3#	0.310	0.477	140/3#	0.211	0.519
119/1#	0.114	0.452	109/3#	0.279	0.156	157/1#	0.149	0.442	141/1#	0.912	0.018
119/3#	0.000	0.400	111/3#	0.088	0.044	157/3#	0.229	0.523	141/3#	0.444	0.785
120/1#	0.060	0.074	112/1#	0.195	0.311				142/1#	0.141	0.325
120/3#	0.324	0.122	113/3#	0.102	0.189				142/3#	0.066	0.062
122/3#	0.764	0.319	116/3#	0.201	0.154				143/1#	0.192	0.373
130/1#	0.469	0.183	121/1#	0.372	0.151				143/3#	0.000	0.486
130/3#	0.141	0.107	121/3#	0.000	0.728				144/1#	0.507	0.240
131/1#	0.315	0.700	123/3#	0.225	0.484				144/3#	0.688	0.607
			124/3#	0.575	0.269				145/1#	0.342	0.254
			125/1#	0.000	0.189				145/3#	0.534	0.559
			126/1#	0.516	0.231				146/1#	0.713	0.315
			126/3#	0.127	0.206				146/3#	0.228	0.646
			127/1#	0.710	0.418				147/1#	0.556	0.436
			127/3#	0.209	0.099				147/3#	0.619	0.284
			128/3#	0.042	0.355				148/1#	0.283	0.187
			129/1#	0.156	0.545				148/3#	0.309	0.259
			129/3#	0.311	0.232				149/1#	0.382	0.091
		
Average	0.27	0.33		0.27	0.34		0.26	0.49		0.46	0.55
Results (CS)			Intermittent Data: SIMFAC(1) > Naïve Method								
			Lumpy Data: SIMFAC(1) > Naïve Method								
			Smooth Data: SIMFAC(1) > Naïve Method								
			Erratic Data: SIMFAC(1) > Naïve Method								

From table 6 we can see that, for intermittent data SIMFAC (1) and Naïve Method have almost the same forecasting accuracy related to demand events; for erratic data and lumpy data, SIMFAC (1) shows better forecasting quality and for smooth data Naïve Method shows better forecasting accuracy. According to Spicher: "For sporadic time series it is not surprising that the Naïve-Forecast performs well. The reason seems to be, that the inherent structure of the underlying process is directly transferred into the forecast, while SIMFAC (1) tries specifying the structure (through extrapolation of the approximation functions) and thus creating the forecast." – It will be a question of further research finding out, for which categories of STS Naïve-Forecast competes well or even beats traditional forecasting methods.

4.3 Forecasting Comparison of Events-Related Values

The events-related values will be compared among SIMFAC (1), Naïve method and Actual. Error metric is Cosine Similarity.

The forecasting results of Naïve method and SIMFAC (1) will be measured against Actuals, to compare which one shows a better similarity. All data are tested, and the results are shown as table 7: since the forecasting difficulty grows with the growth of zero proportion, especially under the error metric CS, which often get zero for intermittent data and lumpy data.

Therefore, for those sporadic data which has relative higher proportion of non-zeros, the SIMFAC (1) is recommended; for those sporadic data having higher proportion of zeros, Naïve Method also possesses a reasonable forecasting accuracy for low budget approaches.

5. Summary

By comparing all the available sporadic forecasting methods, i.e. Croston's method, SBA, WSS, Naïve and a brand-new forecasting method – SIMFAC (1) under four applicable error metrics, i.e. APE, Theil's U, a new proposed metric MEM as well as Cosine Similarity, we got the following conclusions:

- (1) SIMFAC (1), as a brand-new STS method, shows satisfactory forecasting quality when considering the demand values and inter-demand intervals together;
- (2) For the forecasting of average demand values of sporadic data, SBA - as the existing and most widely accepted method, and WSS, as a patented method in USA, performs pretty well in some certain sporadic data classification;
- (3) Naïve Method, as one of the simplest methods, can also be applied in sporadic data, in some cases, i.e. for low budget applications, Naïve method may be the most suitable one;
- (4) For the measurement of forecasting of sporadic data, there is no perfect error metric up to now, APE and THEIL'S U are capable for measuring "Average Demand Values" like the standard error metric for complete time series, MEM is good at the estimation of "Demand Events" while Cosine Similarity (CS) can measure similarity between time series, but CS cannot work well when facing sporadic data with higher proportion of zeros.

6. Relevant Ideas for Further Research

Based on the research of sporadic data up to now, there are at least the following points which can be further studied:

- a) studying the contribution/value of Cosine Similarity in measuring FC-accuracy
- b) using Cosine Similarity for outlier identification and replacement

Since CS can measure the similarity between vectors very well, it is also worth considering this method for identification of outliers

- c) WSS method can be studied further

for being comparable of the results among forecasting methods, only "Average demand value" are taken from the WSS forecasting results. As stated before, WSS shows a demand distribution over forecast period, which contains more information than we took

d) Further improvements of SIMFAC (1)

d1) applying different Approximation Functions

In this essay, the linear approximation is used, and the results perform quite well. But, for some kinds of heavy volatile data, other approximation functions such as Sigmoid-functions / Gompertz-functions might perform better

d2) Analysis of impact of outliers on FC-quality of the SIMFAC approach

d3) Utilizing general data from the spare part business for improving Forecast accuracy of spare part-based STS.

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