

# Optimal Power Consumption Analysis of Two-level Hierarchical Model and Non-hierarchical Model

Ganglin Zhang ,Guangcan Liu ,Weibing Chen and Cheng Yang

Dept. of Electrics & Communication Engineering

Changsha University

Changsha Hunan, 410003, P. R. China

ccsutxgc@126.com

**Abstract**—Energy scarcity is one of the most critical problems that occur in wireless sensor networks compared to traditional networks. However, the problem has been partly solved by building the power consumption model of WSN. This paper is based on a simple wireless sensor network model, it gives the optimal location of the CH nodes in a cluster and the best parameter about how to divided the cluster amongst hierarchy networks. As shown last, the proposed scheme can save up to 95% of power consumption.

**Key words** - wireless sensor network (WSN),power consumption, two-level hierarchical, non-hierarchical

## I. INTRODUCTION

Wireless sensor network (WSN) is composed of a large number of tiny sensor nodes forming a distributed wireless ad hoc sensing network. The nodes can coordinate their actions and perform tasks such as exploration, surveillance, security management[1], or monitoring and tracking “target points” over a specific region, often referred to as the “mission space”.

WSN Has attracted enormous research attention recently because of their wide range of applications and new characteristic compared to traditional networks. Limited power supply for sensor nodes is one of their typical characteristics[2]. It generates an urgent requirement to maximize the lifetime of a sensor network under a tight power constraints as well as quality of service in wireless sensor networks. Energy aware technologies and strategies in hardware and software aspects are well studied and developed e.g.[3], [4]. And clustering scheme proposed for the WSN can minimize the power consumption of the sensor nodes. One of Clustering scheme is the COSMOS model cluster-based heterogeneous Model for Sensor networks) researched in[5]. And there are several clustering-based routing protocols proposed for minimizing the power consumption[6]. In the existing clustering-based routing protocols for the WSN, including the LEACH (Low-Power Adaptive Clustering Hierarchy) scheme [7], the CHs(Cluster Heads) are usually choosed at random, which may increase power consumption and thus shorten network lifetime. In this paper the position of CH nodes and the scale of a cluster are considered very important for the network lifetime. Based on this observation, the optimal location of the CH nodes in a cluster and the best parameter how to divide the cluster amongst hierarchy networks are

proposed. Computer simulation reveals that the proposed scheme can save up to 95% of power consumption.

The remainder of the paper is organized as follows. In Section 2, present the WSN model worked on. In Section 3 models and compares the hierarchical power consumption with the cluster-head positioned at different locations. Section 4 models and compares the power consumption of two-level hierarchical structure.Finally, Section 5 gives concluding remarks and discusses possible directions the work of this paper can be extended.

## II. THE WIRELESS SENSOR NETWORK MODEL

In this paper, the WSN with its sensors nodes is randomly deployed. A node communicates with another one via radio transmitter/receiver. If a node needs to transmit to another node out of its radio range, the message has to be relayed by intermediate nodes.

A wireless sensor network is considered as a directed graph  $G(V,E)$  with a single sink, illustrated in Fig. 1,where  $V$  is the set of all nodes and  $E$  is the set of all directed links  $(i, j)$  where  $i, j \in V$ ,if and only if sensors  $a$  is in  $b$ 's radio transmission range.

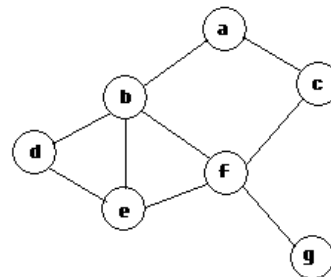


Fig. 1. The physical model of the wireless sensor network.

Power consumption of every node is made up of 3 aspects: receiving, sending and processing data. Result of experimentation in[8] shows that the power required for sending messages is much more than the one required for receiving data. On the other hand the power required to process data is much smaller than that required to send and receive data. Based on the example in Fig. 1: If sensor  $a$  wants to send one piece of data to sensor  $g$ , at least 3 hops are needed; so 3 units of power will be consumed. In the discussion of the following section, only consider data relaying via the shortest path.

### III. POWER CONSUMPTION ANALYSIS OF ONE-LEVEL HIERARCHY

A widely used formula to calculate the power consumption for transmitting a unit (e.g., a packet) data from a to b is as follows:

$$E_{a,b} = d^\sigma \quad (1)$$

Where  $\sigma$  is a constant greater than or equal to 2, and  $d$  is the distance from a transmitting node 'a' to a node 'b'. The power consumption formula has been adopted in many earlier studies [9][10]. According to the model in [11], use formula (1) with  $\alpha = 2$  in this paper's analysis. The total power consumed in one round of transmission from all sensors to a CH node is just the sum of power consumption incurred by all individual sensors. Let the squared cluster contain  $N^2$  nodes, with dimensions  $N \times N$ . If the whole cluster is viewed as non-hierarchy, choosing any node as the CH node would obviously have the different total communication cost. The cost can be calculated as follows (see Fig. 2).

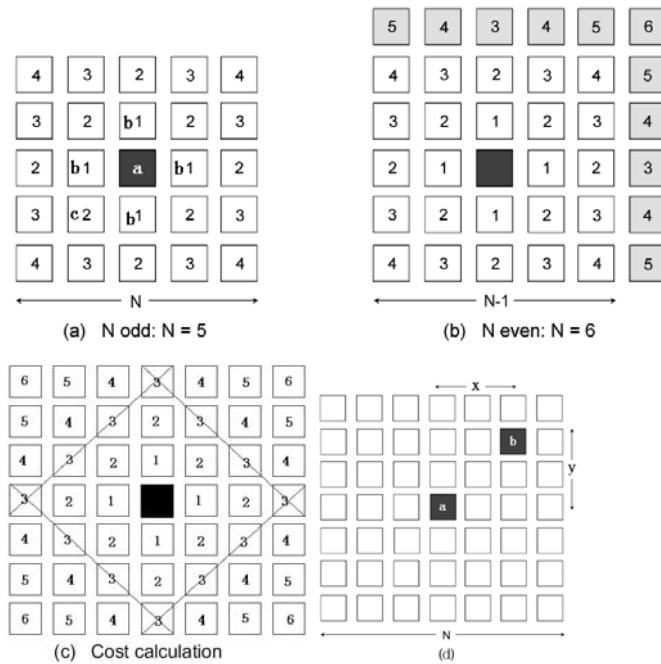


Fig. 2(a). At center is the master node (dark). Numbers in all other nodes represent their communication costs. Fig. 2 (b). A 6x6 cluster. The dark node at the "pseudo center" is the master. Fig. 2 (c). Illustration for cost calculation. Fig. 2 (d). CH node moves from a (0, 0) to b(x, y).

In Fig. 2, the sensors (represented by black squares) are almost evenly deployed in a two dimensional space. Each unit area (or cell) contains one sensor with a high probability. A group of near-by sensors are organized into square-shaped clusters. Assume the black smaller, square-shaped area is the CH nodes, and it can only

communicate directly with its four immediate neighbouring sensors. For example in Fig. 2(a), the sensor 'a' can only transfer to the 4 b sensors. To communicate to sensor c, the data must be relayed by b sensor. And to facilitate analysis; the distance between two adjacent sensors is quantized as one-unit distance. Suppose sensor b wants to send one unit of data to sensor a. The distance from b to a is of 1 unit. Then by formula (1) with  $\alpha = 2$ , the total units of power consumption is  $12 = 1$ . But for transmission from c to a, the data has to be relayed by b sensor, the total units of power consumption will be 12 (from c to b) + 12 (from b to a). So it cost 2 units of power consumption. Following this, Fig. 2(a) is gotten.

Refer to Fig. 2. The total power consumption for one round of (center station to all sensors) transmission is just the sum of all nodes in the cluster. Let  $Ca_o(N)$  denote the total power consumption, where subscript "o" stands for odd and "e" stands for even. CH is in a(0,0), N is odd, and the center node is the CH node. Then have:

$$Ca_o(N) = 4 \sum_{i=1}^{N-1} i^2 + 4 \sum_{i=1}^{N-1} ((N-i)*i)$$

After manipulation:

$$Ca_o(N) = (N^3 - N) / 2 \quad (2)$$

If CH is in a(0,0), N is even, and the center node is the CH node:

$$Ca_e(N) = Ca_o(N-1) + 2 * N / 2 + N + 4 \sum_{i=N/2+1}^{N-1} i$$

After manipulation:

$$Ca_e(N) = N^3 / 2 \quad (3)$$

If CH is in b(X,Y), N is odd, and the edge node is the CH node:

$$Cb_o(N) = 4 \sum_{i=1}^{N-1} i^2 + 4 \sum_{i=1}^{N-1} ((N-i)*i) +$$

$$(X * N * (N+1) / 2 - ((N-1) / 2 - X) * N * X) + (Y * N * (N+1) / 2 - ((N-1) / 2 - Y) * N * Y)$$

After manipulation:

$$Cb_o(N) = (N^3 - N) / 2 + N * X + N * X^2 + N * Y + N * Y^2$$

If CH is in b(X, Y), N is even, and the edge node is the CH node:

$$Cb_e(N) = Ca_o(N-1) + 2 * N / 2 + N + 4 \sum_{i=N/2+1}^{N-1} i$$

$$+ (X * N * (N-1) / 2 - ((N+1) / 2 - X) * N * X)$$

$$+ (Y * N * (N-1) / 2 - ((N+1) / 2 - Y) * N * Y)$$

After manipulation:

$$Cb_e(N) = N^3 / 2 - N * X + N * X^2 - N * Y + N * Y^2$$

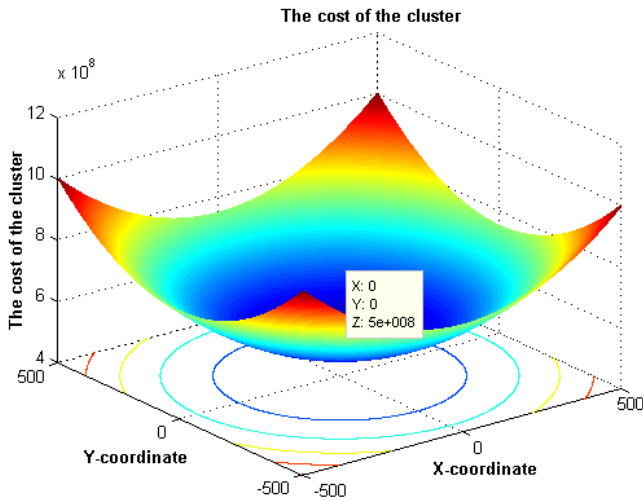


Fig. 3 The cost of a cluster when the cluster-head is located at different locations.

According to Fig. 3, the cost is different when the position  $(b(x, y))$  of the cluster-head is moved from center of mass to the boarder. It shows that the least cost node is  $a(0, 0)$ . When  $N = 1000, X = 0, Y = 0$ , the cost is  $5e+008$ ; when  $N = 1000, X = 500, Y = 500$ , it becomes  $1.001e+009$ .

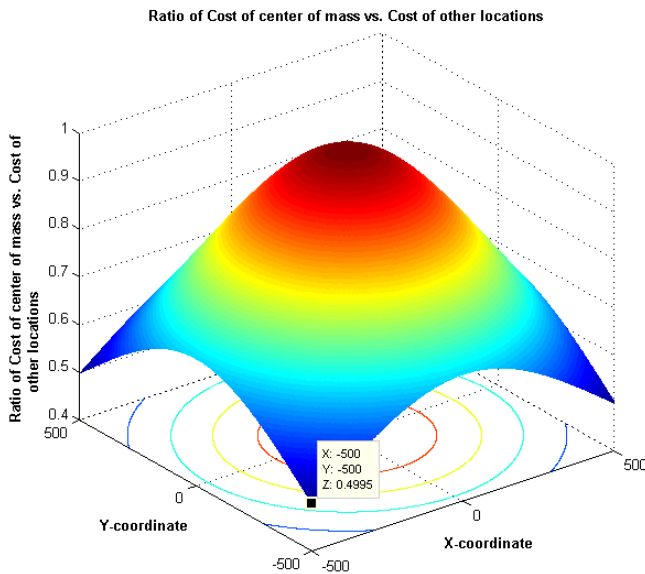


Fig. 4. The ratio of cost of the center of mass vs. the cost of other locations

The saving of power consumption by this CH node location scheme is greatful. The ratio of minimal location  $a(0,0)$  power cost versus  $b(x,y)(x \neq 0, y \neq 0)$  cost is (using "odd N") given by:

$$\frac{Ca_0}{Cb_0} = \frac{(N^3 - N)/2}{(N^3 - N)/2 + N * X + N * X^2 + N * Y + N * Y^2}$$

Fig. 4 plots the same data in terms of the ratio of the

$b(x,y)$  costs/ $a(0,0)$ : When  $N = 0, X = 0$  and  $Y = -500$ , the  $b(x,y)/a(0,0)$  costs ratio is about 100%; when  $N = 1000, X = 0, Y = 0$ , it drops to 49.95%. It shows that the proposed scheme can save up to 50% of power consumption.

#### IV. POWER CONSUMPTION ANALYSIS OF TWO-LEVEL HIERARCHY

In fact, it is not appropriate to adopt a flat structure in WSN: As the CH nodes getting closer and closer, the power will be consumed more and more. What's more, the data processed by the CH node is also sharply increasing. As a result, the nearer the CH node, the shorter the lifetime of nodes. However, this leads the survival time of the network is shorter. So hierarchical structure is not used to real world.

The two-level hierarchy of WSN design is studied in this paper; The whole flat network is divided into  $(N/x)^2$  smaller, squareshaped clusters, and each square-shaped cluster is made up of  $x * x$  nodes, as shown in the following figure. There is still a center station for the whole WSN, located at the center of the WSN, and a CH node in Every squareshaped clusters[12]. The smaller clusters is the a bottom layer network, and the upper layer network consists of all the CH nodes. The data collection of central station is performed in two phases. In the first phase, all clusterheads collect data from sensors in their own clusters. The data is aggregated and/or preliminarily processed in CH nodes. In the second phase, the WSN's center station collects data from all CH nodes.

To maximize the lifetime of the network is always nothing but to reduce the power consumption incurred by WSN. Assuming two-level hierarchy, we need to find out the best way to divide the WSN so that the total cost is minimum. On the other hand, how to choose the optimal locations of CH nodes and X of cluster is very important. The problem about locations is solved above: the best candidate for the cluster CH node position is the center of the cluster, or the one closest to the center of the cluster. Then the next job is to find the optimal X of the cluster.

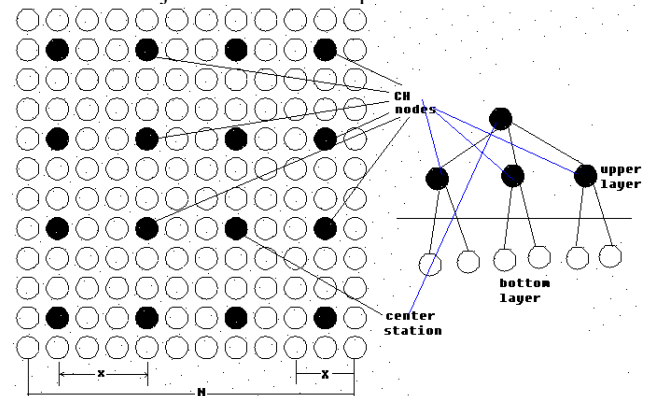


Fig.5. two-level hierarchy. The whole network is divided into  $(N/x)^2$  smaller, squareshaped clusters.

*A. The calculation of power consumption in the first phase of the data collection*

Refer to Figure 5. Suppose the WSN is of dimension  $x \times x$ , so that  $x$  divides  $N$ . Then by Eq. (2) and Eq. (3) which is the power consumption for a round of in-cluster communication, Therefore the total cost for all  $(N/x)^2$  smaller, squareshaped clusters is given by:

$$C_{1o}(N) = N^2(X^2 - 1)/(2X) \quad X \text{ odd} \quad (4)$$

$$C_{1e}(N) = XN^2/2 \quad X \text{ even} \quad (5)$$

*B. The calculation of power consumption in the second phase of the data collection*

As for the second phase, note all CH nodes form a cluster by themselves which is the same as the cluster in phase one. So the same method is adopted to calculate the power consumption. However, the transmission distance from CH node to the nearest CH node is now  $x$  instead of 1 (refer to fig.5). Applying Eq. (2) and Eq. (3) again, the power consumption for the upper network in phase two is given as follows:

$$C_{2o}(N/X) = ((N/X)^3 - N/X)/2 * X^2 \quad N/X \text{ odd}$$

$$C_{2e}(N/X) = ((N/X)^3)/2 * X^2 \quad N/X \text{ even}$$

After manipulation,

$$C_{2o}(N/X) = (N^3 - NX^2)/(2X) \quad N/X \text{ odd} \quad (6)$$

$$C_{2e}(N/X) = N^3/(2X) \quad N/X \text{ even} \quad (7)$$

Combining (4), (5), (6) and (7), we have the expression for total power consumption of the WSN:

$$C_{total}(N, X) = C_1(N, X) + C_2(N, X) = \begin{cases} (N^2X^2 - N^2 + N^3 - NX^2)/(2X) & X \text{ odd, } N/X \text{ odd} \\ (N^2X^2 + N^3 - NX^2)/(2X) & X \text{ even, } N/X \text{ odd} \\ (N^2X^2 - N^2 + N^3)/(2X) & X \text{ odd, } N/X \text{ even} \\ (N^2X^2 + N^3)/(2X) & X \text{ even, } N/X \text{ even} \end{cases} \quad (8)$$

To obtain the minimum  $C_{total}(N, X)$ , we must take the derivative of  $C_{total}(N, X)$  with respect to  $x$ , denoted  $C_{total}(N, X)'_x$ , and solve  $C_{total}(N, X)'_x = 0$  for  $x$ .

$$C_{total}(N, X)'_x = 0 \Rightarrow \begin{cases} N(N-1)(-N+X^2)/(2X^2) = 0 & X \text{ odd, } N/X \text{ odd} \\ N(NX^2 - X^2 - N^2)/(2X^2) = 0 & X \text{ even, } N/X \text{ odd} \\ N^2(X^2 + 1 - N)/(2X^2) = 0 & X \text{ odd, } N/X \text{ even} \\ N^2/2 - N^3/(2X^2) = 0 & X \text{ even, } N/X \text{ even} \end{cases} \Rightarrow \left\{ X_{opt} = \sqrt{\frac{1}{N}} \right. \quad (9)$$

Refer to Eq. (9), the optimal  $X$  of the cluster is  $X_{opt} = \sqrt{\frac{1}{N}} = \sqrt{N}$ . So Combining (9), (4) and (5), we have the expression for minimal total power consumption of the WSN:

$$C_{total}(N) = \begin{cases} \frac{5}{N^2} - \frac{3}{N^2} & X \text{ odd, } N/X \text{ odd} \\ \frac{5}{N^2} - \frac{3}{N^2}/2 & X \text{ even, } N/X \text{ odd} \\ \frac{5}{N^2} - \frac{3}{N^2}/2 & X \text{ odd, } N/X \text{ even} \\ \frac{5}{N^2} & X \text{ even, } N/X \text{ even} \end{cases} \quad (10)$$

Two-level hierarchical Cost vs. Non-hierarchical Cost

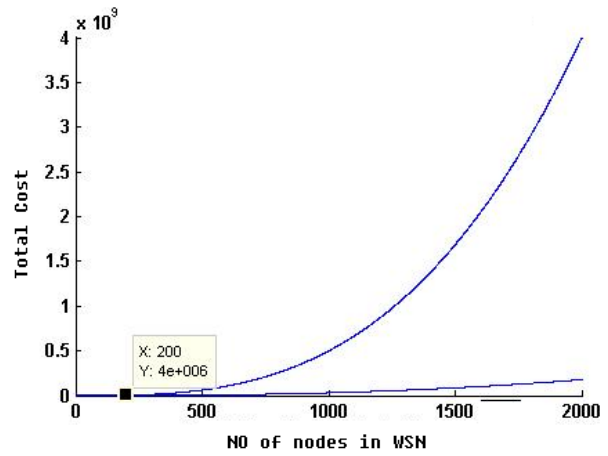


Fig.6. The comparison between minimum two-level hierarchical power consumption and non-hierarchical power consumption, for  $N = 1$  to 2000.

Refer to Fig.6, the cost increases when the  $N$  is increased, but the power consumption of non-hierarchical structure increases more quickly than the two-level hierarchical structure. When  $N = 200$ , the non-hierarchical/two-level hierarchical costs are  $4e+006$  vs.  $5.629e+005$ , respectively; when  $N = 900$ , they are  $3.645e+008$  vs.  $2.427e+007$ ; when  $N = 1800$ , they become  $2.916e+009$  vs.  $1.374e+008$ .

Refer to Eq. (10) and (2), when  $N$  and  $x$  are odd, the ratio of minimal two-level hierarchical power consumption and non-hierarchical power consumption is:

$$\frac{C_{total}(N)}{C_{ao}(N)} = \frac{\frac{5}{N^2} - \frac{3}{N^2}}{(N^3 - N)/2} = \frac{2}{N+1}$$

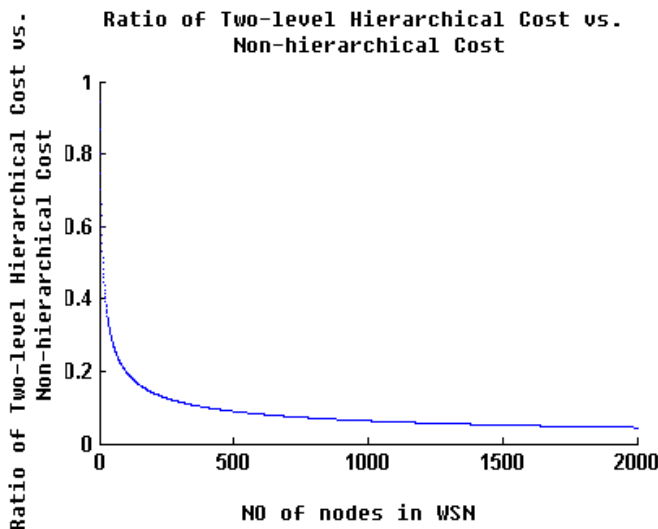


Fig .7.The ratio of two-level hierarchical cost vs non-hierarchical cost

Figure 7 plots the data in terms of the ratio of minimal two-level hierarchical power consumption and non-hierarchical power consumption:it can be seen that the ratio is sharp dropping when the N is increased,When N = 10, the two-level hierarchical/ non-hierarchical ratio is 0.575; When N = 200, the two-level hierarchical/ non-hierarchical ratio is 0.1407; When N = 500, the two-level hierarchical/ non-hierarchical ratio is 0.089; When N = 2000, the two-level hierarchical/ non-hierarchical ratio is 0.0447; It shows that the two-level hierarchical structure can save up to 95% of power consumption when the N is increased.

## V. CONCLUSIONS

In this paper, the problem of power consumption incurred by wireless sensor network (WSN) is considered. At first, the power consumption model by a cluster with Randomly Distributed cluster-head nodes is built. Whereas the simulation results show that the optimal location for CH node is the center of the cluster or the one which is closest to the center of the cluster. Furthermore, the result of the comparison between minimum two-layer hierarchical power consumption and non-hierarchical power consumption indicates that the power consumption of non-hierarchical structure increases more quickly than the two-level hierarchical. Finally, the optimal parameter about how to divide the cluster is found. It is shown that the proposed scheme can save up to 95% of power consumption.

## ACKNOWLEDGMENT

This work is supported by Scientific Research Fund of Hunan Provincial Education Department of China under Grant 10C0412.

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