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The Scheme Characteristics for Students at the Level of Trans in Understanding Mathematics during Etno-Mathematics Learning

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***Abstract*—To improve mathematics understanding, learning must begin with something close to ethno-mathematics. The objective of this study is to describe the characteristics of trans level in understanding mathematics during ethnomathematics learning. The subjects of this study were 10 high school students majoring in mathematics and science in Bengkulu, selected from 215 students based on their cognitive abilities. The research was carried out in a participatory manner in the regular learning process by applying the ethno-mathematics approach. Subjects were interviewed based on the assignments given. Data were analyzed qualitatively by applying genetic decomposition and the fixed-comparison methods. The result indicated that the trans students were able to build thematic linkages between actions, processes, objects, and other schemes. These schemes can be used to solve mathematical problems and that related. It can be concluded that the characteristic of the Trans Level was able to initiate the formation of a mature scheme.**

***Keywords*—scheme; trans level; mathematics; learning**

I. INTRODUCTION

The mathematical learning that is meaningful makes it easy for students to develop mature schemes in the student's body of knowledge. With this in mind, it will be easier to do mathematical processes, both horizontal and vertical mathematical [1-3].

Horizontal mathematical processes start from things that are close to the students' minds or that relate to everyday life such as local culture. Therefore, ethomatematics is suggested to be used as a medium for learning mathematics [4-6].

We take advantage of the *Andun* Dance culture [7] as a starting point in relationship learning and function [8]. This makes it easy for students to reach the concept. In this learning we apply the inquiry learning model [9,10]. The results of Widada et al. were the mathematical representation of the inquiry model of higher learning than students taught with conventional learning models for students given ethnomathematics-based mathematics material [8]. If students are given mathematics, material is not ethnographic, based on mathematics, then the mathematical representation of students is taught with inquiry learning models. Students are taught with conventional learning models.

According to Widada et al. [11] after the students 'initial ability was controlled, the average mathematical understanding of students' ability to learn the realistic learning approach was higher compared to those taught by implementing the conventional learning. Also, based on Fauzan there is an interaction effect of the learning approach and the orientation of mathematical material on the ability of mathematical representation after controlling the student's early ability [12].

The description states that learning ethnomathematics has a positive effect on the ability to understand mathematics. To find out, students' cognitive processes can be determined through the APOS framework (action-process-object-schema) [13-17]. To determine the cognitive structure of students an analysis of genetic decomposition is carried out [18]. There are three levels of student cognition in understanding mathematics, namely Intra Level, Inter Level and Trans Level. This is a hierarchical triad level, intra level as the lowest level, inter level as intermediate level, and trans level as the highest level. This level is functional, not structural. Baker et al. state that: in the preliminary phase of the triad, the intra level, particular events or objects are analyzed in terms of their properties [14]. Explanations at this level are local and particular. In the inter stage, the student is aware of the relationships present and can deduct from an initial operation, once it is understood, other operations that are implied by it or that can coordinate it with similar operations. In the trans level, the student can perceive new global properties that were inaccessible at the other levels.

Other research found that, the student cognition framework in understanding mathematics is categorized in seven levels [16,17,19]. The seven levels in a row were pre-intra, intra, semi-inter, inter, semi-trans, trans, and extended trans levels [17].

Individual mathematical knowledge is its tendency to respond to situations of perceived mathematical problems and their solutions by reflecting them in social contexts and constructing or reconstructing mathematical actions, processes and objects and arranging them in schemes to be used in handling situations [20].

According to Baker et al., a mature scheme is a collection of actions, processes, objects, and other coherent pre-built schemes that are coordinated and synthesized by individuals to form the structures used in problem situations [14]. Cooley et al. [21] state that the APOS theory is the framework was extended to develop a tool for the analysis of the thematization of a schema in general, and of the calculus graphing schema in particular, through the genetic decomposition presented for the analysis [22].

The genetic decomposition is useful for analyzing students' ability to understand mathematics [18] and also the quality of their responses [23]. The intra students having genetic decomposition are only taking actions or processes separately, and cannot determine the relationship of actions, processes or objects [17]. As for the extended trans students, aside from being in the trans level, they can build new structures based on the mature schemes their already have [16,17,24].

The majority of trans level students have unique characteristics [14,16,17]. Therefore, these characteristics need to be studied. Based on the previous description, this paper discusses the characteristics of trans level in understanding mathematics during ethnomathematics learning.

II. Methods

The subjects of this study were high school students majoring in mathematics and science in Bengkulu. A total of 10 subjects were selected from 215 students based on their cognitive abilities. The research was carried out in a participatory manner in the regular learning process by applying the ethnomathematics approach. Subjects were interviewed based on the assignments given. Students are asked to sketch the graph of the function h that meets the following conditions (adoption of Baker et al. [14]).

h continuous;

$h(0) = 2$, $h'(-2)=h'(3)=0$, and $\lim_{x\to 0} h'(x) = \infty$;

$h'(x)>0$, if $-4<x<-2$ and $-2<x<3$;

$h'(x)<0$, if $x<-4$ and $x>3$;

$h''(x)<0$, if $x<-4$, $-4<x<-2$, and $0<x<5$;

$h''(x)>0$, if $-2<x<0$, and $x>5$;

$\lim_{x\to -\infty} h(x) = \infty$; and $\lim_{x\to\infty} h(x) = -2$. [14].

Subjects are explored about their cognitive processes about interconnectivity between property schema, and interval domain schema based on assignments. Data were analyzed qualitatively by applying genetic decomposition. To obtain trans level characteristics, the description of genetic decomposition continued with fixed-comparison methods [25].

III. Result and Discussion

We began to explain the results of the study in the form of pieces of interviews with the subject. He is a high school student who has very good mathematical abilities. Our interviews were conducted in ethnomathematics-oriented mathematics learning. Learning begins with contextual problems about local culture. This is a chart sketch written on "*Batik Besorek*". Students are asked to choose one batik motif, then he sketches in graphical form and the student determines the function or relation formula of the graph.

During ethnomathematics learning, students find various derivative properties of a function to sketch the graph. He utilizes the properties of the first derivative and the second derivative to determine the graphic sketch.

At the end of the session we interviewed students (say "***A***") who completed the sketches correctly (See Figure 1).

We present the interview results and genetic decomposition (Q: Researcher). Pay attention to the first interview piece.

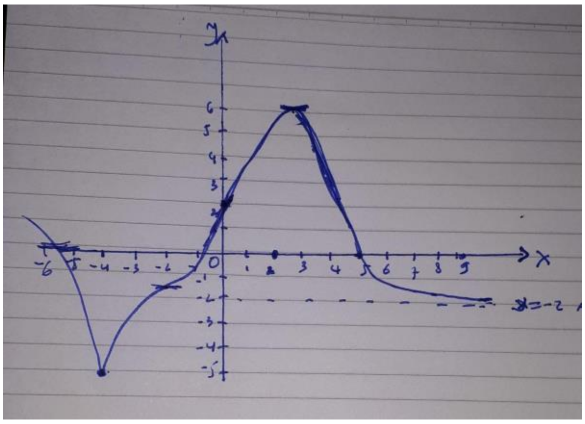

Fig. 1. Sketching graph of function h by subject A.

Q: Okay! Now please give an explanation of what you are doing!

A02: I will see the interval first, where he rises, he goes down, then where he concaves upward, and concaves downward. [***A*** while pointing to the work of Figure 1].

Based on the interview pieces and Figure 1, ***A*** can coordinate cognitively the intervals that are close together and those that overlap on the function domain h into a correct sketch like. ***A*** can coordinate cognitively about all the properties of the function given at intervals in the domain h so that it forms a mature scheme, (See ***A02***). We present the next interview piece.

A03: first for intervals-4 <x <-2; there are two possibilities, namely h '(x)> 0 means that the graph rises to the right, and the fourth condition [fourth row] is h "(x) <0, which means the graph concaves downward. [***A*** sketches the graph as described ***A03***.].

Q: Good!

A04: then I will see for the interval x <-4, there are also two conditions, h '(x) <0 and h "(x) <0, meaning the curve goes down to the right and concaves downward [this will relate] I used to draw this first, I will join it together later. [***A*** classifies the graph for the interval x <-4 as ***A04*** is passed, as ***A03*** continued, and he associates it with ***A05***.].

Q: Okay! Nice!

A05: At interval x <-4 there is also a limit x towards infinite negativity equal to n infinite so that at intervals x <-4, (then the graph will go up to the left). [***A*** emphasizes the sketch of the graph x <-4 with ***A05***, meaning the graph at the best interval continues to the left (down to the right), and concaves downward.].

Q: Yach ...

A06: whereas for x = -4 is a break point because for x <-4 [the curve] is concave down, [so is] for -4 <x <-2, and here is continuous because it is known that h is continuous. [***A*** shows a break point like the argument ***A06***, and in -4 the graph is continuously sketched.].

Based on this piece of interview, ***A*** encapsulates processes about adjacent intervals and overlaps on x <-2 into an object. This object is part of a mature scheme in previous interview pieces. While the encapsulated processes are descriptions of intervals x <-4, 4 <x <-2 see ***A03-A06***. The establishment of the scheme can also be seen from the description of ***A04*** that all of these descriptions will be related to overall unity.

As with the development of the interval scheme, ***A*** also recognizes: the process of all the properties of the function h at all intervals in the domain h, x <-2, so that an object is formed. This object is part of a mature scheme. The encapsulated processes are the description of h '(x), the limit of h (x) for x to negative infinity and h "(x) for x <-4, description of h' (x) and h" (x) for -4 <x <-2, and good coordination is done by L at x = -4 so L concludes that at x = -4 as a break point, see ***A03-A06***.

For more complete characteristics of the subject, we describe the pieces of the interview with the last A.

A07: Next I will see the interval -2 <x <3 I first took [i.e.], -2 <x <0 first, at this interval h "(x)> 0 which means concave upwards, [and at intervals it also curves up to the right because h '(x)> 0], here is also the condition that the limit of h' (x) to x approaches 0 is infinite, meaning that there is a tangent line almost parallel to the y axis and almost coincide. Also here is the condition h (0) = 2, meaning h through (0.2) Because that the limit of h '(x) to x approaches 0 is infinite, so that the graph is almost close together to the y axis but does not coincide correctly, the graph is steep. [A sketches the graph as described ***A07***.].

Q: Okay! Why turn at point 2?

A08: From -4 to -2 he [curve h] rises and concaves downward and from -2 to 0, he concaves upwards and rises, there goes up and concaves down before -2 and concludes upwards after -2 and before 0, so that there turns. [A while sketching the graph of the function h according to ***A08***.].

Q: Why is -2 an extreme point?

A09: [Because] h '(- 2) = 0 and according to the terms h' (x)> 0, if -4 <x <-2 and h "(x)> 0, if -2 <x <0, so at -2 is a turning point. [A while sketching the graph of the function h according to ***A09***.].

Q: Okay!

A10: We continue, for intervals of 0 <x <5, h "(x) <0, meaning [h curve] is concave down but between 0 and 5 there is an interval we need to see, [i.e.] -2 <x <3 Because -2 <x <0 we have, then we see 0 <x <3, where h '(x)> 0 means going up to the right. After here it's almost steep [curve h] goes up to 3. Then for the second condition that for x> 3, h '(x) <0, this means that he [h curve] descends to the right, in 3 said the batik point, this concaves down to 0 <x < 3, this also concaves down to -3 <x <5.

Q: ...... why is it sunken down?

A11: He is sunken down, because from 0 to 5, h "(x) <0 ... so it concaves downward.

Q: Okay! Nice!

A12: Then with the fourth condition, for x> 5, h "(x)> 0 and h '(x) <0, then [curve h] concludes upwards and downwards, here [at intervals (3.5)] goes down to 5 concave up, while the limit of h (x) for x goes to infinity equal to -2, meaning the graph [after 5] cannot touch this axis [line y = -2, as flat asymptote].

Q: There are conditions h '(- 2) = h' (3) = 0, what does that mean?

A13: h '(- 2) = h' (3) = 0 means that the gradient is 0, or the tangent at that point is parallel to the x axis, at x = -2 the point turns, at x = 3 turning points. Oh yeah ... while at point x = -4 the point is broken. [A shows a graphic sketch of the function h according to ***A13***.].

This interview piece confirms that ***A*** has a mature scheme about the derivative properties of a function. ***A*** can consistently encapsulate processes about adjacent intervals or overlaps at x = -2 into an object. This object is also part of a mature scheme in piece 1, which is also associated with objects in piece 2, see ***A08***. The encapsulated processes are descriptions of intervals -2≤x <0, x = 0, 0 <x <3, 3 <x <5, and x> 5, see ***A07- A13***. In this section, A also encapsulates the processes of all the properties of the function h at all intervals in the domain h, namely x≥-2, so that an object is formed. The conceptual object is part of a scheme that matures in Pieces 1. Whereas the encapsulated processes are the description of the turning point in x = -2 description of h'(x) and h"(x) for -2 <x <0, the description of the limit of h '(x) for x approaches 0 is infinite around x = 0, the description of h' (x) and h "(x) Chest interval 0 <x <3, maximal turning point description at x = 3, description of h '(x) and h "(x) for intervals of 3 <x <5, descriptions of h' (x), h" (x) and limits of h (x) for x towards infinite x > 5, see ***A07- A13***.).

The interesting things from the results of the interview based on the task against A is that A can sketch the graph of the function h accurately. A coordinates each domain interval h with all the functions of the given function. For intervals, A can coordinate the first derivative, second derivative and infinite limit and h (x) (see ***A04***). At an interval of -4 <x <-2, A can coordinate the first derivative, and the second derivative (see ***A03***), from the two-interval Coordination and the properties that are in it. A describes the point at x = -4 as a break point (cusp) (See ***A06***). The properties at point x = -2, A coordinates it with the properties of the functions that exist in two adjacent intervals, namely the interval -4 <x <-2 and -2 <x <0, so that it concludes as a · turn point (see ***A08***). Around x = 0, A coordinates all properties that are in close intervals, namely the first derivative, the second derivative, and the limit of the first derivative, which then A concludes the graph is steep, almost closed to the y axis (see ***A07***). For intervals of 0 <x <3, A coordinates the first derivative of ma and second derivatives, and at x = 3, is described as the maximum turning point as a result of coordination between h '(3) = 0. with properties that exist at the interval adjacent to x = 3 (see ***A10***). At 3 <x <5. A coordinates the first and second derivatives, and at intervals x> 5, and with infinite limits (see ***A12***).

Based on the analysis of genetic decomposition, ***A*** can coordinate object objects about intervals that are adjacent or overlapping on domain h so that a coherent interval scheme is formed. Also, ***A*** can coordinate objects about the properties of a given function, so that a coherent function scheme is formed. Therefore, ***A*** can be categorized into a trans level.

The results of the analysis are supporting previous research, including: subjects can achieve new global traits that cannot be accessed at other levels (intra & inter) [26]. According to Clark were able to construct the basic structure of chain rules and can determine their wearability [27,28]. Subjects can consciously construct a strong link between the sequence as a function and as a list. Subjects can use this framework as a line scheme to find new concepts that are related or face another new situation. DeVries, an individual constructing all structures found interrelated in this level is understood and formed a mature scheme [29]. The important functions and characteristics of maturity are used to decide whether an object enters the scope of the scheme or not. At the trans level, change understanding from registering to a rule. At this level, the collections that are formed can be referred to as a mature and possibly thematized scheme [14]. Subjects can coordinate all conditions between intervals on domains and graphs. At this level the subject can find close coordination of traits and intervals, so that it can efficiently construct an accurate graph [16]. Subjects can coordinate the action-process-objects of all the properties of a given function, with all given intervals, with all adjacent intervals even overlapping the domain so that a mature scheme is formed about the sketch of the function graph. S/He can apply the scheme to solve the problem of sketching graphs of functions [17], the subject can establish a link between the actions of processes, objects and other schemes, so that a mature scheme is formed. The scheme can be used to solve problems associated with the scheme.

The above description is synthesized into: An individual who enters the Trans Level, can establish the linkages between actions, processes, objects, and other schemes by doing a retrieval of the previous schema so that a mature scheme is formed.

IV. CONCLUSION

The characteristic of the Trans Level is the formation of a mature scheme. Trans students build thematic linkages between actions, processes, objects, and other schemes. It does retrieval of the previous schema. The scheme can be used to solve mathematical problems and related characters. The scheme is used to classify objects selectively.

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