A New Derivation of TGR with Naked Antenna in Lossy Medium

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***Abstract*—A new derivation of Transient Grounding Resistance(TGR) has been proposed in this paper. Take grounding system as naked antenna in lossy medium, and then get the input resistance of naked antenna in a given frequency based on Helen current integral equations. With inverse Fourier transform, the resistance calculation equations can be translated from frequency domain to time domain. The result can be applied to design a new grounding system and to calculate its TGR .**

***Keywords*—TGR; Grounding System; Naked Antenna**

I INTRODUCTION

In electromagnetic shielding field, protecting the electronic systems against HPM or UWB weapons besides LEMP is a significant branch. And grounding system plays a more and more important role in the safe and reliable operation of electronic systems. The main purpose of grounding system is to distribute the strong and destructive effect such as a lightning strike through the wires and rods, which belong to a protection system.

In recent years, with the developments of computer technology and computational electromagnetics, especially the FDTD (finite-difference time-domain) method for electromagnetic waves, the transient grounding resistance(TGR) has been the newest research focus of grounding system. For example, in paper[1] K.Tanabe calculated the pylon's TGR under lightning current striking with FDTD. In paper[2]~[4], the resistance of grounding systems in the multilayered earth has been analyzed, and in paper[5]~[6] professor He Jinliang has analyzed the performance of grounding systems influenced by the soil ionized under strong current striking. Besides above electromagnetic analysis methods, in paper[7] finite element method (FEM) has applied to calculate the transient resistance of grounding electrode of tower and in paper [8] lightning electromagnetic field of the thin wire structure in lossy half-space is calculated by the fast multipole algorithm. And in paper[9]~[12], transmission-line model has been used to analyze the resistance of grounding system composed of long conductors.

In this paper, a new derivation of TGR with naked antenna in lossy medium has been proposed. Firstly, take grounding electrode as a naked antenna in lossy medium. Then calculate the frequency domain resistance based on Helen current integral function secondly. Considering the pulse current, calculate the current in frequency domain with Fourier transform. In the end, the voltage in time domain has been calculated and the TGR has been inferred.

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II RESISTANCE CALCULATION OF NAKED ANTENNA IN LOSSY MEDIUM

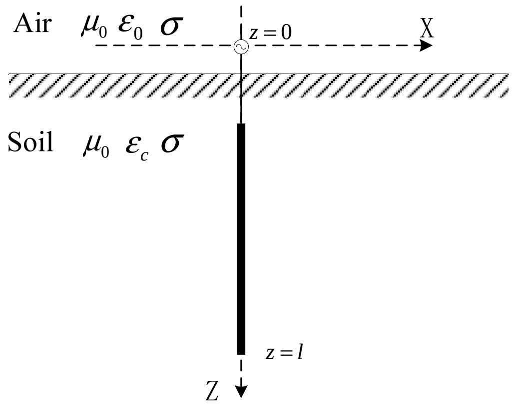

Fig.1. Grounding wire end-driven in lossy medium

In the new method, the grounding system is taken as an naked antenna in lossy medium. For example, the vertical grounding electrode is modeled as an end-driven monopole naked antenna of length l ,placed in lossy medium, just as shown in Fig. 1.

According to the antenna theory and for symmetrical cylinder antenna, the current distribution can be gotten[13] by Helen integral equations just as expression (1):

$$\int_0^l I(z')\frac{e^{-jkr}}{r}dz' = c\cos kz + \frac{2\pi V}{j\eta}\sin k|z| \qquad (1)$$

Where V is the driven point voltage, c is a undetermined constant, k is the propagation constant of the lossy medium, and η is the intrinsic impedance of lossy medium:

$$k = \omega\sqrt{\varepsilon_c\mu} = \omega\sqrt{\varepsilon\mu(1-j\sigma/\omega\varepsilon)} \qquad (2)$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon(1-j\sigma/\omega\varepsilon)}} \qquad (3)$$

and ε_c is complex dielectric constant of lossy medium:

$$\varepsilon_c = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) \quad (4)$$

As we all know, σ is the conductivity of lossy medium and then ε_c , k and η are all complex numbers. And k can be farther expressed as expression (5)

$$k = \beta - j\alpha = \beta\left(1 - j\frac{\alpha}{\beta}\right) = \beta\left(1 - jq\right) \quad (5)$$

Compare expression (2) with (5), and α , β can be defined as:

$$\alpha = \omega\sqrt{\mu\varepsilon}t_\alpha \qquad \beta = \omega\sqrt{\mu\varepsilon}t_\beta \quad (6)$$

Where $t_\alpha = \sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$

$$t_\beta = \sqrt{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \quad (7)$$

Then expression (3) can be expressed as:

$$\frac{1}{\eta} = \sqrt{\frac{\varepsilon}{\mu}}\left(t_\beta - jt_\alpha\right) \quad (8)$$

With expressions (5)~(8), the current integral expression (1) can be farther expressed as:

$$\int_0^l I\left(z'\right)\frac{\mathrm{e}^{-j\beta r}\mathrm{e}^{-\beta qr}}{r}\mathrm{d}z' = c\cos\left(\beta z(1 - jq)\right)$$
$$-2\pi V(t_\alpha + jt_\beta)\sqrt{\varepsilon/\mu}\sin\left(\beta\left|z\right|(1 - jq)\right) \quad (9)$$

Then the sum of n-order polynomial series can be used to take the place of $I\left(z'\right)$ and to solve the equation (9) with approximation. Let

$$I\left(z'\right) = \sum_{m=1}^{n} I_m (1 - \left|z'\right|/l)^m \quad (10)$$

Then in expressions (9) and (10), there are two groups of parameters undetermined which are complex coefficients I_m and constant c. For naked antenna, $z \in [0, l]$. Take n+1 points in the antenna and equation (9) must be workable at every point, and then n+1 complex linear equations can be gotten. Take $z_p(p = 1, 2, \cdots n+1)$ to express these points' coordinates, and let

$$F_m\left(z_p\right) = \int_0^l \left(1 - \left|z'\right|/l\right)^m \frac{\mathrm{e}^{-j\beta r_p}\mathrm{e}^{-\beta qr_p}}{r_p}\mathrm{d}z' \quad (11)$$

Where $r_p = \left[\left(z_p - z'\right)^2 + a_r{}^2\right]^{1/2}$ (12)

In expression (12), a_r is the radius of grounding electrode. With expressions (10)~(12), the expression (9) can be expressed as:

$$\sum_{m=1}^{n} I_m F_m(z_p) - c\cos\left(\beta z_p(1 - jq)\right)$$
$$= -2\pi V(t_\alpha + jt_\beta)\sqrt{\varepsilon/\mu}\sin\left(\beta\left|z_p\right|(1 - jq)\right) \quad (13)$$

z_p can be taken arbitrarily and for the purpose of simplifying the calculation, let z_p equally spaced on the electrode from 0 to l. Then

$$z_p = \left(p - 1\right)l/n \qquad p = 1, 2, \cdots, n+1$$

When $I_1, I_2, \ldots, I_n$ are all solved, expression (10) can be used to calculate the complex current distribution along the naked antenna in lossy medium.

Because the driven point's location is $z' = 0$, the current is

$$I\left(z' = 0\right) = \sum_{m=1}^{n} I_m \quad (14)$$

And then the input resistance is

$$Z\left(\omega\right) = \frac{V}{I\left(0\right)} = \frac{V}{\sum_{m=1}^{n} I_m} \quad (15)$$

With expression (15), the input resistance of grounding electrode has been gotten. But the result is belong to a certain frequency while TGR is in time domain. So the above mentioned result should be evolved.

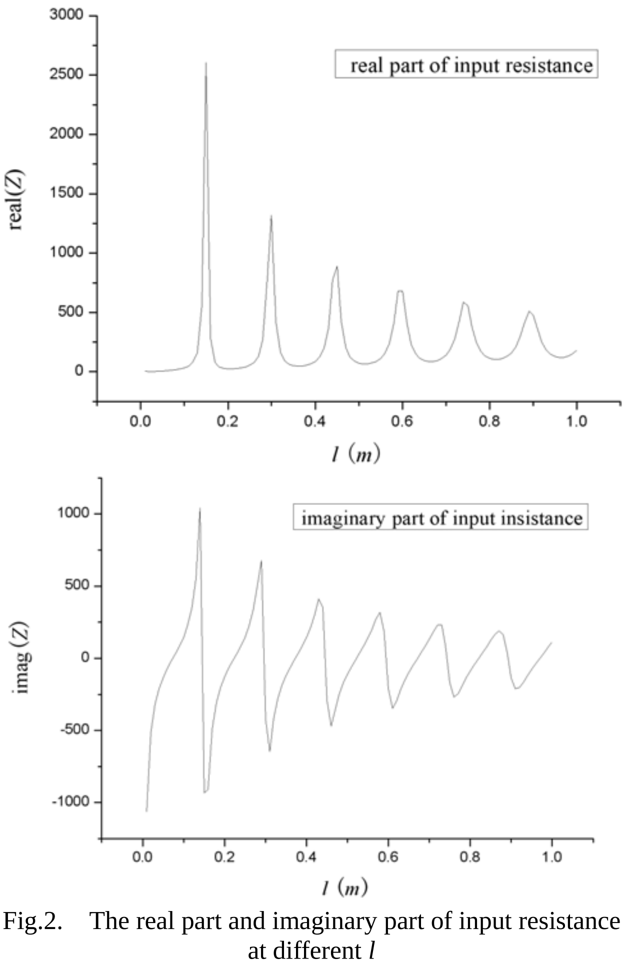

Fig.2. The real part and imaginary part of input resistance at different l

For simplifying the calculation, take a_r as zero and m=n=1 to calculate the input resistance at the point z'=0. When n=1, p only is 1 or 2 and z_p is 0 or l. With expression (11), the result can be gotten:

$$F_1(0) = \ln\gamma + \delta(0) + \sum_{n=1}^{\infty}\frac{\gamma^n}{n\cdot n!} + \frac{1-\mathrm{e}^{\gamma}}{\gamma}$$
$$F_1(l) = \frac{\mathrm{e}^{\gamma}-1}{\gamma} \tag{16}$$

where γ=-jkl. When z_p=0, with (13) the constant c can be calculated $c=IF_1(0)$; and when $z_p=l$, with (13) the input resistance can be gotten:

$$Z(\omega) = \frac{F_1(l) - F_1(0)\cos kl}{-2\pi\sqrt{\varepsilon/\mu}\left(t_\alpha + jt_\beta\right)\sin kl} \tag{17}$$

Take expressions (2), (5), (16) into (17), then $Z(\omega)$ can be expressed as:

$$Z(\omega) = \frac{j(\mathrm{e}^{\gamma}-1) + \mathrm{ctg}kl(\ln\gamma + \delta(0) + \sum_{n=1}^{\infty}\frac{\gamma^n}{n\cdot n!})}{2\pi l\left(\omega\varepsilon - j\sigma\right)} \tag{18}$$

Take ε_r=78, $\sigma/\omega\varepsilon_0\varepsilon_r$ =0.038, f=114MHz to calculate the Z(ω) with (18) and Fig.2 shows the real part and imaginary part of input resistance at different point of l.

III TGR DERIVATION

As mentioned previously, TGR is the transient resistance which is defined as the ration of voltage-current in time domain:

$$Z(t) = \frac{V(t)}{I(t)} \tag{19}$$

Usually, lightning current is represented as a double exponential function, just as:

$$I(t) = I_0(\mathrm{e}^{-at} - \mathrm{e}^{-bt}) \qquad t \ge 0 \tag{20}$$

Where a=37621, 1/s, b=1.13643×10^7, 1/s, I_0=5.4kA. a, b determine the impulse current's rise time while I_0 is the peak current.

Because $I(t)$ is contious and finite points are decimated in time, Fourier transform can be applied to transform the pulse current from time domain to frequency domain :

$$I(\omega) = \int_{-\infty}^{\infty} I(t)\mathrm{e}^{-j\omega t}\mathrm{d}t \tag{21}$$

Considering expression (20) and $t \ge 0$, expression above can be calculated easily :

$$I(\omega) = I_0\left(\frac{1}{a+j\omega} - \frac{1}{b+j\omega}\right) \tag{22}$$

In part II, the resistance of electrode in frequency domain has been calculated as expression (15). Then the voltage in frequency domain $V(f)$ can be calculated:

$$V(\omega) = Z(\omega)I(\omega) \tag{23}$$

where $I(\omega)$ has been calculated as expression (22).

Transform $V(\omega)$ to time domain with inverse Fourier transform, $V(t)$ can be calculated:

$$V(t) = F^{-1}\left[V(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega)\mathrm{e}^{j\omega t}\mathrm{d}\omega \tag{24}$$

Combined with expression (18), expression (24) can be expressed as:

$$V(t) = \frac{I_0}{4\pi^2 l}\int_0^{\infty}(\frac{1}{a+j\omega} - \frac{1}{b+j\omega})\frac{\mathrm{e}^{j\omega t}}{\omega\varepsilon - j\sigma}[j(\mathrm{e}^{-jkl}-1) + \mathrm{ctg}kl(\ln(-jkl) + \delta(0) + \sum_{n=1}^{\infty}\frac{(-jkl)^n}{n\cdot n!})]d\omega \tag{25}$$

The direct calculation of expression (25) is very complex because the analytical expression of (11) is non-holonomic while z'=0. But we can short-cut the calculation with many characters of Fourier's transformation, such as frequency shifting property, convolution property, time shifting property and so on. Then the TGR of grounding electrode can be calculated with expression (19).

IV CONCLUSION

In this paper, a new derivation of TGR based on antenna theory in lossy medium has been proposed. For designing grounding system and calculating it's TGR, this method must be workable. Because antenna theory and computing method are mature, the derivation can be applied to many complex grounding system while FDTD is limited. The next work is to simulate the analytical expression and to compare the result with the one of FDTD.

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