Denoising of Hyperspectral Remote Sensing Image using Multiple Linear Regression and Wavelet Shrinkage

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Abstract

Hyperspectral remote sensing image is easily contaminated by noise, which will affect the application of hyperspectral image, such as target detection, classification and segmentation, etc. Therefore, a denoising method of hyperspectral remote sensing image based on multiple linear regression (MLR) and wavelet shrinkage (WS) is proposed. Firstly, the residual image and the predicted image are obtained via MLR. Secondly, WS is performed on the residual image to remove the noise in the spatial domain. Lastly, a final denoised image is obtained by the predicted image and the corrected residual image. The experimental results show that the proposed method can improve signal-to-noise ratio (SNR) of the hyperspectral image efficiently.

Keywords: hyperspectral remote sensing image, denoising, multiple linear regression, wavelet shrinkage

1. Introduction

Hyperspectral remote sensing images can be viewed as three-dimensional data consisted by one-dimensional spectral information and two-dimensional spatial information. The reliability of the information delivered by hyperspectral remote sensing applications highly depends on the quality of the captured data [1]. Although over last decades the imaging spectrometers are developed rapidly, hyperspectral remote sensing image is still affected by many complex factors during the process of acquisition and transmission, which will introduce a mass of noises. It will affect the target detection, classification and segmentation of hyperspectral image, so it is necessary to study the denoising method in hyperspectral remote sensing image [2]. Currently, denoising methods in hyperspectral remote sensing image are mainly divided into three types: noise reduction in spectral domain, such as minimum noise fraction (MNF) proposed by Green, et al. [3], noise reduction in the spatial domain, such as wavelet shrinkage proposed by Donoho and Johnstone [4], and noise reduction in both the spectral domain and the spatial domain. Among them, the combination of spectral and spatial denoising achieves the better performance. For example, Arkinson, et al. present a denoising method via wavelet transform in spatial domain and Fourier transform in spectral domain [5]. A hybrid of spatial and spectral wavelet shrinkage (HSSNR) method is proposed by Othman and Qian [1].

Considering the strong spectral correlation of the hyperspectral image the proposed method in the paper performs

the MLR in the spectral domain, and then WS based on BayesShrink is performed to the residual image obtained via MLR to remove the spatial correlation. Experimental result shows that the proposed method outperforms HSSNR in terms of SNR.

2. Noise model and the proposed method

2.1. Noise model

The noise model and some parameters are given according to the reference [1]. The noise term is modeled as additive Gaussian white noise, and spatially stationary in each band, but the variance of the noise varies from band to band. That is to say, the level of the noise is dependent on the average amplitude of each band, but spatially stationary in each band.

An important parameter to measure the noise level is signal-to-noise ratio (SNR). Here, the SNR is defined as follows

$$SNR = 10\log_{10}\left(P_X / P_N\right) \tag{1}$$

where P_X is the power of the pure signals, and P_N is the noise power in the noisy signals, that is estimated as

$$SNR = 10\log_{10}\left(\frac{\sum_{i=1,j=1,k=1}^{M,N,B}\left|x_{i,j}^k\right|^2}{\sum_{i=1,j=1,k=1}^{M,N,B}\left|\tilde{x}_{i,j}^k - x_{i,j}^k\right|^2}\right) \tag{2}$$

where $x_{i,j}^k$ stands for the pixel value of the position (i,j) in band k, $\hat{x}_{i,j}^k$ is the estimated value of $x_{i,j}^k$, $i = 1,2,\cdots,M$, $j = 1,2,\cdots,N, k = 1,2,\cdots,L$.

2.2. MLR model

MLR is widely used to remove the spectral correlation for hyperspectral images [6]. We assume that the datacube has $P = M \times N$ pixels in each band and L bands through the spectrum. Let X denote a $P \times L$ matrix of the L spectral observed vectors of size P. In this paper, the $P \times 1$ vector X_i is the i-th column vector of the matrix X. $\hat{X}_i$ is vector predicted for the signal X_i of band i pixel. All bands (not including itself) are utilized to perform MLR. That is,

$$\begin{aligned} \hat{X}_i &= X_{\lambda i}\beta_i \\ \xi_i &= X_i - \hat{X}_i (i = 1,2,\ldots,L) \end{aligned} \tag{3}$$

where the $P \times (L-1)$ matrix $X_{\lambda i}$ is consisted of the all column vector of X (not including the i-th column vector), $X_{\lambda i} = [X_1,\cdots,X_{i-1},X_{i+1},\cdots,X_L]$. β_i is the regression vector of size $(L-1)\times 1$, and ξ_i is the i-th column vector of the residual image, here, $\hat{\xi} = [\hat{\xi}_1,\hat{\xi}_2,\ldots,\hat{\xi}_n]$. For $i = 1,2,\ldots,L$, the least squares estimator of the regression vector β_i is given by

$$\hat{\beta}_i = \left(X_{\lambda i}^T X_{\lambda i}\right)^{-1} X_{\lambda i}^T X_i \tag{4}$$

The noise is extracted from the residual image

$$\hat{\xi}_i = X_i - X_{\lambda i}\hat{\beta}_i \tag{5}$$

2.3. Wavelet Shrinkage

Wavelet transform provides a sparse representation for two dimensional stationary images, which provides a useful tool to remove the spatial correlation. To remove the noise in spatial domain, the noisy signal is transformed into the wavelet

domain, obtaining the wavelet coefficients W, then shrinkage by a threshold function is performed on W, and the coefficients are shrunk as $\tilde{W}$. Lastly an inverse wavelet transform is applied to obtain the denoised image.

As the soft-threshold function is able to keep continuity and smoothness of the signals, and yields visually pleasant images, it is widely applied in image denoising [7]. Therefore, the method applied in this paper is based on the soft-threshold function.

The soft-threshold function of the threshold T is defined as:

$$\eta_T(x) = \mathrm{sgn}(x) \cdot \max(|x| - T, 0) \quad (6)$$

The threshold T is estimated based on BayesShrink, which minimizes the Basyes' risk estimator function assuming a generalized Gaussian prior [8]. The threshold T_{Bayes} is given by

$$T_{Bayes} = \frac{\hat{\sigma}^2}{\hat{\sigma}_x} \quad (7)$$

where $\hat{\sigma}$ and $\hat{\sigma}_x$ are the estimated standard deviations of the noise and the pure signal, respectively, and are given by

$$\hat{\sigma} = \frac{\mathrm{Median}(|d_j|)}{0.6745} \quad (8)$$

$$\hat{\sigma}_x = \sqrt{\max(\hat{\sigma}_y^2 - \hat{\sigma}^2, 0)} \quad (9)$$

where $\{d_j\}$ are the wavelet coefficients at the finest scale and $\hat{\sigma}_y$ is the standard deviation of the noisy signal.

2.4. Proposed Algorithm

The algorithm processes as follows:

- Obtain the residual image $\hat{\xi}$ and the predicted image $\hat{X}$ ($\hat{X} = X - \hat{\xi}$) via MLR utilizing all bands;
- The residual image $\hat{\xi}$ is transformed into the wavelet domain, and then the wavelet coefficients are shrunk by the soft-threshold function. Inverse wavelet transform is performed to obtain the corrected image $\tilde{\xi}$;
- The final denoised image $\tilde{X}$ is obtained from the corrected image $\tilde{\xi}$ and the predicted image $\hat{X}$, that is, $\tilde{X} = \hat{X} + \tilde{\xi}$.

3. Experiment Result

In order to illustrate the superiority of the proposed algorithm in our paper, the simulated experiment is carried out on AVIRIS image, Jasper Ridge, Lunar Lake and Low Altitude provided by JPL, NASA. The size of datacube we extracted from the AVIRIS data for testing is $256 \times 256 \times 224$ (width $\times$ height $\times$ band). The proposed method is compared with the HSSNR method [1].

The compared result shows that the two methods both improve the SNR of hyperspectral image, but the proposed method performs much better.

Table. I Comparison of SNR on AVIRIS image (unit: dB)

	Jasper Ridge	Lunar Lake	Low Altitude
Original SNR	27.7815	27.7815	27.7815
HSSNR	33.7245	34.4997	33.0064
The proposed method	38.4142	40.3703	36.9320

4. Conclusion

A denoising method of remote sensing image based on multiple linear regression and wavelet shrinkage is proposed in this paper. The useful signal is extracted from the residual image by WS. And the features of the data cubes during the denoising process are kept. Experimental result shows that the proposed method improves the quality of hyperspectral remote sensing images significantly in terms of SNR.

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6. References

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