Research on Rotating Vibration of Rotor System with Bent Shaft

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Abstract. In rotor systems, especially high load-carrying bearing-rotor system, bend usually arises slightly across the shaft for different reasons, and brings out unexpected effects on rotating vibration. This article puts forward mathematical model on rotating vibration response of a rotor system with a bent shaft. Then through calculation in simulation its studies system's response with different bend settings. The result shows that bend have effect on shaft vibration response magnitude.

Keywords: rotating vibration; bent shaft; bearing displacement.

1. Introduction

The shaft which works as a main transmission part in various rotor systems is sometimes not as straight as designed for multiple reasons. Especially in high load-carrying bearing-rotor system, such as in marine propulsion shafting, shaft is assembled from a line to a curve, in order to reduce the load of bearings which are closed to heavy propeller. As a result, the shaft bears extra bend forces across the shaft, which are parallel to the section. However, in many analyses, the shaft is thought to a straight line with not any bend. Only the force from out-put devices is the unique load on system. This simplification would obviously affect accuracy of vibration calculation and are not propitious for dynamic analysis. This paper puts forward mathematical model on rotating vibration response of a rotor system with a bent shaft.

2. Dynamic Models

In rotor systems, bend usually comes from the force by bearings, such as the shaft in fig.1. Bend force occurs along the axial direction in the bent part, parallel to the middle section.

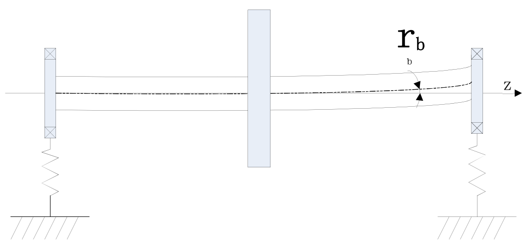


Fig.1 bent shaft

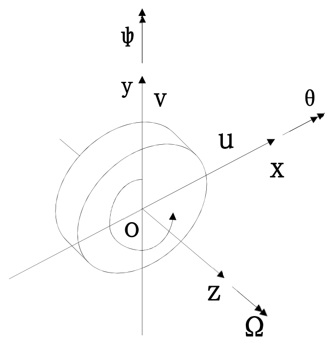


Fig.2 Coordinate system

For $\boldsymbol{q} = [u \quad v \quad \theta \quad \psi]^T$ is state vector of the node (such as in Fig.2), r_b is amount of shaft bend in the node. While the shaft rotates in speed Ω, r_b has the same rotating speed around axial line. Node's displacement is:

$$\begin{cases} u = u_e + r_b \cos \Omega t \\ v = v_e + r_b \sin \Omega t \end{cases} \tag{1}$$

where u_e and v_e are elastic displacement in the node. The elastic force by bend is:

$$\begin{cases} \mathrm{f_x} = \mathrm{ku_e} = \mathrm{k(u - r_b \cos \Omega t)} = -\mathrm{m\ddot{u}} \\ \mathrm{f_y} = \mathrm{kv_e} = \mathrm{k(v - r_b \sin \Omega t)} = -\mathrm{m\ddot{v}} \end{cases} \tag{2}$$

where k is stiffness of shaft. For $r = u + jv,\ \varphi = \psi - j\theta,\ j^2 = -1$, equation becomes:

$$\mathrm{m\ddot{r} + kr = kr_b e^{j\Omega t}} \tag{3}$$

where the equation right is the bend force in the node.
State vector $\boldsymbol{q(t)}$ and force vector $\boldsymbol{Q}(t)$ could be written in a transform:

$$\boldsymbol{q(t)} = [re^{j\Omega t} \quad \varphi e^{j\Omega t}]^T = \Re(\boldsymbol{q}_0 e^{j\omega t}) \tag{4}$$

$$\boldsymbol{Q}(t) = [kr_b e^{j\Omega t} \quad 0]^T = \boldsymbol{Q}_0 \cos \Omega t = \Re(\boldsymbol{Q}_0 e^{j\Omega t}) \tag{5}$$

For vibration of a bend rotor system, its FEA general form of equation is:

$$\mathbf{M\ddot{q}} + (\Omega\mathbf{G} + \mathbf{C})\mathbf{\dot{q}} + \mathbf{Kq_e} = \mathbf{0} \tag{6}$$

where $\mathbf{q}$ is nodes' total displacement vector, $\mathbf{q_e}$ is the elastic displacement vector, $\mathbf{q_b}$ is the bend displacement vector. Thus, $\mathbf{q} = \mathbf{q_e} + \mathbf{q_b}$. Equation (6) becomes:

$$\mathbf{M\ddot{q}} + (\Omega\mathbf{G} + \mathbf{C})\mathbf{\dot{q}} + \mathbf{Kq} = \mathbf{Kq_b} = \Re(\mathbf{Kq_{b0}}\mathrm{e}^{\mathrm{j}\Omega \mathrm{t}}) \tag{7}$$

Assuming the solution of form (4)(5), where $\boldsymbol{q}_0$ is complex displacement, equation becomes:

$$\mathbf{q}_0 = [(\mathbf{K} - \mathbf{\Omega^2 M}) + \mathrm{j}\Omega(\Omega\mathrm{G} + \mathrm{C})]^{-1}\mathbf{Kq_{b0}} \tag{8}$$

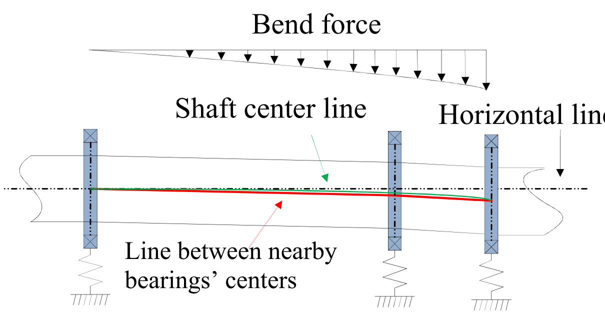


Fig.3 bend force

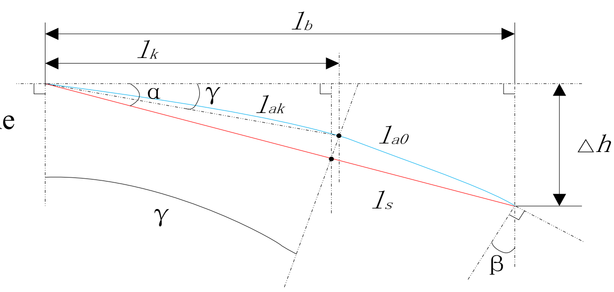


Fig.4 bend displacement

For bend usually comes from the force by bearings, we could assume that the bend force is distributed in bent part along the line between two nearby bearings' centers. The force's magnitude depend on each node's bend displacement (such as in Fig.3).

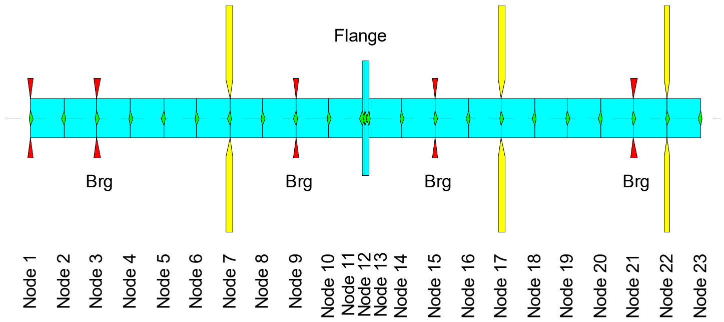
Taking a bent part for example, the bend displacement is shown in Fig.4, the both sides are two nearby bearings middle section such as in Fig.3. In this case, there are two lines: the geometric center line of bent shaft (blue curve) and slant distance between two bearings' center point (red line). For either node k in bent part, $\Delta\mathrm{h}$ is height difference between both sides, $\mathrm{l_b}$ is horizontal distance between both sides, $\mathrm{l_s}$ is slant distance (red line), $\mathrm{l_k}$ is its axial position from one bearing, α and β are red line's cutting angles depending on bend by bearing, γ is curve's cutting angle. Solving the trigonometric functions, bend displacement and force vector are:

$$\mathbf{q}_{b0k} = \begin{Bmatrix} 0 \\ l_k(\tan\alpha_k - \tan\gamma_k) \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

$$\mathbf{Q}(\mathbf{t}) = \Re\left(\mathbf{K}\mathbf{q}_{b0}e^{j\Omega t}\right) = \Re\left\{\mathbf{K}_k \begin{Bmatrix} 0 \\ l_k(\tan\alpha_k - \tan\gamma_k) \\ 0 \\ 0 \end{Bmatrix} e^{j\Omega t}\right\} \tag{10}$$

3. Influence Characteristics of Bent Shaft

Consider an anisotropic rotor system, shown in Figure 1. Two shafts are 1.0m long and they are combined by a flange which is 0.02m long. There are four bearings and three disks with a thick of 0.02m and diameter of 0.2m.



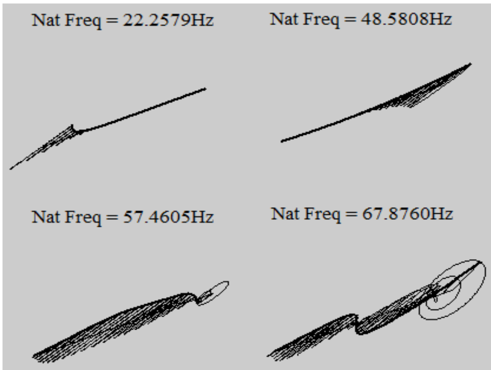


Fig.5 rotor system's model

Fig.6 nature frequencies

The rotor's model is shown in Fig.5, the 1st to 4th nature frequencies (NF) and natural mode of rotating vibration are shown in Fig.6. To obtain a bent shaft, the two bearings in the right (node 17 and 22) are set a 0.3mm permanent displacement higher than left ones, and rotor has not any other outer load. In this case, bend part mainly occurs in middle part (from node9 to node 15). Bend force and response are calculated in equations (9) (10), and the rotor speed range is from 0 to 4000rpm with a step value of 100rpm. The result is shown in Figure 7.

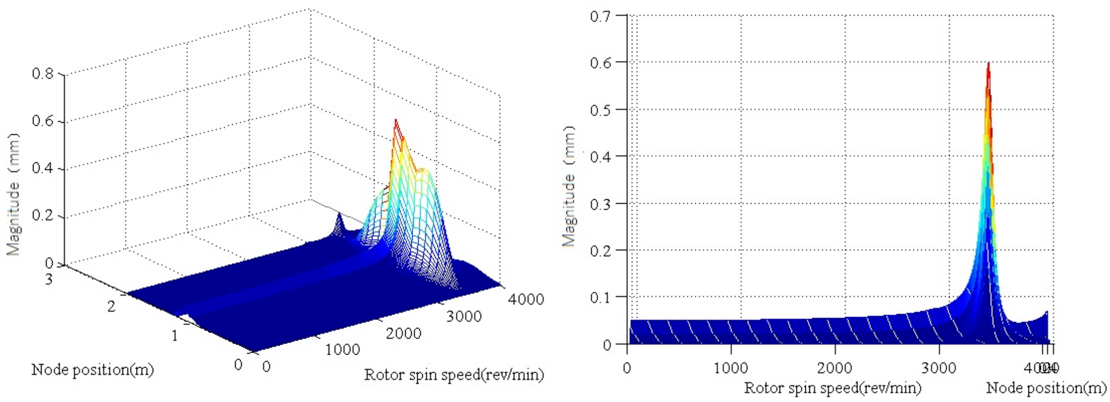


Fig.7 shaft's lateral response magnitude (bearing bend 0.3mm)

As shown in Fig.7 left, x axial is rotor speed, y axial is node's position in length, z axial is magnitude. Bend force focus on middle bent shaft between normal bearing and set bearing. While rotor speed increases to 2nd NF, bend force arises and gets to crest near 3rd NF.

It could be concluded that while speed approaches to the 2^{nd} NF, resonance happens with increasing vibration, which effects shaft's camber. While speed increasing to the 3^{rd} NF, bend force arises further and leads to more intense vibration till rotor speed overcome NF.

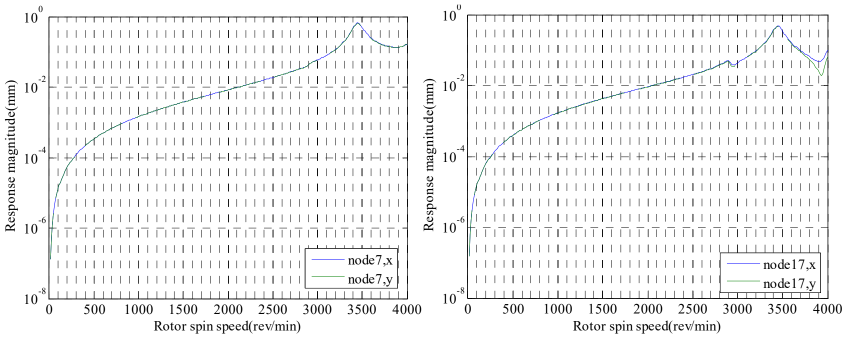
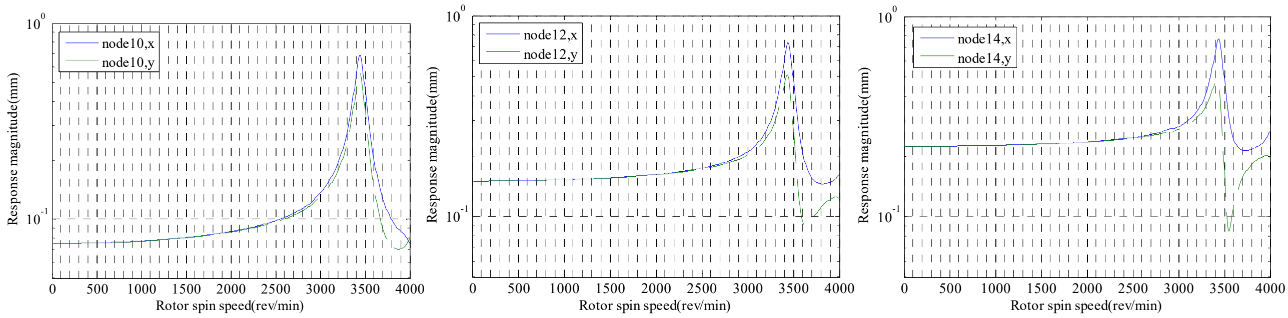


Fig.8 disks' response magnitude

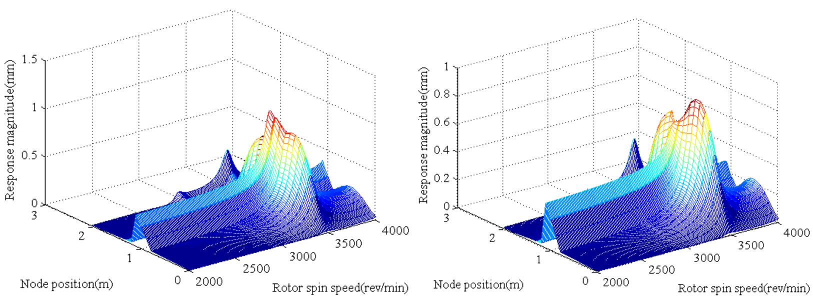
Response magnitude of disks (node 7 and 17) is shown in Fig.8. In this case, bend occurs in the middle part and disks rotate in straight part. The magnitude is tiny as rotor speed starts from zero, and arises smoothly till speed reaches the 2nd and 3rd NF. When speed is out of NF, disks' response is similar to initial value. Bent shaft seems has little influence on straight part.



node10 node12 (flange) node14

Fig.9 bent part's response magnitude

There are three nodes of bent part in symmetrical positions. Node 10(near the default part) and node 14 (near the higher part) are in the middle of bent part between bearing and flange, node 12 is in the middle section of flange. As shown in Fig.9, these three nodes' response is approach to default bend displacement when rotor speed starts from zero. Magnitudes arise remarkably with increasing speed and their numerical values are much larger than ones of disks in straight part. And the node 14 which is closed to bent part has significant difference between vibrations in x axis and y axis, compared to node 10. We can assume that bent part is in the elastic deflection at the first as bend set, its nodes have deviated balance positions and unbalance force and moment appear across the bent shaft, which leads to acute vibration.



lateral response (x axial) vertical response (y axial)

Fig.10 shaft's lateral response magnitude (bearing bend 0.4mm)

For further result, bend now is set from 0.3mm to 0.4mm. As response in high speed (2000 to 4000 rpm) shown in Fig.10, shapes of curve is similar to ones in Fig.7, and the value arise with the increased bend. Also the difference exists between directions: vibration's peak value lays abroad to the bent part in lateral meanwhile to the straight part in vertical. It could be concluded that permanent bend set only in vertical has led to such difference.

4. Summary

For rotor system with bent shaft part, this paper gives out model and analysis on rotating vibration response towards some bend set. The features include the following:

1. Bend force mainly effects bend part's response of rotating vibration and effects less in other straight parts
2. Amount of bend mainly determines vibration's response magnitude linearly. Mode of vibration is relatively stationary and has difference between lateral and vertical directions.
3. Multiple bend would lead to complicated rotating vibration.
4. It is necessary to improve rotor system's stability by alignment the shaft with bearings' load and displacement.

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