A Fast Algorithm for Monostatic RCS based on LU Decomposition

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Abstract— **Compared with bistatic RCS, the monostatic RCS calculation is more difficult for the needs of surface current on scatterer induced by the incident wave of every single angle, so the calculation burthen is very large. Based on the character of "impedance matrix of MOM is invariable whether the angle of incident wave was changed", this paper proposes a fast algorithm for monostatic RCS based on LU decomposition. Numerical results show out that this algorithm can compress the computation burthen of monostatic RCS significantly, and this algorithm is easy to operate.**

Keywords-MOM, RWG basis, monostatic RCS, LU decomposition

I. INTRODUCTION

The calculation for monostatic is complicated than bistatic RCS because of the needs of surface current on scatterer or equivalent field on Huygens extrapolated boundary for each incident angle. If the scatterer was complicated, the calculation burthen will be very large. So the studies almost focus on bistatic RCS more than monostatic one[1-2]. In recent years, some scholars have made some progress in monostatic RCS calculation[3-6], but there is some space to ideally in the region of accuracy or computation burthen.

In the beginning of 1980's, Rao and other scholars proposed the famous Rao-Wilton-Glisson basis (RWG-basis) function[7]. The RWG-basis caught lots of attention for its flexibility and practicability since it was proposed, and has been applied successfully in many fields[8-10]. The impedance matrix of MOM based on RWG-basis has a characteristic that the impedance matrix is invariable whether the incident angle changes. Based on this specialty, references [11] proposed a monostatic RCS calculation method based on Gauss Elimination method. This method simplified the computation process of monostatic RCS but not easy to implement. This paper proposes an algorithm based on LU decomposition for monostatic RCS calculation. This new algorithm transforms the original equation into triangle matrix equation by LU decomposition of the impedance matrix, and it cut down the computation burthen of the matrix equation significantly. Simulation results show out that this algorithm can shorten the computation time significantly.

II. MOMENT EQUATIONS BASED ON RWG-BASIS

Based on the boundary conditions of tangential electric field continuality on metal surface, so there will be $\hat{n}\times\overline{E}(\overline{r})=0$, which is $\hat{n}\times(E^i+E^s)=0$. Where E^i is the incident plane wave, the scattering field induced by E^i can be express as $E^s=-j\omega A-\nabla\Phi$, so the fields integral equation can be express as:

$$-E_{\tan}^i=(-j\omega A-\nabla\Phi)_{\tan} \tag{1}$$

where A and Φ are the magnetic vector potential function and electric scalar potential function respectively.

Formula (1) can be solved by moment method and the induced surface current J can be expressed as $J=\sum_{n=1}^{N}I_n f_n(r)$, where I_n is current coefficient to be solved, and it is corresponding to the normal current density of the common edge, f_n is RWG-basis, and *N* is the amount of common edges.
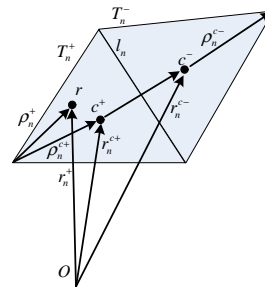

Fig 1 RWG-basis Function

RWG-basis is sub domain base function, and each basis function is associated with an interior edge of the triangle patches pair and it is zero everywhere on the surface of scatterer, except in the two triangles $T_n^\pm$ attached to that interior edge l_n. Fig1 shows two such triangles, T_n^+ and T_n^-, corresponding to the nth edge of a triangulated surface modeling. The basis function f_n is used to approximately represent the surface current. In reference [7], the vector basis function associated with the nth edge was defined as

$$f_n(r) = \begin{cases} \dfrac{l_n}{2A_n^+}\rho_n^+, & r \in T_n^+ \\[2mm] \dfrac{l_n}{2A_n^-}\rho_n^-, & r \in T_n^- \\[2mm] 0, & others \end{cases} \quad (2)$$

In formula (2), l_n is the length of the edge and $A_n^\pm$ is the area of triangle $T_n^\pm$. The plus or minus designation of the triangles is determined by the choice of a positive current reference direction for the nth edge, the reference for which is assumed to be $T_n^+ \to T_n^-$. ρ_n^+ is the vector on T_n^+ which is from vertex of l_n to any points in T_n^+, ρ_n^- is the vector on T_n^- which is from vertex of l_n to any points in T_n^-, r is the vector form origin to any points. The function basis f_n out of $T_n^\pm$ are all 0.

Using Galerkin's method to take formula (1) and f_m for inner product, we can get

$$(E^i, f_m) = j\omega(A, f_m) + (\nabla\Phi, f_m) \quad (3)$$

and then arrange formula (3) as $\mathbf{ZI}=\mathbf{V}$ form as

$$z_{mn} = l_m\left[j\omega\left(A_{mn}^+ \cdot \frac{\rho_m^{c+}}{2} + A_{mn}^- \cdot \frac{\rho_m^{c-}}{2} \right) + \Phi_{mn}^- - \Phi_{mn}^+ \right] \quad (4)$$

$$v_m = l_m\left[E_m^{c+} \cdot \frac{\rho_m^{c+}}{2} + E_m^{c-} \cdot \frac{\rho_m^{c-}}{2} \right] \quad (5)$$

where

$$A_{mn}^\pm = \frac{\mu}{4\pi}\int_s f_n(r')\frac{e^{-jkR_m^\pm}}{R_m^\pm}ds' \quad (6)$$

$$\Phi_{mn}^\pm = -\frac{1}{4\pi j\omega\varepsilon}\int_s \nabla \cdot f_n(r')\frac{e^{-jkR_m^\pm}}{R_m^\pm}ds' \quad (7)$$

$$R_m^\pm = \left| r_m^{c\pm} - r' \right| \quad (8)$$

$$E_m^\pm = E^i\left(r_m^{c\pm} \right) \quad (9)$$

Impedance matrix $\mathbf{Z}$ shows the interactions of edge current, z_{mn} is the influence of I_n to I_m via radiation fields. Dimension of the impedance matrix is equal to N, the number of common edge. From formula (4), we know that $\mathbf{Z}$ is independent of incident field E^i and it is depend on the

scatterer itself only. Components correlated with incident field E^i are all on the right side of formula (5).

III. FAST ALGORITHM FOR MONOSTATIC RCS BASED ON LU DECOMPOSITION

Moments equations $\mathbf{ZI}=\mathbf{V}$ can be classified as linear equations as $\mathbf{Ax}=\mathbf{b}$, where $\mathbf{A}\in R^{m\times m}$. Reference [12] points out that if $\mathbf{A}$ was nonsingular and its LU decomposition existed, then its LU decomposition is unique. So in solving linear equations $\mathbf{Ax}=\mathbf{b}$, we can factorize $\mathbf{A}$ as $\mathbf{A}=\mathbf{LU}$, where $\mathbf{L}$ is a lower triangular matrix and $\mathbf{U}$ is an upper triangular matrix, and then we can transmit $\mathbf{Ax}=\mathbf{b}$ as $\mathbf{LUx}=\mathbf{b}$, and then we can solve it in a very simple way. In addition, because LU decomposition of $\mathbf{A}$ is unique, and the impedance matrix of RWG moment equations is invariable whether the incident angle was changed. The computation of monostatic RCS carrying on the procedure of LU decomposition only once, and for other incident angles, the results of LU decomposition can be used directly. So this algorithm can reduce the computation burthen significantly.

There are two steps to solve linear equations by LU decomposition[13]. Firstly, solve equation $\mathbf{Ly}=\mathbf{b}$, where $\mathbf{L}$ is lower triangular matrix, it can be solved by successive forward substitution method. Secondly, solve equation $\mathbf{Ux}=\mathbf{y}$, where, $\mathbf{U}$ is upper triangular matrix, it can be solved by successive backward substitution method. So the LU decomposition process for $\mathbf{A}$ can be expressed as follow:

Hypothesis $\mathbf{A}=\{a_{ij}\}$, if there was $\mathbf{A}=\mathbf{LU}$, where, $\mathbf{L}=\{l_{ij}\}$ is lower triangular matrix, $\mathbf{U}=\{u_{ij}\}$ is upper triangular matrix, then the corresponding elements in $\mathbf{L}, \mathbf{U}$ can be expressed as formula (10) and (11), where, element u_{kj} in row k in $\mathbf{U}$ can be expressed as:

$$u_{kj} = a_{kj} - \sum_{r=1}^{k-1} l_{kr}u_{rj}, \quad j = k, k+1, \ldots, n \quad (10)$$

where $k = 1, 2, \ldots, n$; element l_{ik} in column k in $\mathbf{L}$ can be expressed as:

$$l_{ik} = \frac{a_{ik} - \sum_{r=1}^{k-1} l_{ir}u_{rk}}{u_{kk}}, \quad i = k+1, \ldots, n \quad (11)$$

where $i = 1, 2, \ldots, n$, and $\sum_{r=1}^{0} = 0$.

Using formula (10) and (11) alternatively and successively, we can solve all the elements in $\mathbf{L}$ and $\mathbf{U}$, and all these elements can be saved in the corresponding position in matrix $\mathbf{A}$. This process can be programmed easily, and during the computation process, the elements in matrix $\mathbf{A}$ can be replaced by elements in matrix $\mathbf{L}, \mathbf{U}$ successively. And it is no need to use new matrix to save $\mathbf{L}, \mathbf{U}$, so the storage is more efficient.

Based on the LU decomposition in the former step, $\mathbf{Ly} = \mathbf{b}$ and $\mathbf{Ux} = \mathbf{y}$ can be solved by successive substitution method, the computation method can be expressed as follow:

$$y_i = b_i - \sum_{r=1}^{i-1} l_{ir} y_r, \quad i = 1, 2, ..., n \tag{12}$$

$$x_i = \frac{y_i - \sum_{r=i+1}^{n} u_{ir} x_r}{u_{ii}}, \quad i = n, n-1, ..., 1 \tag{13}$$

The original equation can be solved by formula (12) and (13). For solving RWG moment equations, the impedance matrix should be transformed from $\mathbf{ZI} = \mathbf{V}_1$ to $\mathbf{ZI} = \mathbf{V}_2$ if the angle of incident wave changed, where $\mathbf{V}^{(2)} = \left(v_1^{(2)}, ..., v_n^{(2)} \right)^T$. In these two equations, that the matrix $\mathbf{Z}$ on the left side is invariable and $\mathbf{V}$ on the right side changes only. So we can using $\mathbf{L}$ and $\mathbf{U}$, which are known already, to solve the new equation for simplify the computation procedure of monostatic RCS.

IV. NUMERICAL RESULTS

A. Monostatic RCS of Rectangle Metal Plate

Fig2 shows the monostatic RCS of rectangle metal plate, which is located in XOY plane, and calculated by different algorithm. Where the incident angle represents the angle between incident wave vector $\hat{k}$ and $\mathbf{X}$ axis, and it is changed in XOY plane; Electric field E_i is parallel to XOY plane and its amplitude is 1; Frequency of incident wave is $300MHz$ and the size of the metal plate is $1m \times 1m$. Simulation results show out that the RCS results of Gauss Elimination method and LU decomposition method consistent with each other, which proves the correctness of this new algorithm.
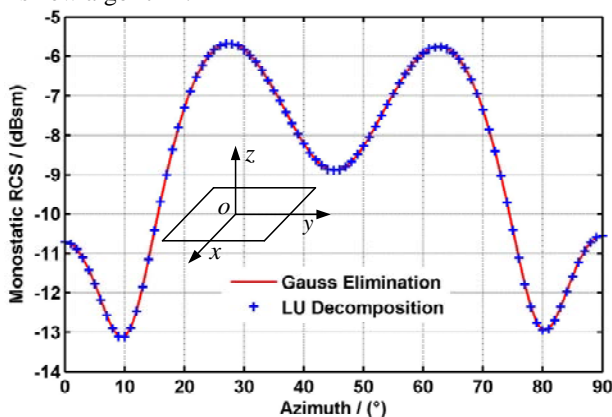

Fig2 monostatic RCS of Rectangle metal plate

B. Monostatic RCS of Metal Cubic

Fig3 shows the monostatic RCS of metal cubic in different algorithm. Where the incident angle represents the angle between incident wave vector $\hat{k}$ and Z axis, and it is changed in YOZ plane; The direction of Electric field E_i is $\hat{X}$ and its amplitude is 1; Frequency of incident wave is $300MHz$ and the size of metal cubic is $1m \times 1m \times 1m$. Simulation results show out that RCS computed by Gauss Elimination method and LU decomposition method consistent with each other very well, and they both consistent with the result implemented by Ansoft HFSS.
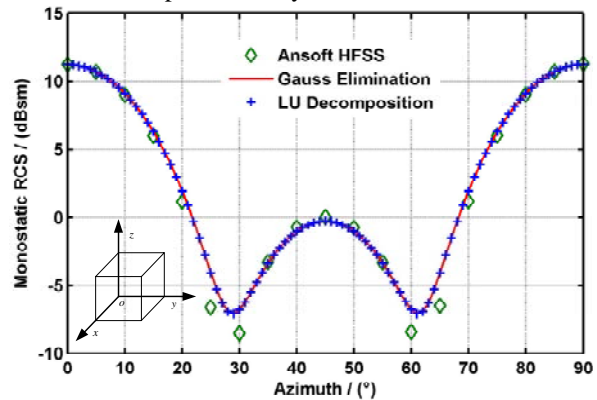

Fig3 monostatic RCS of metal cubic

C. Analysis of Computation Burthen

If solving the equation completely by LU decomposition, the computation burthen is equivalent to Gauss Elimination method [13], so there is no advantage by adopting this method. But for monostatic RCS, the procedure of LU decomposition for impedance matrix is carried on only once and this will compress the computation burthen remarkably.

Fig4 shows the computation times of Gauss Elimination method and LU decomposition method expended in former simulation of metal cubic respectively. We can draw a conclusion that compared to traditional Gauss Elimination method, the proposed algorithm in this paper can compress the computation time to almost 10% of the Gauss Elimination method.
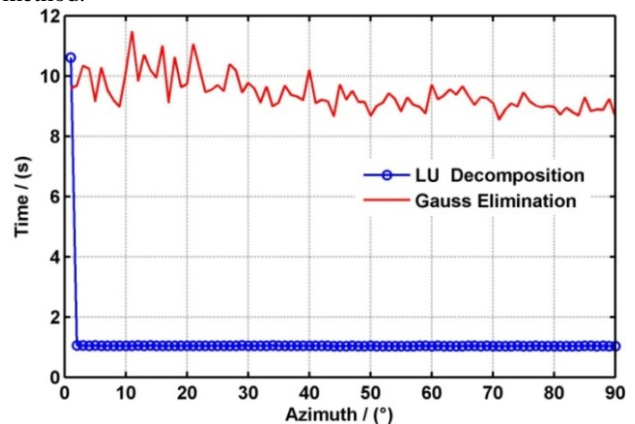

Fig4 computation time

V. CONCLUSION

Based on the character of "impedance matrix of MOM is invariable whether the angle of incident wave was changed", this paper proposes a fast algorithm for monostatic RCS based on LU decomposition. The proposed algorithm predigest a complex matrix equation into two triangle matrix equations by LU decomposition, and predigest the computation procedure of monostatic RCS in further by using the invariable characteristic of the impedance matrix. Numerical results show out that the proposed algorithm can compress the computation burthen of monostatic RCS significantly, and it is very easy to operate.

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