The one warehouse and N retailers problem with uncertain demand

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Abstract

The major aim of this paper is to provide a solution to one warehouse and N retailers problem with uncertain demand. The 'static-dynamic uncertainty' strategy is used. The objective function is to minimize expected average total costs incurred by the total system comprising of retailers' replenishment cost, retailers' inventory holding cost, warehouse's replenishment cost, warehouse's inventory holding cost and lost-sale cost. We use integer-ratio policies. We generated 15 samples for each 5 retailers by using ProModel simulation software for Heuristic I and Heuristic II. The results of stochastic model compared with the deviation from optimum solution for these different heuristics.

Keywords: supply chain management, one warehouse N retailers problem, integer-ratio policies and uncertain demand.

1. Introduction

In supply chain management (SCM), delivering products from a warehouse to retailers plays a very important role. Due to their applicability to real world situations, one warehouse and *N* retailers problem have been extensively analyzed in the literature [1]. B. Abdul-Jalbar et al. proposed a detailed literature review analysis [2]. Monthatipkul and Yenradee proposed a new inventory control system called the inventory/distribution plan (IDP) control system for a one-warehouse multi-retailer supply chain [3].

Bookbinder and Tan showed that the mathematical structure of the static uncertainty model was equivalent the deterministic model in their study [4].

B. Abdul-Jalbar et al. dealt with the classic one-warehouse N retailers problem [5].

Integer-ratio policies have the advantage of giving more freedom in determining the replenishment intervals since they do not restrict them to be powers of two multiples of some base planning periods [1].

The main contribution of this study is the progress of one warehouse and N retailers supply chain system under uncertain demand. This system is shown in figure-1.

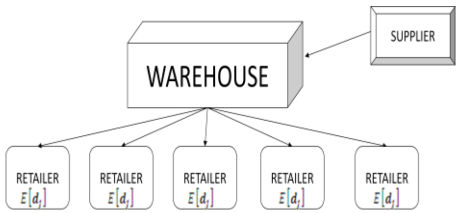

Fig. 1: One warehouse and N retailers supply chain system

2. Problem Formulation

Warehouse replenishes its orders from external supplier and supplies all orders of retailers in the system. We assumed that the demand was stochastic. Lost sales were allowed. The notation in this paper is as follows:

Indices:

$\boldsymbol{J}$ Retailers, $\forall\, j = 0,1,2, \dots .N$

Parameters:

$\boldsymbol{f_j}$ Number of times that retailer *j* places an order during t_0.

$E[\boldsymbol{f_j}]$ Expected number of times that retailer *j* places an order during t_0 .

$\boldsymbol{d_j}$ Demand per unit time at retailer j

$E[\boldsymbol{d_j}]$ Expected demand per unit time at retailer j

$E[\boldsymbol{sd_j}]$ Expected demand per unit time at retailer j with α service level.

$\boldsymbol{k_j}$ Fixed replenishment cost per order at retailer *j*

$\boldsymbol{k_0}$ Fixed replenishment cost per order at the warehouse

$\boldsymbol{h_j}$ Inventory holding cost per unit per unit time at retailer *j*

$\boldsymbol{h_0}$ Inventory holding cost per unit per unit time at the warehouse

$\boldsymbol{t_j}$ Replenishment interval at retailer *j*

$E[\boldsymbol{t_j}]$ Expected replenishment interval at retailer *j*

$\boldsymbol{t_0}$ Replenishment interval at the warehouse

$E[\boldsymbol{t_0}]$ Expected replenishment interval at the warehouse

$\boldsymbol{Q_j}$ The quantity of products replenished to retailer *j*

$E[\boldsymbol{Q_j}]$ The expected quantity of products replenished to retailer *j*

$\boldsymbol{TC_j}$ The average total costs incurred by retailer *j*

$E[\boldsymbol{TC_j}]$ The expected average total costs incurred by retailer *j*

$\boldsymbol{C_0}$ The average total costs incurred by the warehouse

$E[\boldsymbol{C_0}]$ The expected average total costs incurred by the warehouse

$\boldsymbol{C_T}$ The average total costs incurred by the total system

$E[\boldsymbol{C_T}]$ The expected average total costs incurred by the total system

$\boldsymbol{ls_j}$ Lost sale cost per unit and per unit time at retailer *j*

$\boldsymbol{LS_j}$ The quantity of products unsatisfied at retailer *j*

$\boldsymbol{r_j}$ The ratio between the replenishment and the inventory holding costs for retailer *j*.

$\boldsymbol{r_S}$ The ratio between the replenishment and the inventory holding costs for the total system

$\boldsymbol{Z}$ Standard normal value corresponding to α service level

$\boldsymbol{\alpha}$ service level

$\boldsymbol{CV}$ Coefficient of variation of demand per unit time at retailer j

$\boldsymbol{\sigma}$ The standard deviation of the demand per unit time at retailer j with α service level

The objective function is to minimize expected average total costs. When the demand in the supply chain system is not supplied, it is assumed as a lost sale. The total cost is minimised under a minimal service level constraint.

The objective function of the total system cost for deterministic case is calculated by equation 1.

$$C_T = \sum_{j=0}^{N} C_j = \frac{k_0}{t_0} + \left\{ \boldsymbol{t_0} > \boldsymbol{t_j} \,\middle|\, \boldsymbol{h_0} \sum_{\boldsymbol{t_0} > \boldsymbol{t_j}; \boldsymbol{j}=\boldsymbol{1}}^{\boldsymbol{N}} \frac{(\boldsymbol{t_0} - \boldsymbol{t_j})\boldsymbol{d_j}}{\boldsymbol{2}} \right\} + \sum_{j=1}^{N} \left[\frac{k_j}{t_j} + \frac{h_j d_j t_j}{2} \right] \quad (1)$$

When this integrated function is decomposed, we cannot write warehouse holding inventory costs normally. Which retailer's order interval is longer and which one is shorter is not known

After solutions have been found, objective function can be written exactly. The integer values of optimal solutions must be determined firstly and then optimal solutions must be calculated by using these values. The process of calculation for one warehouse N retailers problem for deterministic case was proposed by Şenyiğit and Akkan [1]. We used the heuristic proposed by Şenyiğit and Akkan to get optimum solution (lower bound).

The demands per unit time at retailer j are assumed to be stochastic and normally distributed for stochastic model as Bookbinder and Tans' study. Lead time is zero.

We assume $E[\boldsymbol{sd_j}]$ as $\boldsymbol{d_j}$, $E[\boldsymbol{C_j}]$ as $\boldsymbol{C_j}$, $E[\boldsymbol{t_j}]$ as t_j , $E[\boldsymbol{f_j}]$ as $\boldsymbol{f_j}$, $E[\boldsymbol{Q_j}]$ as $\boldsymbol{Q_j}$ according to the static-dynamic uncertainty strategy [4].

We assume $E[\boldsymbol{sd_j}]$ is calculated by equation 2 for each retailer [4].

$$E[\boldsymbol{sd_j}] = E[d_j] + \text{ZCV}E[d_j] = E[d_j](1 + \text{ZCV}) \quad (2)$$

We assume, the objective function of the stochastic model, $E[\boldsymbol{C_T}]$ is calculated by equation 3.

$$E[\boldsymbol{C_T}] = \sum_{j=0}^{N} E[C_j] = \frac{k_0}{E[t_0]} + \left\{\boldsymbol{t_0} > \boldsymbol{t_j} \left| \boldsymbol{h_0} \sum_{\boldsymbol{t_0} > \boldsymbol{t_j}; \boldsymbol{j=1}}^{\boldsymbol{N}} \frac{(E[t_0]-E[t_j])E[d_j]}{\boldsymbol{2}} \right.\right\} + \sum_{j=1}^{N} \left[\frac{k_j}{E[t_j]} + \frac{h_j E[d_j] E[t_j]}{2}\right] + \sum_{j=1}^{N} \boldsymbol{ls_j}\ \boldsymbol{LS_j} \quad (3)$$

We assume all constraints in deterministic case in Şenyiğit and Akkan study [1] are the same at the stochastic model in this study, according to the static-dynamic uncertainty strategy.

3. Application

In this section, we present a one-warehouse and five-retailer problem with uncertain demand data set (Table-1). We adapt Abdul Jalbar et. al data [5] to our study.

We take into account CV and α in numerical analysis. We assumed that CV=0.5 and α=0.80. Lost sale cost is $1. There are 5 (N=5) retailers in the system. We produce 15 samples for analyzing stochastic model performance.

Table.1 Data Set

	j=0	*j=1*	*j=2*	*j=3*	*j=4*	*j=5*
h_j	31	181	427	425	142	134
k_j	124	102	118	116	453	74
$E[d_j]$		801	392	386	4530	784

Stat Fit and Pro Model simulation software are used for realized demand per unit time at retailer j. These realized demands and optimum solution values are shown in Table 2.

Table.2 Results for Heuristic I

d_1	d_2	d_3	d_4	d_5	***Opt.***	***Dev.%***
594	290	286	3358	581	42321.64	16.18
479	234	231	2707	468	38010.09	29.36
693	339	334	3922	679	45744.19	7.49
122	60	59	691	120	19211.89	155.93
641	314	309	3626	627	43990.67	11.77
363	178	175	2055	356	33117.22	48.47
150	73	72	846	146	21243.59	131.45
646	316	312	3654	632	54239.58	9.35
466	228	225	2637	456	37508.20	31.09
585	286	282	3306	572	42007.99	17.05
838	**410**	**404**	**4740**	**820**	**50287.60**	**1.59**
445	218	215	2516	435	36652.98	34.15
65	32	32	370	64	14060.09	249.71
742	363	358	4195	726	47321.86	3.90
140	69	68	794	137	20588.99	138.81
					Ave.	59.09

We called optimum solution as lower bound for each row (see Table.2). We calculate the deviation from the lower bound by equation 4. We calculated **σ** by equation 5 for generating realized demand by simulation.

$$Dev.\% = \frac{E[\boldsymbol{C_T}]-LB}{LB} \quad (4)$$

$$\text{CV} = \frac{\boldsymbol{\sigma}}{E[\boldsymbol{sd_j}]} \quad (5)$$

In Heuristic I, we assumed that $E[d_j]$ are realized demand. We designed our policy according to this assumption. We calculated $E[\boldsymbol{C_T}]$ as \$49169.29 except 11th row which $E[\boldsymbol{C_T}]$ is \$49488.29 since there are lost sales in the system. So, there is \$319 lost sale cost. We did not take service level constraint into account in Heuristic I.

In Heuristic II, we assumed that $E\left[d_j\right](1+\mathrm{ZCV})$ is the realized demand for assuring service level α. We calculated $E[\boldsymbol{C_T}]$ as \$58605.21. There are no lost sales in the system for this case.

Table.3 Results for Heuristic II

No	**1**	**2**	**3**	**4**	**5**
Dev.%	38.48	54.18	28.12	205.05	33.22
No	**6**	**7**	**8**	**9**	**10**
Dev.%	76.96	175.87	8.05	56.25	39.51
No	**11**	**12**	**13**	**14**	**15**
Dev.%	16.54	59.89	316.82	23.84	184.64

4. Conclusions and future works

The stochastic models due to one warehouse N retailers problem have the best performances in terms of percentage deviation from the lower bound (1.59% for Heuristic I, 8.05% for Heuristic II) while the stochastic model has the worst performance with 249.71% deviation for Heuristic I and 316.82% deviation for Heuristic II.

The stochastic model cost performance rises while the differences between the realised demands and the expected demand get smaller as stochastic model cost get closer to the deterministic model (lower bound) cost.

Our aim is to get a solution to uncertain demand. We realized it. Different retailers numbers, CV and α value can be investigated. The system can be modelled by simulation software (ARENA, PROMODEL, etc.).

For future work, new heuristics for one warehouse N retailers problem under uncertain demand could be developed. Lead-time and price uncertainty can be incorporated into problem formulation. Defect rates can be taken into account. Fuzzy modelling can be adapted to the problem.

5. Acknowledgements

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6. References

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