Method and approach of Calculus teaching reform

Wangyuan Dong

College of science, Huazhong Agricultural University, Wuhan, Hubei, P. R. China

Abstract

In this paper, the present situation of the teaching contents of Calculus seriously lagging behind the subject development is analyzed. And the necessity and feasibility of introducing exterior differential forms and contemporary mathematics in University Mathematics teaching are discussed.

Keywords: Calculus, Teaching reform, Exterior differential form.

The teaching of Calculus has lasted about three centuries since it began from the first system calculus textbook "Infinitesimal Analysis"(G. F. A. L' Hospital 1696). During the 300 years, Calculus has undergone several major changes from the methods to contents. In contrast, the teaching of Calculus still remains traditional. Today, when we face the future, how to reform the teaching of Calculus is a new problem. We hope that this reform will not only maintain the classic basic contents, but also reflect the development of modern mathematics in the course of teaching. This new topic needs to be considered by all mathematical workers and educators.

1. The development of Calculus

In the history (see [1], [2]), Calculus has gone through three stages of development. The first stage was about 1660-1670 of the 17th century. This was the stage of establishment of the calculus, and Calculus gradually developed into an independent discipline.

In the second stage the basic calculus theory was established by Cauchy, Weirstrass and Riemann. The Cauchy definition of limit and $\varepsilon-\delta$ method were entrenched and has been used until now. There is almost no difference between the symbols used now in most textbooks and those in the years of Weirstrass.

In the third stage of the development of Calculus, the exterior differential form was built (see [3]). Exterior differential form was established by Crassmann, Poincare and Cartan in the early days of the twentieth century.

2. Reform of course name and course content

2.1. Scientific name and accurate positioning

In the first grade of University, Physics, Chemistry and Biology are called General Physics, General Chemistry and General Biology respectively. But in most universities of China, mathematics is called advanced mathematics. However, where is "advanced" reflected? If we look at the contents of the textbook, we can find that it only contains the content of the second stage of the development of Calculus, not related to many disciplines of Mathematics that have obtained great development in the last century. It seems to be wrong to name it advanced mathematics. Therefore, the advanced mathematics teaching

should be reformed from scientific nomenclature. It appears to be more accurate and scientific to name it calculus.

2.2. Choice and complementary of the teaching contents

From the teaching contents of view, there are many problems worthy to study, research and practice. For the purpose to add new contents, we must reduce the class hours of old curriculum, because the total class hours are limited. In this respect, many universities have made a bold attempt. As for Agricultural and Forestry Colleges, Calculus course needn't too subtle $\varepsilon-\delta$ method, unilateral limit and the infinite limit strictly proved. The chain rule of compound function derivation can not speak. Engineering University also does not need to talk about all kinds of strange series convergence tests, such as integral test method. Of course, a teacher who not speaks $\varepsilon-\delta$ approaches has some problems to adjust his teaching methods. In fact, this is entirely possible. From the birth of Calculus to the establishment of calculus foundation, after about 200 years, during this period the $\varepsilon-\delta$ method has not been born. The imparting of Calculus has not only been taught, but also indeed promoted the development of Physics, Mechanics and Astronomy, and make them gain great progress.

If we give up the $\varepsilon-\delta$ method, it is necessary to add the content of the third stage of the development of Calculus, this is the exterior differential form. As everyone knows, the formula

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

was called the fundamental theorem of Calculus (see [4]). Without this formula there would be no Calculus. This formula has pointed out the connection between differential and integral. This means that the differential and integral are inverse relation. In the high dimensional space the embodiment of this relationship are Green formula, Stokes formula and Guass formula.

If we hope to express this relation in a more concise, clear way, the exterior differential form is the most appropriate choice. Let $\Omega \subset R^n$ be a domain, $\partial\Omega$ be its boundary, ω be n order exterior differential form, and then we have

$$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega \qquad (1)$$

here $d\omega$ denote the exterior differential for ω. When ω is zero order exterior differential form, formula (1) is the Newton-Leibniz formula. When ω is first order exterior differential form and Ω is a domain of two dimensional plane, it is the Green formula. When ω is first order exterior differential form and Ω is a curved surface of third dimensional space, it is the Stokes formula. When ω is second order exterior differential form and Ω is a domain of third dimensional space, it is the Gauss formula. It must be pointed out that the formula (1) is established for high dimensional space. It is also the establishment when Ω be a differential manifold.

The formula (1) is the most essential theorem in Calculus. It is the peak and end point of Calculus. It is the most profound theorem in Calculus. It is the end of Calculus and entrance of modern mathematics. It does not belong to the scope of Calculus if go forward again.

Formula (1) is rare in the simple, deep theorems of mathematics. Fetching it to Calculus teaching is not only necessary but is also feasible. The development of mathematics education and the development of human thought is a process from elementary stage to advanced stage. It is a continuous process of the senior instead

of the elementary. Psychology also reveals the dialectical relation of unity of opposites of human thinking about memory and forgetting. Only to forget some old things, one could accept and remember some new things. For example, the current primary school mathematics has many travel problems and ergonomics problems. By using four operations of primary school they can resolved them, but students often find it difficult. When junior high school students learn algebraic equation, the solution of this kind of problems is not only simple, but also clear. Equation of Junior high school is more advanced than senior primary school arithmetic, but the advanced is easier than the low-level, and more effective. And the advanced has wider application range. In Calculus teaching to speak of exterior differential form is a typical example of senior (is actually acceptable) to replace the base (phase it is difficult). When we have the formula (1), then Green formula, Stokes formula and Guass formula may be forgot temporarily. When needed, it is very easy to deduce by the formula (1).

2.3. Cotemporary Mathematics and Calculus teaching

In recent years, a lot of engineering colleges, agricultural colleges and medical colleges have made many efforts in introducing modern mathematics. Even so, Mathematics is still far behind other disciplines to update the knowledge structure. The mathematical knowledge of our graduates of high school only remains at the level of the seventeenth Century. But in high school physics, students can learn nuclear physics, quantum theory and Einstein's theory of relativity. For general college students, their mathematical courses are calculus, linear algebra and probability statistics. These are mathematics of the eighteenth Century and nineteenth Century. Even the students of department of mathematics don't have a whole concept of last century mathematics at the time of graduation. This situation of teaching content seriously lagging behind the contemporary progress is very easy to cause the students' misconceptions. Firstly, students think that calculus and linear algebra are already the most advanced mathematics. Therefore, mathematics has now come to the end. Secondly, they think that other mathematics is abstract, difficult and useless symbol accumulation. But in fact, mathematics has got great progress and development in the last century (especially in the last 50 years). For instance, the famous Fermat conjecture was proved in 1995. This is a brilliant academic success. And mathematics has been widely and deeply used in science, technology, management and other various fields.

In the same field, the mathematical tools used are updated very fast. As in the error correction code surface, when BCH code appears in the Sixties of Twentieth Century, the mathematical tools of communication engineering are updated from the traditional Fourier analysis and statistics to modern algebra. Soon after, Algebraic Geometric code was found in 1982, it is better than that of BCH codes. It could foreknow that in the future communication engineers must study a little algebra theory and algebraic geometry. Maybe as Mr. Yau Shing-Tung said: "The engineers who are engaged in Computer Graphics need to break the limits of the industry, learn a bit of differential geometry". All the facts show that, for the twenty-first Century college students, their mathematical knowledge structure must have changed a lot. In order to make their mathematical knowledge serving their respective professional field, college teachers must teach them some knowledge of Contemporary Mathematics. If university graduates' mathematical structure still remains at the present levels,

it will be difficult to solve the problems in the future. Therefore, it is an urgent task to add the Contemporary Mathematics to College Maths teaching for the teaching reform.

Support by teaching research provincial project of colleges and universities in Hubei province: Teaching reform and practice to the courses of analysis of information and computing science specialty in Agricultural and Forestry College (No. 2012173), and Optimization and practice of advanced mathematics teaching in Agricultural Colleges Based on Specialty Group (No. 2010134).

References

[1] M. Kline, Mathematical Thought from Ancient to Modern Times, New York: Oxford University Press, 1972.

[2] C. Boyer, A History of Mathematics. John Wiley & Sons, Inc. 1968.

[3] C. V. Westenholz, Differential form in mathematical physics, North-Holland Publishing Company, Amsterdam, 1978:141-208.

[4] B. Armstrong, D. Davis, Brief Calculus（second Edition, Higher Education Press, Beijing, 2004.