3rd Asian Education Symposium (AES 2018)

Identifying of Rigorous Mathematical Thinking on Olympic Students in Solving Non-routine Problems on Geometry Topics

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***Abstract*—Mathematics is often contested in certain events such as provincial, national and international Olympic. When solving mathematical problems, a subject needs mathematical abilities, carefulness and accuracy (rigor). RMT (Rigorous Mathematical Thinking) defines accuracy in mathematics, so RMT is needed in solving math problems. RMT involves several cognitive functions, namely level 1 (qualitative thinking), level 2 (quantitative thinking with accuracy), and level 3 (abstract relational thinking). The aim of this research is to determine description of Rigorous Mathematical Thinking skills at each level in Olympic students. The subjects of this research are 8 students in one of the favorite senior high schools in Cirebon. This type of research is a qualitative descriptive methods and data retrieval techniques that are carried out by giving written tests and interviews to Olympic students, which are then analyzed by fixed comparison techniques based on RMT cognitive function. While checking the validity of the data using triangulation with sources and peer examination through discussion. Based on the results of the research, the average use of RMT cognitive function from 8 subjects is 44.69%, the results show that S1 is at level 1, and S2, S3, S4, S5, S6, S7 and S8 are at level 3.**

Keywords—rigorous mathematical thinking; cognitive function; mathematics Olympiad; geometry

I. INTRODUCTION

Mathematics learning process in education faces the challenge of preparing students in superior quality, they must have mathematical thinking skills, interpersonal skills, and adaptive skills [1]. Mathematics course must be learned by every educational level even in college has many course relating to mathematics. All this time, mathematics is often tested in certain events such as provincial, national, and international Olympic Games. Mathematics is also one of the subjects that are nationalized. Schools usually have determined students who can take part in math Olympiad, they have mathematical skills that are superior to other students or we call them as mathematically talented students. According to Krutetskii, mathematically talented students are students who think flexibly and abstractly, and are able to generalize mathematics material [2]. Mathematically talented students are able to solve high-level problems and inductive thinking [3]. Students who have been born with special talents have passion and dedication to their craft, and particularly, the way in which they identify, confront, and take pains to remedy their weaknesses [4]. Therefore, mathematically talented students need to be involved in complex and challenging tasks [5]. According to Lenart the mathematically talented students have original ideas, curiosity, and commitment to solving problem [2]. When solving mathematical problems, students should have mathematical skills and accuracy. The prerequisite for being precise and logical is rigor [6].

Geometry is "mathematical science" with logical and abstract structures [7]. Geometry is a branch of mathematics that deals with points, straight lines, numbers, fields, space, spatial, and the relationships between them [8]. According to Stamper, geometry is first taught in a higher class, then gradually taught to students who are new [9]. One of the roles of geometry in mathematics is to foster thinking processes because geometry has complex elements. According to NCTM [10] a good understanding of geometry will influence higher thinking or abstract mathematical concepts. According to Idris, the lack of understanding in geometry learning can hamper student performance [11]. Therefore, the thought process that utilizes cognitive processes with a higher level of abstraction needs the ability to Rigorous Mathematical Thinking.

The theory of Rigorous Mathematical Thinking (RMT) is first coined by James T. Kinard. RMT defines accuracy in mathematics, when students are involved in complex problems, students are driven by a strong desire, persistence, and an understanding concepts to understand problems [12]. There are three levels of cognitive function in RMT, namely level 1 (qualitative thinking), level 2 (quantitative thinking with accuracy), and level 3 (abstract relational thinking) [6]. These three levels of cognitive function define mental processes of general cognitive skills to a higher level of mathematical knowledge [6]. In this research, Rigorous Mathematical Thinking is a high-level thinking skill that involves several cognitive functions. It requires an abstract and complicated thinking stage, students can determine and apply the right concepts to solve problems. According to Kinard, the skill of Rigorous Mathematical Thinking is needed to improve the accuracy and function of students' cognitive skills abstraction

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[13]. The development of Rigorous Mathematical Thinking skill is indispensable so that students better understand the concepts learned and can apply them in various situations, to train students 'sharpness in focus, perception, critical power and can develop and improve students' high-level thinking skills. Previous research about rigorous mathematical thinking based on cognitive style show that the subject of impulsive male has a better RMT ability than the other three research subjects (reflective male, implusive female and reflective female) [6]. The RMT approach can also enhance Students' Mathematical Creative and Critical Thinking Abilities rather than the expository approach [14]. However, the previous research has not examined the ability of Olympic students or mathematically talented students. Based on the discussion above, the purpose of this research is to determine description of Rigorous Mathematical Thinking skills at each level in Olympic students.

II. Methods

The method used in conducting this research is a qualitative descriptive approach. The subjects of this research are 8 students of Mathematics Olympiad of SMA Negeri 2 Cirebon. The technique used by the researcher in collecting data is written tests and interviews. The instrument in this research is a Rigorous Mathematical Thinking skills test (written) and interview guidelines. Data analysis in this research uses a fixed comparison method by steps are as follows:

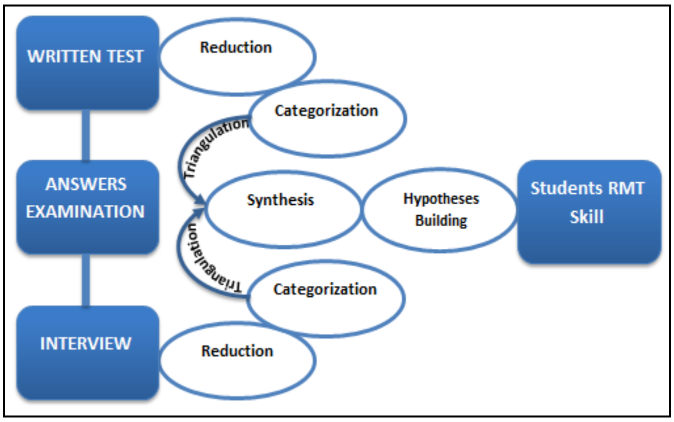

Fig. 1. Data analysis technique and data validity check.

III. Results and Discussions

A. Analysis Phase

Based on the analysis process that is done by the researcher, where S1, S2, S3, S4, S5, S6, S7 and S8 are subject 1, subject 2, subject 3, subject 4, subject 5, subject 6, subject 7, and subject 8, it results the conclusions regarding the use of RMT cognitive functions by the following subjects.

TABLE I. Use of Subject Cognitive Functions

Level	S1	S2	S3	S4	S5	S6	S7	S8	Total
1 (%)	71.43	85.71	92.86	85.71	85.71	64.29	78.57	85.71	81.25
2 (%)	0.00	35.29	52.94	52.94	35.29	17.65	64.71	35.29	36.76
3 (%)	0.00	10.71	25.00	42.86	10.71	3.57	21.43	14.29	16.07
Total (%)	23.81	43.90	56.93	60.50	43.90	28.50	54.90	45.10	44.69

TABLE II. Materials and Questions [15].

Case
There are five points in the *ABCD* rectangle with oneunit area, therefore three of the five points are not collinear. Determine the minimum number of triangles choosen from the five points with an area of no more than ¼.
Notes
a. Lemma: the area of a triangle in a rectangle no more than half the area of a rectangle.
b. In the *ABCD* rectangle, if there are three points such that the area of the triangle with the three points as a node is not more than ¼, then these three points are called good triple.
c. Convex polygon: the length of the ribs can be the same or not, the angles can be the same or not, each angle is less than 180° and the shape is convex.
Questions
1. Draw a rectangle *ABCD* with *E, F, H, &* G each being the midpoint of *AB, CD, BC & AD*, and *O* is the intersection point of the *EF &GH* line segment.
2. Mention the rectangle obtained from the picture!
3. From picture number 1, suppose there are at least two points (eg *M & N)* of the five points in the *AEOG* rectangle, illustrate the two points!
4. From picture nuumber 3, suppose that *P & Q* are the points on the *OHCF* rectangle, and the last point of the five points is *R*. If *R* is on *OFDG*, illustrate it and determine the minimum total of good triple! Then mention it!

Table 2. Cont.

5. From picture number 4, if *R* is in *OHCF*, *MNPQR* "s convex polygon will be formed contained in the *AEHCFG* convex hexagon. Calculate the area of the convex hexagon and draw a rectangle with the five points!
6. From picture number 5, if the convex polygon produced by *M, N, P, Q, & R* is a convex pentagon, illustrate and determine the minimum total of good triple from *MNPQR* pentagon!
7. From picture number 5, if the convex polygon produced by *M, N, P, Q, & R* is a convex quadrilateral. For example, rectangles $A_1A_2A_3A_4$ and the fifth point is A_5, where $A_i \in \{M, N, P, Q, R\}, (i = 1, 2, 3, 4, 5)$. Draw a convex quadrilateral and make a line segment A_5A_i $(i = 1, 2, 3, 4)$ and specify the minimum total of good triple from the quadrilateral $A_1A_2A_3A_4$!
8. From picture number 5, if the convex polygon produced by *M, N, P, Q, & R* is a triangle, for example $\Delta A_1A_2A_3$ and the rest A_4 & A_5, where $A_i \in \{M, N, P, Q, R\}, (i = 1, 2, 3, 4, 5)$. Draw $\Delta A_1A_2A_3$ and make line segments A_4A_i $(i = 1, 2, 3)$, as well as A_5A_i $(i = 1, 2, 3)$ and specify the minimum total of good triple from $\Delta A_1A_2A_3$!
9. From question number 1-8, the answer to the above case is...
10. If in the *ABCD* rectangle, the *M & N* points are placed on the ribs of the *AD & AB* of the *ABCD* rectangle such that *AN:NB = AM:MD = 2:3* and *E* are the midpoints of *AD*. Illustrate and mention how many *good triple* will be gotten from those five points *M, N, B, C, D*!

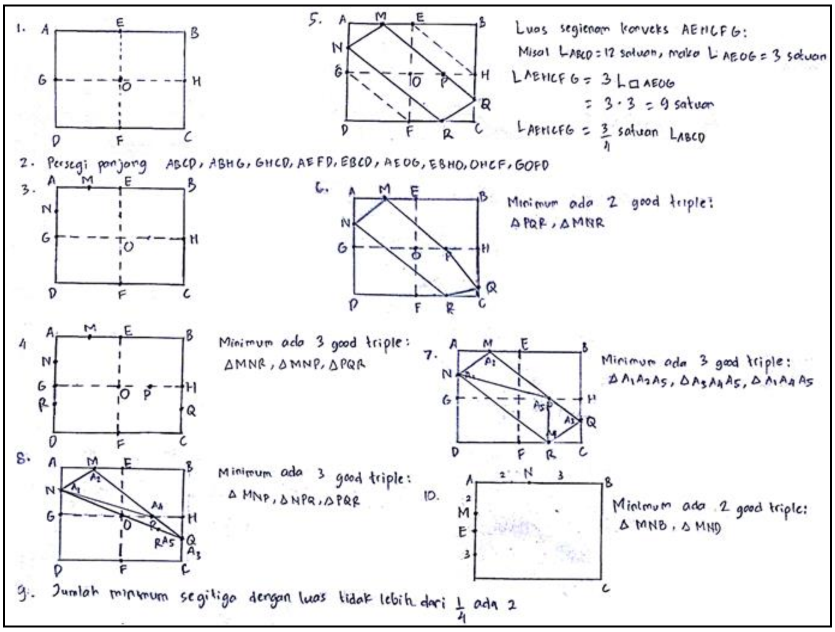

Fig. 2. Answers of one subject at level 3.

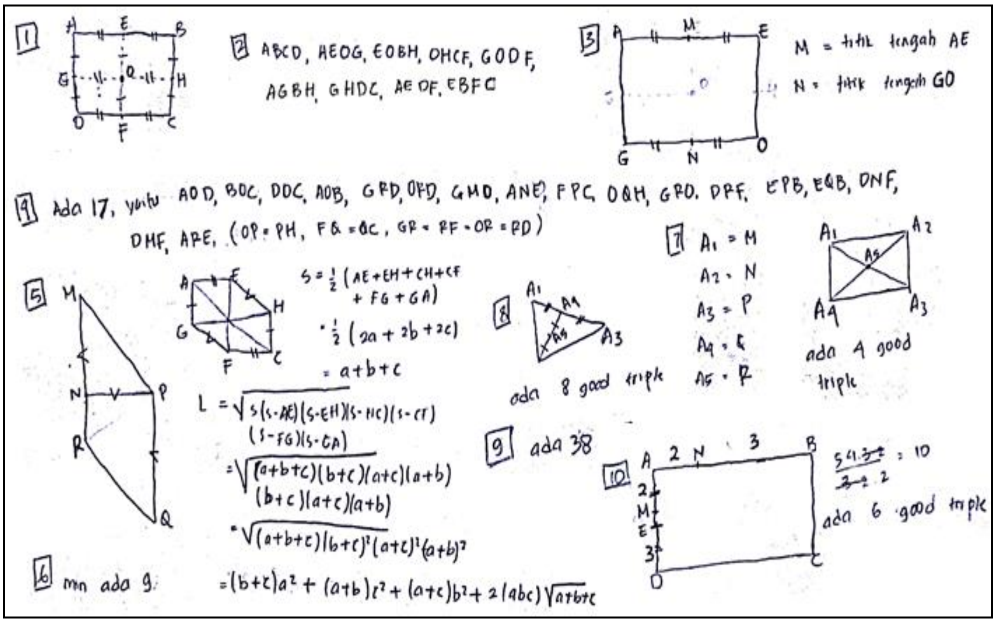

Fig. 3. Answers of one subject at level 1.

1) Student's RMT skill: The description of the RMT cognitive function used by the subject are as follows.

a) Subjects at level 3:

TABLE III. USE OF COGNITIVE FUNCTIONS LEVEL 1 QUALITATIVE THINKING

Cognitive Function	The answer results of S2, S3, S4, S5, S6, S7 and S8
Labeling	Can write each vertex in a rectangle with *A, B, C* and *D*, write points *E. F. G* and *H* at the midpoints *AB, CD, AD* and *BC*, and write down the midpoint of the rectangle with point *O*, the subject can also write points *A, B, C, D* on a rectangle and can place points *M, N* and the midpoint *E*.
Visualizing	Can draw a rectangle whose name is given and add a line segment *EH* and *GH*, the subject can draw a fifth *MNPQR*, the subject can also draw a rectangle whose name is given.
Comparing	Can mention all rectangles that are different in size and position from the picture 1.

Table 3. Cont.

Searching systematically to collect and complete information	Can decide the area of the rectangle and its parts to calculate the area of the hexagon (S2, S3, S4, S5 and S8), the subject can also calculate the area of the triangle to determine the good triple (S2 and S3). However (S5, S6, and S8) has not been able to calculate the area of the triangle to determine good triple.
Using more than one source of information	Use the definition of good triple, combination and lemma to determine good triple.
Encoding	Can write points A_1, A_2, A_3, A_4, A_5 on the convex polygon drawn and the subject can write the ratio $AN:NB = AM:MD = 2:3$ in the picture. However (S6) cannot write points A_1, A_2, A_3, A_4, A_5 on the convex polygon drawn.
Decoding	Can interpret the member symbols $A_i \in \{M, N, P, Q, R\}, (i = 1, 2, 3, 4, 5)$, and A_5A_i $(i = 1, 2, 3, 4)$ that is the subject makes a segment line A_5A_i $(i = 1, 2, 3, 4)$ in convex quadrilateral (S2, S3, S4, S5, S7 and S8).

TABLE IV. USE OF COGNITIVE FUNCTIONS LEVEL 2 QUANTITATIVE THINKING WITH ACCURACY

Cognitive Function	The answer results of S2, S3, S4, S5, S6, S7 and S8
Conversing constancy	Can identify the minimum good triple remains the same as number 4 even though the five points differ in position and produce different polygons in numbers 7 and 8 (S4 and S7), and the subject uses this cognitive function at number 6 (S2).
Quantifying space and spatial relationships	Can calculate the area of hexagon (S2, S3, S4, S5, S6 and S8), the subject also calculates the good triple triangle area (S2).
Analyzing	Can put the points *M, N, P, Q* and *R* in the rectangle that is asked in the question, not on rectangular ribs at numbers 3, 4, and 5 (S3) on (S7) only at numbers 3 and 4.
Integrating	Understand the question command by repeating the picture in the requested number completely and adding or moving the fifth point on numbers 3, 4, and 5 (S3, S4, S5 and S8), on (S7) only at numbers 4 and 3, (S2) only at number 4, (S6) only at numbers 4 and 5.
Generalizing	Can analyze good triple at numbers 10 and 6 (S4), (S7) at numbers 4, 7, 8, and 10, (S2, S3, S5, S6, and S8) at number 10 only.
Being precise	Can conclude correctly the answer of the question asked, which is choosing the *good triple* of the least number 4, 6, 7, and 8 except (S6).

TABLE V. USE OF COGNITIVE FUNCTIONS LEVEL 3 ABSTRACT RELATIONAL THINKING

Cognitive Function	The answer results of S2, S3, S4, S5, S6, S7 and S8
Activating prior mathematically related knowledge	Use some of the concepts they have learned to solve this case, such as combinations, calculating areas, collinear and drawing convex polygons.
Providing mathematical logical evidence	Can explain how to find a good triple that is obtained from half a section or two rectangular plots in numbers 4 and 6 (S4), on (S7) only at number 4, not yet seen in numbers 6, 7, and 8 and (S2, S5, S6, and S8) have not used this cognitive function.
Articulating mathematical logical evidence	It is not used by the subject yet.
Defining the problem	Can understand what a case means, which is to choose the good triple number of at least 4, 6, 7 and 8, but the subject has not been able to make a convex quadrilateral and the convex triangle from the MNPQR point that is ordered, (S5 and S8) can make the convex quadrilateral of the MNPQR point that the question instructs.
Hypothetical thinking	Can explain how they determine good triple by means of mathematics at numbers 4, 6, 7, and 8 (S3 and S4), on (S7 and S8) only at number 4.

Table 5. Cont.

Inferential thinking	Can explain how to obtain a good triple minimum with lemma at numbers 4 and 6 (S4), on (S7) only at number 4, then it is not seen in numbers 6, 7, and 8.
Projecting and restructuring relationships	Determine a good triple of the half of the rectangle (S4 and S7).
Forming proportional quantitative relationships	It is not used by the subject yet.
Mathematical inductive thinking	It is not used by the subject yet.
Mathematical deductive thinking	It is not used by the subject yet.
Mathematical relational thinking	Divide 4 ABCD rectangles to determine good triple so the good triple possibility is at M & N at number 10 (S3 and S4). Calculate the area of a rectangle then divide by four, and calculate the area of the triangle to determine the good trilpe number 10 (S2).
Elaborating mathematical activity through cognitive categories	It is not used by the subject yet.

b) Subject at level 1:

TABLE VI. USE OF COGNITIVE FUNCTIONS LEVEL 1 QUALITATIVE THINKING

Cognitive Function	The answer results of S1
Labeling	Can write each vertex in a rectangle with *A, B, C,* and *D*, write points *E, F, G* and *H* at the midpoints *AB, CD, AD* and *BC*, and write down the midpoint of the rectangle with point *O* , the subject can also write points *A, B, C, D* on a rectangle and can place points *M, N* and the midpoint *E*.
Visualizing	Can draw a rectangle whose name is given and a line segment *EH* and *GH* is added, the subject can also draw a rectangle whose name is given, but the subject cannot draw a fifth *MNPQR*.
Comparing	Can mention all rectangles that are different in size and position from the picture 1.
Searching systematically to collect and complete information	It is not used by S1 yet.
Using more than one source of information	Use the definition of good triple, combination and lemma to determine good triple.
Encoding	Can write points A_1, A_2, A_3, A_4, A_5 on the convex polygon drawn and the subject can write the ratio $AN:NB = AM:MD = 2:3$ in the picture.
Decoding	Can interpret the member symbols $A_i \in \{M, N, P, Q, R\}, (i = 1, 2, 3, 4, 5)$, and A_5A_i $(i = 1, 2, 3, 4)$ that is the subject makes a segment line A_5A_i $(i = 1, 2, 3, 4)$ in the convex quadrilateral.

IV. Conclusion

Based on the results of the analysis and discussion of this research, namely a written test of Rigor Mathematical Thinking skills and interview, it can be concluded from 8 subjects, 7 subjects are at level 3 RMT cognitive function, and 1 subject is at level 1 RMT cognitive function. Overall, the subjects have used RMT cognitive function, but must continue to be trained to use all cognitive functions of level 3 RMT, Olympic coaches can provide more varied Olympic questions so that they can increase Olympic student RMT. The results of Rigorous Mathematical Thinking skills analysis can be taken into consideration for the Olympic coach to be able to include subjects in City, Provincial, National and International Olympics.

Acknowledgment

This research was supported and sponsored by the advisor as well as the lecturer at the faculty of Teaching and Education in Universitas Swadaya Gunung Jati Cirebon. I am also very grateful to the teachers and students of SMA Negeri 2 Cirebon.

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