Multiple-Regime Models and Exchange Rate Forecasting

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Abstract

We extend the basic random walk Markov-Switching model in two ways and evaluate the out-of-sample forecasting performance on the Japanese yen during 1995-2004. First, we estimate both a two- and also a three-regime Markov switching models. Second, we add four exogenous variables as suggested in the monetary theory. According to the modified Diebold-Mariano forecast equivalence test, the result shows that our modified models, a three-regime random walk model and a two-regime monetary model, outperform a simple random walk for the yen. However, the interpretation of coefficients in the two-regime monetary model is unclear and the exchange-rate disconnect puzzle still remains a subject for further investigation

Keywords: Markov-switching, exchange rate

1. Introduction

The exchange-rate disconnect puzzle is heavily studied in international macroeconomics. Meese and Rogoff [1] show that, even with the aid of ex post data on the fundamentals, the several macroeconomic exchange-rate models cannot beat a naïve random walk in the out-of-sample forecasting. Several subsequent papers also document the limited prediction ability of monetary models. Do multi-regime models improve the forecasting ability? We try to answer this question using Markov-Switching (MS) models in this study.

Aside from the monetary models, some studies examine the forecasting performance of regime-switching time-series models. Engel and Hamilton [2] compare the out-of-sample performances of a random walk model and a two-regime Markov switching random walk model. They find that the mean square errors from the Markov switching model are lower than the mean square errors from the random walk model in many cases. Engel [3] documents that the Markov switching model does not generate superior forecasts to a random walk model. However, he finds that the Markov switching model is better in predicting the direction of the exchange rate movements.

This paper extends the basic Markov switching model that is used in Engel and Hamilton [2] in two ways. First, we add one more regime in the MS model and evaluate whether a three-regime model can improve the forecast performance. Second, we add four exogenous variables that are suggested in the sticky-price monetary model in Frankel [4]. In Meese and Rogoff [1], they suggest several possible reasons for the worse performance from structural models and one of them is structural instability. Although the structural change is modeled in the form of Markov switching in Engel and Hamilton [2] and Engel [3], the Markov-Switching model in these studies is the two-regime random walk models. They focus on the structural change of the exchange rate movement itself rather than the structural change of fundamental variables[1].

The objective of this paper is to evaluate the performance of Markov-Switching random walk and Markov-Switching monetary models on Japanese yen (JPY). We find that, according to the modified Diebold-Mariano test, the multi-regime random walk models and a two-regime monetary model outperform a naïve random walk significantly in the out-of-sample forecasting.

2. Methodology

Although Meese and Rogoff [1] show that a random walk without drift outperforms a random walk with drift and the former can beat other structural monetary models in out-of-sample forecasting, Engel and Hamilton [2] argue that the use of a random walk with drift is a more reasonable approach when the drift is

[1] For structural change in fundamentals, M. Frommel, R. MacDonald and L. Menkhoff, "Markov switching regims in a monetary exchange rate model," *Working Paper*, 2004.[5] propose a two-regime Markov switching model allowing a structural change in exogenous variables but not in the variance. However, our model allows different variances in different regimes.

estimated to be significantly different from zero. Thus, we use a random walk with drift as our benchmark and it can be expressed as:

$$\Delta e_t = \mu + \varepsilon_t \quad (1)$$

Next, the monetary model used in this study is expressed as the following form:

$$e_t = a_0 + a_1(m_t - m_t^*) + a_2(y_t - y_t^*) + a_3(i_t - i_t^*) + a_4(\pi_t^e - \pi_t^e{}^*) + u_t \quad (2)$$

where e is the logarithm of the exchange rate, (m - m*) is the logarithm of the ratio of the domestic (Japan) money supply to the foreign (US) money supply, (y - y*) is the logarithm of the ratio of domestic to the foreign real income, (i - i*) is the short-term interest rate differential and $(\pi^e - \pi^e{}^*)$ is the expected long-run inflation differential. We use the long-term interest rate as the proxy of the expected inflation. The equation that we estimate is in the form:

$$\Delta e_t = \alpha_0 + \alpha_1 \Delta m_t + \alpha_2 \Delta y_t + \alpha_3 \Delta i_t^s + \alpha_4 \Delta i_t^l + \varepsilon_t \quad (3)$$

Δm_t is the relative percentage change in money supply over a period, which is calculated as:

$$\Delta m_t = [\ln(m_t) - \ln(m_{t-1})] - [\ln(m_t^*) - \ln(m_{t-1}^*)] \quad (4)$$

Δy_t is also calculated in the similar way in order to reflect the relative percentage change of real income. We do not take logarithm for interest rate variables because they are in the form of percentage already. Note that, in Frommel, MacDonald, and Menkhoff [5], the variation is evaluated at annual change instead of monthly change.

For simplicity, we demonstrate a two-regime setting and most notations follow Hamilton [6]. It is easy to extend the model to a three-regime set up.

$$y_t = \mu_{st} + x_t^{'} \beta_{st} + \sigma_{st} \varepsilon_t \quad (5)$$

where $x_t^{'}$ is the vector of the predetermined variables, which can be the lags of dependent variables or the exogenous variables. y_t is the dependent variable which follows a first-order Markov chain: $\{s_t\}_{t=1}^T$, $s_t = 1$ or 2. s_t is the unobservable state variable that is independent of past y_t and conditional on s_{t-1}. The parameters (μ, β, σ) are subscripted by s_t, indicating that their true values are shifting between two sets of possible parameter values: (μ_1, β_1, σ_1) and (μ_2, β_2, σ_2) depending on underlying macroeconomic conditions. We estimate the parameters by the EM algorithm[2]. The transition probability, p_{ij}, is the probability that the regime switches from i to j:

$$\Pr(s_t = j \mid s_{t-1} = i) = p_{ij}, \text{ for } i, j = 1,2. \quad (6)$$

Let **P** be the constant transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} \quad (7)$$

For concision, the Markov chain can be represented as a vector autoregressive process:

$$\xi_t = \begin{cases} [1 \;\; 0]' & , \text{if } s_t = 1 \\ [0 \;\; 1]' & , \text{if } s_t = 2 \end{cases} \quad (8)$$

Let $\theta = (\mu_1, \beta_1, \sigma_1, \mu_2, \beta_2, \sigma_2)$, the parameters to be estimated. The m-step ahead forecast probabilities are calculated as:

$$E[\xi_{t+m} \mid Y_t; \theta] = \hat{\xi}_{t+m|t} = \mathbf{P}^m \cdot \hat{\xi}_{t|t} \quad (9)$$

where Y_t represents the information set up to time t. The forecasted value of dependent variable y_t is calculated by the weighted average of two forecasted valued conditional on each regime.

$$E[y_{t+m} \mid Y_t; \theta] = \begin{bmatrix} E[y_{t+m} \mid s_{t+m} = 1, Y_t; \theta] \\ E[y_{t+m} \mid s_{t+m} = 2, Y_t; \theta] \end{bmatrix}^{'} P^m \hat{\xi}_{t|t} \quad (10)$$

3. Empirical Results

The data are obtained from IMF International Financial Statistics CD-ROM. The exchange rate, USD/JPY, from 1973:7 to 2004:12 is examined. We first normalize the exchange rate so that the exchange rate equals to unity in 1973:7 for all currencies, and we use 100 times the natural logarithm of the normalized exchange rates in estimation. The industrial production is used as the proxy of the output. The money market rate is used as the short-term interest rate because of the availability during the whole sample. Last, the long-term government bond rate is used as the expected inflation.

Table 1 shows the estimation for single- and multiple-regime random walk models during 1973:7-2004:12. For the one-regime random walk (RW), the constant term is significantly positive, indicating that the JPY is appreciating over the period. The estimation of the two-regime random walk model (MS2RW) increases the significance of constant terms. Moreover, the regime one is associated with the periods of appreciation and higher volatility and the regime two is associated with the periods of depreciation and lower volatility. The diagonal terms of the transition probabilities imply the persistency of the regimes. The estimates of transition probabilities imply the regime two is not persistency – if the JPY is in the regime two this month, the probability of switching to the regime

[2] EM stands for expectation-maximization iteration process and it was first proposed by Dempster, Lair, and Rubin A. P. Dempster, Lair, N. M., and Rubin, D. B., "Maximum likelihood for incomplete data via the EM algorithm.," *Journal of the Royal Statistical Society, Series B,* **39**(1): pp.1-38, 1977. The logic is, first, to form an expected log likelihood function given an initial value to the unobservable variable. Second, maximize the expected log likelihood function and iterate until converge. We use the EM algorithm to deal with the unobservable state variable, s_t.

one next month is 56%. The ergodic probability of a regime represents the proportion of that regime in the sample and it shows that around 72 percent of time, the JPY is in the regime one.

Table 1. Estimation for random walk models

Model	RW	MS2RW		MS3RW		
Regime		1	2	1	2	3
Constant	0.25*	0.51**	-0.42**	0.38*	-0.58***	0.30
	(0.17)	(0.25)	(0.19)	(0.23)	(0.13)	(0.27)
Sigma	3.29***	3.80***	1.08***	3.77***	0.47***	1.30***
	(0.09)	(0.18)	(0.20)	(0.16)	(0.13)	(0.22)
Transition probabilities		0.78***	0.22***	0.83***	0.17***	0.00
		(0.07)	(0.07)	(0.06)	(0.05)	(0.03)
		0.56***	0.44***	0.91***	0.00	0.09
		(0.13)	(0.13)	(0.16)	(0.14)	(0.13)
				0.09	0.00	0.91***
				(0.12)	(0.14)	(0.07)
Ergodic probabilities		0.72	0.28	0.74	0.13	0.13
LLF	-984	-969		-960		
BIC	-996	-1004		-1031		
MSC	-988	-981		-986		

The number in the parenthesis is the standard deviation.

***: significance at 1% level; **: significance at 5% level

*: significance at 10% level

Table 2. Estimation for monetary models

Model	MM	MS2MM		MS3MM		
Regime		1	2	1	2	3
Constant	0.25	0.68**	-0.64**	-0.03	-0.49	1.03**
	(0.17)	(0.28)	(0.19)	(0.17)	(0.32)	(0.45)
Money	0.14	0.18	0.09	0.03	0.09	0.05
	(0.13)	(0.20)	(0.14)	(0.15)	(0.22)	(0.36)
Industrial production	0.00	0.03	-0.07**	0.00	-0.08**	0.08
	(0.03)	(0.04)	(0.03)	(0.03)	(0.04)	(0.08)
Short-term interest rate	0.31	0.50	-0.16	0.57	0.96**	-0.27
	(0.21)	(0.35)	(0.33)	(0.30)	(0.44)	(0.82)
Long-term interest rate	0.00	0.06	-0.11	-1.89***	3.28***	-1.99
	(0.43)	(0.68)	(0.53)	(0.50)	(0.87)	(1.64)
Sigma	3.28***	3.81***	1.21***	0.99***	2.43***	4.00***
	(0.10)	(0.19)	(0.20)	(0.17)	(0.29)	(0.28)
Transition probabilities		0.74***	0.26***	0.66***	0.00	0.34**
		(0.08)	(0.08)	(0.11)	(0.10)	(0.14)
		0.53***	0.47***	0.00	0.86***	0.14
		(0.12)	(0.12)	(0.06)	(0.07)	(0.09)
				0.18**	0.12	0.70***
				(0.08)	(0.09)	(0.11)
Ergodic probabilities		0.67	0.33	0.22	0.37	0.41
LLF		-983	-964		-954	
BIC		-1018	-1047		-1096	
MSC		-995	-1012		-1067	

The last three columns of Table 1 show the estimates of the three-regime random walk (MS3RW). Unlike MS2RW, whose estimates are all significant, the constant of the regime three and some transition probabilities estimates are insignificant. The constant terms of the regime one and three are both positive but it is insignificant in the regime three. The main difference between the first regime and the third regime is the volatility – the regime one has highest volatility among the three regimes. The regime two in MS2RW indicates the periods of depreciation and low volatility. The transition probabilities show that the persistency is high for the regime one and three but extremely low for the regime two – the probability of switching to the regime one is 91% but 0% for staying in the regime two. The estimates also show that it never switch from the regime one to three and the regime three to two.

Table 3. Out-of-sample forecasting comparison

(A) Mean Squared Forecast Error (MSFE)

	Forecast Horizon		
Model	1	3	6
RW	11.935	11.950	11.932
MS2RW	11.871	11.968	11.936
MS3RW	11.615	11.843	11.962
MM	11.849	11.787	11.707
MS2MM	11.560	11.893	11.722

(B) Modified Diebold-Mariano statistics (p-value in parentheses)
H_0: MSFE1 = MSFE2, H_a: MSFE1>MSFE2

	Forecast Horizon		
Model	1	3	6
RW vs. MS2RW	1.450*	-2.696	-1.562
	(0.075)	(0.996)	(0.939)
RW vs. MS3RW	1.369*	1.016	-1.673
	(0.087)	(0.156)	(0.952)
MM vs. MS2MM	1.546*	-1.111	-0.862
	(0.062)	(0.866)	(0.805)
RW vs. MM	0.554	0.939	0.702
	(0.290)	(0.175)	(0.242)
RW vs. MS2MM	1.557*	0.389	0.679
	(0.061)	(0.349)	(0.249)
MS2RW vs. MS2MM	1.329*	0.509	0.692
	(0.093)	(0.306)	(0.245)
MS2RW vs. MS3RW	1.099	1.154	-1.452
	(0.137)	(0.125)	(0.925)
MS3RW vs. MS2MM	0.224	-0.264	0.724
	(0.412)	(0.604)	(0.235)

Table 1 also presents two model selection criteria – Bayesian Information Criterion (BIC) and Markov Switching Criterion (MSC). The latter is recently developed in Smith *et al.* [8]. Although the BIC favors the single-regime random walk, the MSC prefers the two-regime random walk. Note that the three-regime model is preferred to one-regime random walk according to the MSC.

The monetary theory suggests the signs are negative for the change of money supply, positive for industrial production, positive for the short-term interest rate differential, and negative for the long-term interest rate differential. In Table 2, we find little evidence to support the monetary theory. For the one-regime monetary model (MM), the only significant estimate is the volatility. The industrial production and the long term interest rate differential play no role here. For the two-regime model (MS2MM), the only significant estimate is the industrial production term for the second regime but the sign is contrary to the theory. The transition probabilities suggest higher persistency for regime one.

4. Out-of-sample evaluation

In this section, we evaluate the forecasting performance among different models by the Mean Squared Forecast Error (MSFE) and the modified Diebold-Mariano test proposed in Harvey, Leybourne, and Newbold [9] . The latter is used because we want to know whether the smaller MSFE is statistically significant. Suppose we want to compare forecasts of two models, 1 and 2. $\{\varepsilon^1\}$ and $\{\varepsilon^2\}$ indicated the forecasting errors from model 1 and 2 respectively. We choose squared error loss function: $L(\varepsilon^1)=(\varepsilon^1)^2$. The loss differential: $d_t = L(\varepsilon^1_{t+h|t}) - L(\varepsilon^2_{t+h|t})$. The modified Diebold-Mariano statistics is[3]:

$$MDM = \frac{\tau^* + 1 - 2p + p(1-p)/\tau^*}{\tau^*} \cdot \frac{\bar{d}}{(\gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j)^{1/2}} \sim t_{\tau^*-1}(0,1)$$

where $\gamma_j = \mathrm{cov}(d_t, d_{t-j})$, τ^* is the number of forecasts.

We choose the last ten years as our forecasting period, 1995–2004, and re-estimate at each time point as the data expand[4]. The ex post values of the four exogenous variables are used to construct the forecasts. In Table 3, all the other models have smaller MSFE than a random walk for the one step-ahead forecasting. Note that, for one-step-ahead forecasting, MS2MM has the lowest MSFE. According to the modified Diebold-Mariano test for the one-month-ahead forecast, we find that MS2RW and MS3RW outperform RW significantly. Although MS3RW has lower MSFE than MS2RW, the outperformance is insignificant. Moreover, MS2MM outperforms RW, MM, and MS2RW significantly for one-month ahead forecast. The results suggest that our modifications, adding a third regime and/or including exogenous variables, help forecast the movement of the JPY during 1995 – 2004.

5. Conclusion

To investigate whether multiple-regime models perform better in forecasting, we extend the basic random walk Markov-Switching model in two ways: add one more regime and include the exogenous variables in the main equation. Our sample is the Japanese yen during 1973:7-2004:12. Our main findings are following: the mean and the volatility both play significant roles in determining regimes under two-regime random walk. Moreover, the high volatility is associated with the regime of positive mean (appreciation periods). For the multi-regime monetary models, however, the signs of coefficients do not all conform to the monetary theory. In the out-of-sample forecasting, we find some evidence of forecasting improvement in our modified models. The three-regime random walk and the two-regime monetary model outperform the random walk model significantly in one-month ahead forecasting. However, we should interpret the performance with caution. The forecasts made by the monetary models depend on the perfect foresight of exogenous variables. Moreover, although some monetary Markov-Switching models perform better than a random walk, the signs of some estimates are opposite to what the theory suggests and it is worthy future investigation.

[3] The second part of MDM is the original version in F. X. Diebold and R. S. Mariano, "Comparing Predictive Accuracy," *Journal of Business and Economic Statistics,* **13**(3): pp.253-263, 1995.

[4] When evaluate forecasting, we incorporate data from 1973:7 for each estimation and forecasting rather than shifting a fixed estimation window.

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