Parameter Estimation of LFM Signal in the Presence of non-Gaussian Noise

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Abstract

In this paper, a method is proposed for chirp rate and initial frequency estimation in the presence of non-Gaussian noise. In this approach, the noise is considered a component of signal phase. After extracting the phase of received signal, we estimate the parameters of interest by using the linear estimators. Since time-frequency simultaneous searches are not done, this approach has low computational complexity. Hence, online and offline implementation are available. Also, simulations confirm the high performance of the proposed method.

Keywords: chirp, parameter estimation, LFM, non-Gaussian noise.

1. Introduction

As it is well known, chirp signals are applied in several research frameworks like communications, physics, sonar, radar. The parameter estimation of these signals has been interested many researchers. For example, Wigner-Ville distribution[1], Wigner-Hough distribution[2], estimation by using the radar ambiguity function[3] are some of methods proposed for estimation of chirp signals. But, all of these methods consider the Gaussian noise.

However, using non-Gaussian models of noise is very important in communication area. Middleton shows that in urban area, the man made noise which is non-Gaussian, affects the transmitted signal[4]. He proposed[5] analytical models for these noises, named

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class A, B and C while among these models, class A model of non-Gaussian noise is very applicable.

Also, other researchers[6] have investigated other non-Gaussian probability density function (pdf), where the most important issue in this context is Electromagnetic Compatibility (EMC). Moreover, these models are applied in several areas like underwater acoustic, radar, remote sensing. One of the main components of non-Gaussian model is sparse high amplitude noise. Car movement, existence of several electrical and electronically systems in our environment, switching circuits, and etc cause these shot noises.

But, in the major amount of previous researches in the Linear Frequency Modulation (LFM) signal detection and parameter estimation, the noise was considered to be Gaussian. However, a few papers considered non-Gaussian noise while there were several disadvantages in these researches. For example, Shamsunder and Giannakis[7] had proposed some bounds for LFM signal estimator in the presence of non-Gaussian noise. But their method was proposed only for estimation of three parameters simultaneously. Also, this approach has high computational complexity since it uses cyclostationarity of received signal. Although they proposed a suboptimum detector with low computational cost, it does not have a good performance. In this paper, we proposed a new method while two above problems will be solved and it can be implemented online.

The remainder of this paper is organized as follows. In section II, the chirp signal estimation by using phase characteristics of received single is reviewed in the presence of Gaussian noise. We will extend this approach to the non-Gaussian channel and estimate the desired parameters in section III. Section IV and V are reserved for simulation and conclusion, respectively.

2. Parameter Estimation by Using Phase Component of the Signal

In this method, the phase of received signal is modeled as a polynomial embedded in noise. Hence, first we can extract the phase and then linear regression model can be applied to parameter estimation. This method is suitable for accurate radar like high performance tracking radar[8].

Here and below, the signal of interest (SOI) is defined as following

$$x_n = A\exp[j(2\pi(\frac{\alpha}{2}n^2 + fn) + \phi)] + \varepsilon_n, \quad n = n_0, n_0+1, \ldots n_0+N-1. \tag{1}$$

where A, α, f, ϕ are magnitude, frequency rate, initial frequency, and phase, respectively. Also ε_n is complex Additive White Gaussian Noise (AWGN).

The received signal can be written as follows[8]

$$x_n = A\exp[j(2\pi(\frac{\alpha}{2}n^2 + fn) + \phi + w_n)], \quad n = n_0, n_0+1, \ldots n_0+N-1. \tag{2}$$

Here, w_n is real AWGN. Needless to say, the phase of the above signal is $x_n = 2\pi\alpha n^2 + 2\pi fn + \phi + w_n$. Fig. 1 is a simple scheme for extracting the phase of received signal. Extracted phase can be applied to estimate frequency rate and initial frequency via linear estimation.

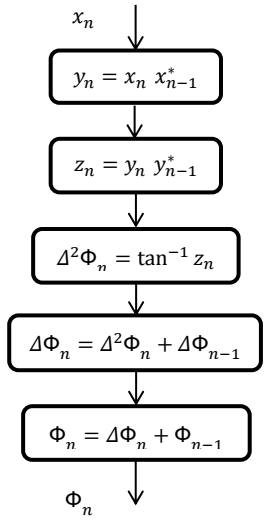

Fig. 1. The extraction process of phase of SOI.

3. Chirp Signal Parameter Estimation in the Presence of non-Gaussian Noise

In this section, we consider the received signal is contaminated with non-Gaussian noise. It will be proved that we are able to extract the phase information from (1) while the noise is non-Gaussian.

First, the received signal must be rewritten as follows

$$x_n = A\exp[j(2\pi(\frac{\alpha}{2}n^2 + fn) + \phi)]\times (1 + \varepsilon_n \exp[-j(2\pi(\frac{\alpha}{2}n^2 + fn) + \phi)]). \tag{3}$$

where, ε_n is complex non-Gaussian noise. Since the magnitude is not known, suppose that $A = \exp(jE_a)$ is a noise multiplied to the signal. If $u_n = \varepsilon_n \exp[-j(2\pi(\frac{\alpha}{2}n^2 + fn) + \phi)]$, we have

$$\underline{u} = \Gamma \underline{e}, \underline{u} = [u_{n0}, u_{n0+1}, ..u_{n0+N-1}]^T. \quad (4)$$

Here $\underline{e} = [\varepsilon_{n0}, \varepsilon_{n0+1}, ...\varepsilon_{n0+N-1}]^T$ and Γ is a diagonal matrix where its elements are as follows

$$\Gamma_{ii} = \exp(-j(2\pi(\frac{\alpha}{2}(n_0 + i - 1)^2 + f(n_0 + i - 1)) + \phi)). \quad (5)$$

As you can see, the noise vector $\underline{e}$ is mapped to a new vector via matrix Γ. Since Γ is diagonal and its elements are deterministic, the vector $\underline{u}$ has non-Gaussian distribution with the covariance matrix which is identical to the covariance of $\underline{e}$. Suppose that $u_n^{(i)}$ is the imaginary part of u_n and E_e is the error of estimation. Hence,

$$1 + u_n = \exp(ju_n^{(i)}) + \exp(jE_e). \quad (6)$$

According to (4)-(7), we have

$$x_n = \exp[j(2\pi(\frac{\alpha}{2}n^2 + fn) + \phi + u_n^{(i)} + E_a + E_e + E_m)]. \quad (7)$$

where E_m is the measurement or sampling noise. After extracting the phase component of the signal via scheme shown in Fig. 1, Φ can be written as following

$$\Phi = 2\pi(\frac{\alpha}{2}n^2 + fn) + E. \quad (8)$$

Here, $E = \phi + u_n^{(i)} + E_a + E_e + E_m$. Since, the total noise of the system is the summation of several unknown and independent noise components, according to the Central Limit Theorem (CLT), the pdf of E is Gaussian. Therefore, although the distribution of $u_n^{(i)}$ is not Gaussian, the parameter estimation of SOI can be done via linear estimation.

4. Simulation

The signal model of (1) is generated with $A = 1, \alpha = 0.1, N = 10, f = 0.5, \phi = 1$ and added to non-Gaussian pdf for 20 different signal to noise ratios, each of which is done for 10000 trials and averaged.

First, we apply Middleton's Class A model of noise defined by $f_n(n) = (1 - eps)N(\mu, \sigma^2) + eps.N(\mu, \kappa\sigma^2)$ where $0 < eps < 1$ and $N(\mu, \sigma^2)$ denotes to the Gaussian pdf with mean μ and variance σ^2. In another simulation, double exponential noise ($f_n(n) = \frac{\lambda}{2}\exp(-\lambda n), \lambda > 0$) was served.

Fig. 2 and Fig. 3 show the error of chirp rate estimation in the presence of Class A and double

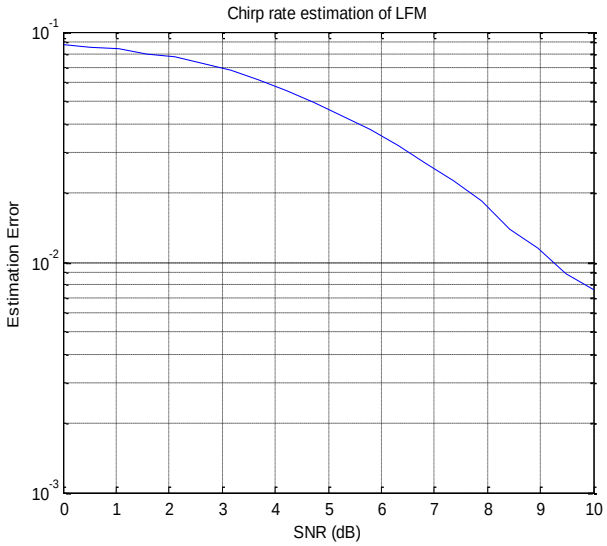

Fig. 2. Chirp rate estimation error in the presence of Middleton's Class A. ($\alpha = 0.1$, $f = 0.2$, $\phi = 0.1$, $\kappa = 1000$, $eps = 0.01$)

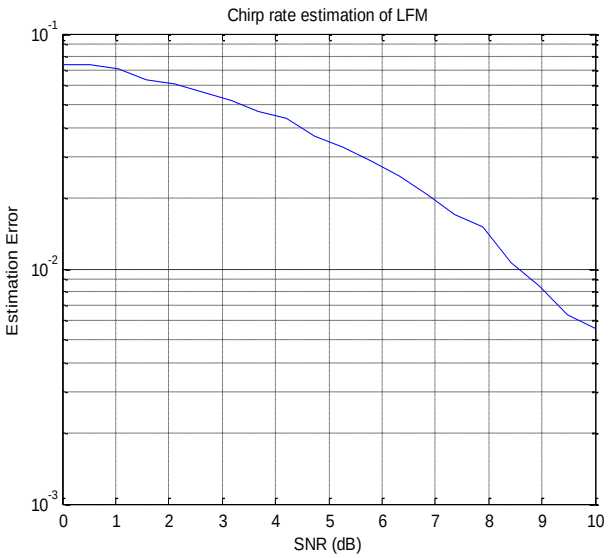

Fig. 3. Chirp rate estimation error in the presence of double exponential noise.

exponential noise, respectively. The estimation of initial frequency is simulated for these two models of non-Gaussian noise and the corresponding performance is depicted in Fig. 4 and Fig. 5. Here, the performance is defined by $10\log(1/(1/M)\sum_{k=1}^{M}(\theta-\hat{\theta}_k)^2)$ where M is the number of trials and $\hat{\theta}_k$ is the parameter of interest.

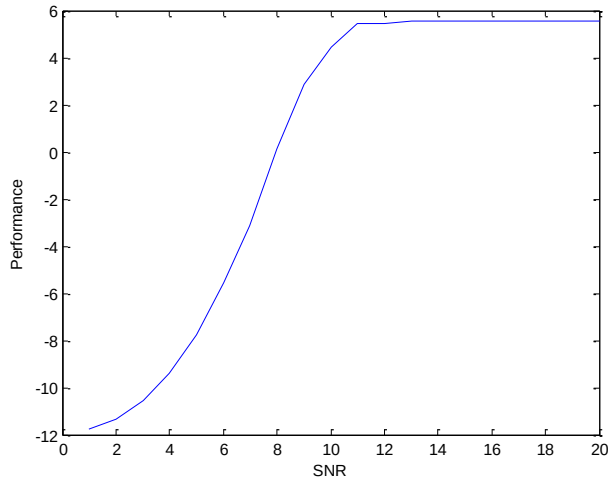

Fig. 4. The performance of initial frequency estimation in the presence of double exponential noise.

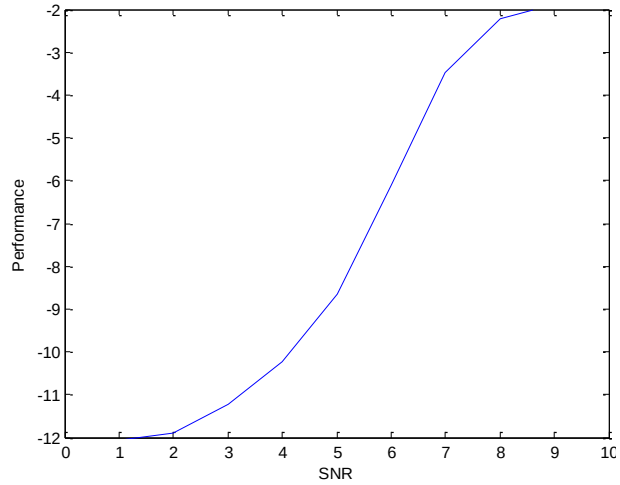

Fig. 5. . The performance of initial frequency estimation in the presence of Middleton's Class A.
$(\alpha = 0.1, f = 0.2, \phi = 0.1, \kappa = 1000, eps = 0.01)$.

5. Conclusion and Future Work

The chirp rate and initial frequency estimation were examined and a method for linear estimation based on phase extraction is proposed. This approach does not need to search time-frequency plane graticule and hence its computational complexity is low enough to implement online. Also, it was shown that this method achieve a good performance.

For future researches in this framework, we want to investigate the analytical differences of this approach for all of the non-Gaussian models of noise, like Middleton's Class A, double exponential noise, Caushy noise and so on.

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