

Is Algebraic Thinking Suitable for Indonesia Elementary School Curriculum?

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Abstract—Recently most research leads to how students understand processes mathematical idea. Algebra is the most topics that discussion in international aspect. Their debate for the topic algebra is about how the students are thinking algebraically and how to solve the student's difficulties while learning algebra. The current issue appears in elementary school is algebraic thinking. Algebraic thinking is about a way of students thinking to make generalizations, solve the problem, make the prediction, justify the model and prove the mathematical ideas. Mostly algebraic thinking is still discussed in middle school, but for the transition from elementary to middle school aspect algebraic thinking is not much. This paper presents the condition around Indonesia curriculum, and the material content afterward gives the country that has been developed algebraic thinking. It is recommended for further research in Indonesia in how to promote algebraic thinking in elementary and middle school as the transition of arithmetic to algebraic thinking.

Keywords—*algebra; algebraic thinking; content curriculum; Indonesia elementary school*

I. INTRODUCTION

An essential subject matter in students' mathematics learning is algebra. In another hand, many students have been experiencing enormous difficulties in learning algebra [1–4]. The student's difficulties with algebra is that caused to the time and the ways of introducing it in schools. The result of the international test, The Trend in International Mathematics and Science Study (TIMSS), showed that Indonesia participant got the lower scores in the domain of algebra. For example, at least only 37% of the Indonesian participants were able to solve algebra in a linear equation content [5]. This phenomenon gave some researchers and mathematics educator that have been great attention paid to solve student's difficulties in learning algebra.

Arithmetic and algebra differ primarily in that arithmetic focus in computational fluency and algebra focus in relation. For enhancing the elementary mathematics, it needs integration of algebra to lead a wealth contribution in identifying of certain algebra. One of the aspect issues from algebra is how the students are thinking algebraically [6–9]. The earlier research in elementary school, there are three

necessary algebraic thinking. They are about doing and undoing mathematical processes, identifying and representing functional rules, and thinking about computations independently of particular numbers [8,10–14]. More recently, algebraic thinking is a form of a mathematical reflection and action that highlights the need to recognize dominant forms based on alphanumeric symbolism [15].

There are many interpretations approaches to defining algebraic thinking, but in this paper, we will discuss algebraic thinking in elementary school subsequently we will relate with Indonesia curriculum elementary school. This article is an overview of algebraic thinking in elementary school then research integrated it. To get similarity about algebraic thinking in both Indonesia and another country, we choose the Singapore curriculum as a support suggestion. Singapore, as a developed country, has been developed algebraic thinking with a variety of students' experiences by use of a problem-solving approach, a generalizing approach and functional approach [9]. Furthermore, the aspect of the Singapore curriculum involves the goal specification, content coverage and the last that is process coverage.

II. ALGEBRAIC THINKING IN ELEMENTARY SCHOOL

In the past decade, discussions about school algebra have changed significantly. The most gatekeeper in school mathematics is characterized by algebra [8] and accomplishing the goal of algebra; students should have experience and preparation that making the students established more formal study algebra in the later grades [16]. Therefore, some curriculum developers, policy makers, and educational researchers are just beginning to explore the kinds of mathematical experiences elementary, and what the students need to prepare them for the formal study of algebra at the later grades [17]. Because algebra is one of the focus points in mathematics education research; researcher and mathematics educator begin to identify and focus on how elementary school curriculum can develop students' algebraic thinking [18].

There are many apparently and broadly usual reasons for developing algebraic thinking ideas in the earlier grades from research on learning [8,19–21]. It means that to promote the algebraic ideas for students is not a simple case, because

developing algebraic concepts need smooth fundamental transition arithmetic to algebraic thinking. The transition from arithmetic to algebraic thinking is difficulties experience to solve their problem [8,11,12,22,23]. In a more specific way, students need adjustments thinking to construct their mathematical ideas. Considering the problem of students, three distinct aspects of algebraic thinking can identify, as the role of studying algebra in an earlier grade, in elementary mathematics instruction, such as generalization, concepts of equality and thinking with unknown quantities [24]. These three aspects provide the framework for recognizing the students thinking algebraically.

The generalization is the extended definition of algebra. The focused subject for generalization is when the students find the patterns. The clue of the word in our mind is how to discover and replicate the mathematical pattern, it is the most critical thing to invent mathematics for all. Students should analyze the structure of the patterns then they making grows or changes from the patterns, organizing the information as systematically and examining the generalization about the mathematical relationship.

Then, the concepts of equality hint that about the relationship between arithmetic and algebra by noting the equal sign. Most of the students will be misled about these expressions. For example, when the problem stated " $5+24=?$ " the students directly can answer this problem. In the other side, if the students are given the problem " $8+4=?+5$ ", their understanding of the answer mostly write 12 and 17 [25]. It is a problem experienced by the most elementary school students. Equality is the crucial concept for algebra achievement, and the equal sign is the most prevalent symbol in understanding mathematical ideas. In elementary school, the equal sign as an operational symbol rather than a relational symbol [26].

Thinking about unknown quantities, we will most frequently connect with the word "variable" or unknown quantity. Most of the variable used is letters or symbols to represent the unknown quantity. However, it is possible if the students work with expressions that have an unknown subject. How they express problems without specific numbers, it can show that they are thinking algebraically. They need the trick to solve the problem. The example of the problem with an unknown quantity is as follows. This problem is suitable for the student in 3rd to 5th grades. Suppose, Habi has some number of pieces of candy in his bowl. Naya has three more pieces of candy than Habi has. Then Habi's mother gives him five more pieces of candy. Now, who has more? Then Habi gives Naya one piece of his candy. Now, who has more? How many more?

From the problem, students can solve it without creating algebraic expressions that contain variables. They may show their answer by drawing the picture to represent the number of candies Habi has. The image may be a circle, square or other picture that students think. Then they could manipulate the image without ever specifying what is in the picture. While the problem is given to the students, it shows that the question asks not for the specific amount of number but asks for a comparison between two quantities.

III. SOME ALGEBRAIC RESEARCH INTEGRATING ALGEBRAIC THINKING

The mathematics education community, for many years, has been investigated the difficulties of students learning with algebra. One of the aspects the most crucial major was about algebraic thinking. Recent findings had indicated that many of the problems young students experiences was when students began formal algebra and showed their algebraic thinking in middle school [15,27–30].

Kieran, in 1996, proposed definition algebraic thinking and gave the attention algebraic thinking in the elementary school. She developed about early learning algebra and algebraic activities in school algebra. She commented that there are the three-part model activities in developing algebraic thinking. They are the generational, the transformational and the global, meta level, mathematical activities [13].

Some articles journal and conference is appearing in Indonesia research. Meanwhile, not much researcher talk about this topic. Studies showed that students at junior high school in five provinces in Indonesia have encounter difficulties in understanding of the basic algebra concept like linked on operating and skill to solve form algebra [31]. The students were tough to distinguish between like or unlike terms, difficulty to understand the meaning of coefficient so that they did not seem able to resolve the operation of algebra properly. The other research showed that the problem of the Indonesia students was not encompassing while learning algebra but the most difficulties while encountering the initial algebra in early grades. Another study also showed that there was three aspect of the incapability of the Indonesia students while learning algebra, namely; (1) related operations; (2) associated rules; and (3) related properties [5]. This research concluded that understanding algebraic expressions, the most frequent difficulties, should pay attention while learning algebra in early grades. Students were not able to develop their expertise algebraically while doing procedural and conceptual understanding. There was the attention to learning algebra. Most mathematics teaching in Indonesia still seems to be traditionally teaching [32]. The Indonesia students should be prepared to learn algebra through experiences with informal algebra in elementary school.

IV. THE SINGAPORE CURRICULUM

Singapore that is one of the countries in Southeast Asian, a neighbor of Indonesia, has a fantastic achievement in international assessment. In TIMSS 2015, Singapore got the first rank for its student's performance in 4th grade. As the nearest countries from Indonesia, Singapore has to an influence on education in Southeast Asian.

In Singapore, students begin the formal study of algebra much earlier [9]. The Singapore curriculum gives the concepts of algebra for the students when they are in 6th grade (age 12+). At this level, the emphases are on the developing of algebraic concepts and algebra manipulations skills [21]. The goal for teaching algebra is related to teachers' guide. However, curricular material is not mentioned algebraic thinking explicitly emerging, but activities lead to the development of algebraic thinking. The development of

algebraic thinking is proposed by components of patterns, relations, and function. In elementary school, algebraic thinking includes for thinking style such as classifying, comparing, sequencing, analyzing parts and whole, identifying patterns and relationships, induction, deduction and spatial visualization [21]. The development of algebraic thinking using of three approaches, a problem-solving, generalizing, and functional approaches, supports the students' activities namely; analyzing parts and wholes, generalizing and specializing, and doing and undoing.

The big ideas related to algebraic thinking is the concept of the unknown. Students are in the first time using rectangles concept to represent of concept unknown rather than using abstract letters. The second big idea in the curriculum is about exploring the structure of number patterns. Students can repeat and grow the number patterns for complementary thinking in processes of generalizing and specializing. The last big ideas are the notion of letters as variables rather than as unknowns. It means that students should pay attention to how a change the input effect of the value of the function.

The processes of primary mathematics curriculum by the thinking process are likely; recognizing and articulating the parts that together form a whole. Next methods are the generalizing. It is one the identifying the patterns by detecting a similarity and difference, making distinctions, repeating and ordering, and classifying and labeling. The last, doing and undoing is the particular goal that helps gain insight into the nature of the operation.

The Singapore curriculum in elementary school engages the students in algebraic thinking. It has a nature curriculum that content covered at a lower level, higher level until complex level. The approaches also give comprehensive and detailed supporting activities to promote algebraic thinking.

V. THE INDONESIA CURRICULUM

Since 1968, the curriculum in Indonesia has changed for several years [33]. In this time, Indonesia uses the 2013 Curriculum, called K13. The development of the K13 is a further step in the development of the Competency-Based Curriculum, which was initiated in 2004 and the 2006 Curriculum. The K13 is compiled by developing and strengthening attitudes, knowledge, and skills in a balanced manner, and concerns four standards of curriculum change, namely the Graduate Competency, Content, Process, and Assessment. Graduates Competency Standards are criteria regarding the qualifications of graduates' abilities which include attitudes, knowledge, and skills. Content Standard is a criterion regarding the scope of the material and the level of competence to achieve the capability of graduates at certain levels and types of education. Process Standard is a criterion concerning the implementation of learning in an education unit to achieve Graduates Competency Standards. Educational Assessment Standards are criteria regarding mechanisms, procedures, and instruments for assessing student learning outcomes. Learning system about modifications to the K13 policy has an impact on four things, namely thematic-integrative learning models, scientific approaches, active strategies, and authentic assessments [34].

In elementary school, thematic-integrative learning model is one of the approaches that recommendations of the government, but the competencies are not yet to developing of algebraic concept. In middle school, K13 use discovery, problem-based, and project-based learning models. However, the teacher can use other models that related the student's characteristics. The curriculum and learning processes do not provide a comprehensive dan promoting developments in algebraic thinking. Indeed, the traditional approaches to learning algebra are still dominant in school.

In material textbook mathematics, it still gives students situation formal form and abstract, and mathematical contextual ideas are like the counterfeit problem. The students are learning algebra in 7th grade then domains topic not mention lead in the big concept of algebra. There is no transition for arithmetic and algebra. This situation makes students difficulties while learning algebra.

One the reflection from Indonesia curriculum is the content that is used in the TIMSS. In 2015, Indonesia sent participant in 4th grade. Most students got the lower score for domain number pattern. Only 51% of the Indonesia students could solve the task rather than the international average is 69%. We had under the global standard. From this result, it requires a reflection on mathematics educators in Indonesia how to adapt the content that exists internationally with Indonesia.

VI. CONCLUSION

Study on how to teach and learn algebra still need development not only in international aspect but also in national perspective. Indonesia, mainly, is not much discussion about algebraic thinking in either middle school or elementary school. Much research needs to discuss this topic. The example topic that we can talk about it from the curriculum in elementary school and middle school, and there is a gap content that curriculum shows it. The comprehensive and discussion Indonesia content between elementary and middle school should pay attention. The material should be continuing, so that the transition from arithmetic to algebraic can reduce difficulties of the students. We conclude for the next question that can Indonesia curriculum, particularly in elementary school, develop algebraic thinking as the necessary goal for early grade?. It requires further research and gets insight into this problem.

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