

Avionics System Design of Vertical Take off and Landing UAV

Amit Joshi

Department of Aerospace Engineering, University of Petroleum and Energy Studies, Energy Acres P.O. Bidholi, Via Nanda Ki Chawki, Premnagar, Dehradun
Dehradun, Uttarakhand 248007, India

Sakshi Kukreti

Department of Aerospace Engineering, University of Petroleum and Energy Studies, Energy Acres P.O. Bidholi, Via Nanda Ki Chawki, Premnagar, Dehradun
Dehradun, Uttarakhand 248007, India
E-mail: sakshikukreti18@gmail.com
www.upes.ac.in

Abstract

This paper presents the details of avionics system design of a quadrotor. It uses the direction cosine matrix approach to identify the coordinates of the quadrotor and then using PID controller it tries to minimize the drifting error. The aim of the paper is to develop a system which uses a small amount a processing power and still remains stable. This reduces the cost of the system and allows the users to develop the system further more. PID controller has been implemented in the algorithm as it is the simplest controller widely used in rotor industry.

Keywords: DCM, Quadrotor, Orientation Kinematics, PID, Multirotor

1. Introduction:

The Quadrotor is a rotary type aircraft which uses four horizontally mounted propeller blades to fly. The basic principle behind the flight of a quadrotor is cumulative thrust produced by its four blades. A simple diagram of a quadrotor is shown in figure 1.

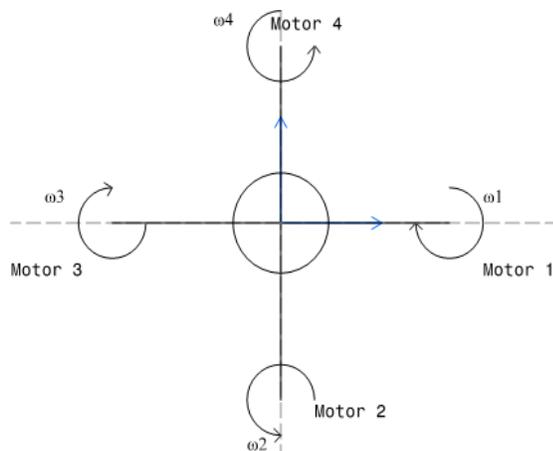


Figure 1: Basic representation of a Quadrotor

In the diagram shown above, ω represents the angular velocity of a motor which can be increased or decrease using electronic speed controllers. The movements of the quadrotor such as roll (θ), pitch (Φ), and yaw (Ψ) can be described as functions of angular velocity ω .

2. How Quadrotors Fly:

Quadrotors fly on the principle of cumulative thrust and torque imbalance. In a quadrotor motor1 and motor3 rotate clockwise, while motor2 and motor4 rotate anticlockwise. This way the torque produced by each pair of motors is cancelled. And the net thrust produced by these four motors lifts the quadrotor up.

Changing the angular velocity of motor 1 and motor 3 results in movement along the Y-axis, whereas the movement in the Z-axis is simply controlled by the combined thrust of all the rotors. To rotate the Quadrotor clockwise about the Z-axis, the angular velocities of motor 1 and motor 3 need to be

decreased while motor 2 and motor 4 need to be increased.

The thrust force from each motor can be described to be related to the size of the rotor blade and the torque caused by the rotor in the air. The flight controller in the quadrotor uses a closed loop control system from which the error value is calculated using PID controller. A quadrotor composes of at least 12 state variables, three degrees of freedom (DOF) in each position. The inputs given to the flight controller consist only of the acceleration and rotation in the three axes. Also the flight dynamics of Quadrotors are typically nonlinear due to the aerodynamics of the vehicle flight as well as from coordinate transforms.

The flight controller in quadrotor uses two control systems; one is used to control rotation of quadrotor in a global frame whereas second one is used for up and down movements. First control system controls the major changes in thrust control whereas the second system compensates the force of gravity as the quadrotor moves in the global frame.

3. Orientation Kinematics

Orientation kinematics calculates the orientation of body relative to a coordinate system. Let us assume our object is attached to a body frame say object frame (Oxyz) and movement, orientation of the object will be calculated or measured relative to a global frame (OXYZ) (Fig. 2). Initially both frames have same origin O.

Let us assume i, j, k are unit vectors in x, y, z direction respectively and I, J, K are unit vectors in X, Y, Z direction. Now I, J, K can be represented with respect to global frame as:

$$I^G = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, J^G = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, K^G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Similarly

$$i^b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, j^b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, k^b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Now writing global coordinates in terms of body coordinates

$$i^G = \{i_x^G \ i_y^G \ i_z^G\}^T$$

$$\text{Or } i^G = \begin{bmatrix} i_x^G \\ i_y^G \\ i_z^G \end{bmatrix}$$

$$\text{Or } i_x^G = |i| \cos(X, i)$$

$$\text{Or } i_x^G = \cos(I, i)$$

Where |i| is length of unit vector and cos(I,i) is the angle between unit vector I and i.

As |I|=|i|=1;

$$i_x^G = \cos(I, i) = |I||i| \cos(I, i) = I \cdot i$$

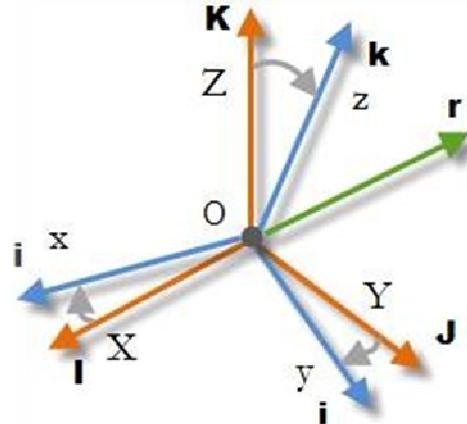


Figure 2: Representation of Frames

It should be noted here that rotation of vector does not modify the angle between vectors as long as both vectors are in same system.

Hence

$$I \cdot i = I^b \cdot i^b = I^G \cdot i^G = \cos(I^G, i^G) = \cos(I^b, i^b)$$

Similarly

$$i_y^G = J \cdot i, i_z^G = K \cdot i$$

Hence in global frame

$$i^G = \begin{bmatrix} I \cdot i \\ J \cdot i \\ K \cdot i \end{bmatrix}, j^G = \begin{bmatrix} I \cdot j \\ J \cdot j \\ K \cdot j \end{bmatrix}, \text{ and } k^G = \begin{bmatrix} I \cdot k \\ J \cdot k \\ K \cdot k \end{bmatrix}$$

Hence a complete set of global coordinates of our object can be represented as

$$[i^G \ j^G \ k^G] = \begin{bmatrix} I.i & I.j & I.k \\ J.i & J.j & J.k \\ K.i & K.j & K.k \end{bmatrix} = \begin{bmatrix} \cos(I,i) & \cos(I,j) & \cos(I,k) \\ \cos(J,i) & \cos(J,j) & \cos(J,k) \\ \cos(K,i) & \cos(K,j) & \cos(K,k) \end{bmatrix} \begin{bmatrix} A_x^b \\ A_y^b \\ A_z^b \end{bmatrix} \cos(I^G, A^G) = |I^b| |A^b| \cos(I^b, A^b)$$

Above matrix gives cosines of angles of all possible combinations of movements of object and global vectors. This matrix is known as Direction Cosine Matrix commonly known as DCM.

Similarly we can get DCM for body frame and it is transverse of DCM of global frame

$$DCM^b = [DCM^G]^T$$

Now we will implement an important property of matrix. If A is a square matrix then

$$A.A^T = 1$$

Using this property

$$DCM^b.DCM^G = [DCM^G]^T.DCM^G = I$$

Where 'I' is Identity Matrix. This proves that DCM matrices are orthogonal to each other.

DCM is used to determine global coordinates of an arbitrary position of object if we know the present coordinates in object frame. Now assuming

$$A^b = \begin{bmatrix} A_x^b \\ A_y^b \\ A_z^b \end{bmatrix} \text{ is an arbitrary vector then its coordinates}$$

in the global frame can be calculated using DCM as follows:

$$\text{Assuming global coordinates of } A^b \text{ are } A^G = \begin{bmatrix} A_x^G \\ A_y^G \\ A_z^G \end{bmatrix}$$

$$\text{Now } A_x^G = |A^G| \cos(I^G, A^G) \text{ (as } A_x^G \text{ is projection of } A^G \text{ onto X axis)}$$

As rotation of a vector does not change its scale and angle between two vectors if both are subjected to same rotation in same system.

Now as $|A^G|=|A^b|$, $|I^G|=|I^b|=1$ and $\cos(I^G, A^G) = \cos(I^b, A^b)$

$$\text{Therefore } A_x^G = |I^b| |A^b| \cos(I^b, A^b)$$

$$A_x^G = I^b.A^b = I^b \cdot \begin{bmatrix} A_x^b \\ A_y^b \\ A_z^b \end{bmatrix}$$

$$\text{As } I^b = \begin{bmatrix} I.i \\ I.j \\ I.k \end{bmatrix}$$

$$\text{Therefore } I^b.A^b = \begin{bmatrix} I.i \\ I.j \\ I.k \end{bmatrix} \cdot \begin{bmatrix} A_x^b \\ A_y^b \\ A_z^b \end{bmatrix}$$

Similarly we can get

$$A_y^G = A_x^b.I.i + A_y^b.I.j + A_z^b.I.k$$

$$A_z^G = A_x^b.J.i + A_y^b.J.j + A_z^b.J.k$$

$$A_z^G = A_x^b.K.i + A_y^b.K.j + A_z^b.K.k$$

Writing above equations in matrix form

$$[A^G] = [DCM^G][A^b]$$

4. CONCLUSION

(a): Above equation shows how we can calculate global coordinates of a object vector if we know direction cosine matrix of that object and coordinates of that object in object frame.

Direction cosine matrix which is provided by IMU (Inertial Measurement Unit) needs to be updated as the quadrotor keeps on changing its position with respect to time. It is done by measuring angular velocity about an axis of rotation.

Let us assume quadrotor is at an arbitrary position A. Vector A is defined as A(t) at time t (Fig.3) . After a small time interval dt it is denoted as $A' = A(t + dt)$

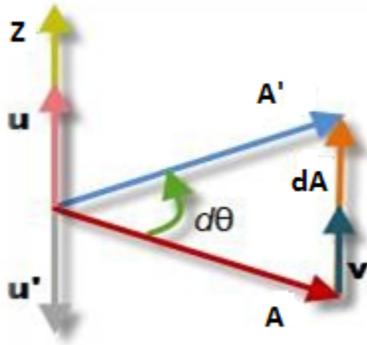


Figure 3: Angular Velocity and Rotation

And $dA = A' - A$

Now let us assume quadrotor was rotating about z axis and u is a unit vector in z direction. Since z is axis of rotation and it is perpendicular to the plane of rotation so u is perpendicular to A and A'.

$$u = \frac{(A \times A')}{|A \times A'|} = \frac{(A \times A')}{|A||A'| \sin d\theta}$$

Or $\omega = \frac{(A \times A')}{|A|^2 \sin d\theta}$ (as rotation doesn't change magnitude hence $|A|=|A'|$)

Now velocity of vector A can be defined as

$$V = \frac{dA}{dt} = \frac{(A' - A)}{dt}$$

As $dt \rightarrow 0, d\theta \rightarrow 0$ therefore angle between A and dA (say ϕ) can be found as $\phi = \frac{(\pi - d\theta)}{2}$

Now as $d\theta \rightarrow 0, \phi \rightarrow \frac{\pi}{2}$

Which implies A is perpendicular to dA when $dt \rightarrow 0$ and A is perpendicular to V since V and dA are co-directional

Now angular velocity is rate of change of angle and A is perpendicular to V since V and dA are co-directional

Now angular velocity is rate of change of angle θ about axis of rotation hence

$$\omega = \left(\frac{d\theta}{dt}\right) u$$

$$\omega = \left(\frac{d\theta}{dt}\right) \times \left(\frac{A \times A'}{|A|^2 \sin d\theta}\right)$$

as $dt \rightarrow 0, d\theta \rightarrow 0$, and for small angles $\sin d\theta \sim d\theta$

$$\omega = \left(\frac{A \times A'}{|A|^2 d\theta}\right) \times \frac{d\theta}{dt}$$

$$\omega = \left(\frac{A \times A'}{|A|^2 dt}\right) = \left(\frac{A \times (A + dA)}{|A|^2 dt}\right) = \frac{A \times A + A \times dA}{|A|^2 dt}$$

$$\omega = \frac{A \times \left(\frac{dA}{dt}\right)}{|A|^2}$$

$$\omega = \frac{A \times V}{|A|^2}$$

Using above equation angular velocity ω can be calculated if linear velocity V, and global coordinates of object are known.

Now assuming Gyroscope gives the raw outputs of angular velocities about x, y and z axis as ω_x, ω_y and ω_z . Now when gyroscope is queried after a short interval of time dt, the gyroscope tells that during this time interval global frame (earth) rotated about gyro's x axis (object frame) by $d\theta_x = \omega_x dt$, about y axis by $d\theta_y = \omega_y dt$, and about z axis by $d\theta_z = \omega_z dt$.

These rotations cause a linear displacement which can be expressed as

$$dA_1 = V_1 dt = (\omega_x \times A) dt$$

$$dA_2 = V_2 dt = (\omega_y \times A) dt$$

$$dA_3 = V_3 dt = (\omega_z \times A) dt$$

Combined displacement will be $dA = dA_1 + dA_2 + dA_3$

And resultant linear velocity can be expressed as

$$V = \frac{dA}{dt}$$

5. Algorithm and Filtering:

Assuming K is Zenith vector in global frame, I denotes North and J is constrained to point west. As obtained above, gyroscope can measure the angular velocity vector at any instance t, and hence linear velocity and displacement can be calculated using above expressions obtained. Using accelerometers we can sense gravitation, thus sensing Zenith vector K. Zenith vector is always pointing opposite to the direction of gravity of earth.

A magnetometer measures the North of Earth hence measuring I^b .

Now we have obtained K^b and I^b using accelerometer and magnetometer respectively which can be expressed as:

$$K^b = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \text{ (Accelerometer outputs)}$$

$$I^b = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \text{ (Magnetometer outputs)}$$

And J^b can be calculated using $J^b = K^b \times I^b$

Which after combining, gives a complete DCM matrix. It is further utilized in calculation of position of the object in 3D. Gyroscope is used to provide to provide a more precise orientation by obtaining a DCM at different time interval thus fine tuning the readings and reducing the error.

Now let DCM of global frame obtained by accelerometer and magnetometer be:

6. Frame Specifications and CAD Drawings:

The frame is made of aluminum alloy which is light in weight and has enough strength to endure the forces produced during the flight. The booms (rods) of the quadrotor are of 25 inch in length and have 0.3*0.75 sq. in. cross section. Two square plates of 0.05 inch thickness and of 4.9 inch length have been used as bottom and top covers to hold booms and other electronics. Below are the CAD drawings and actual pictures of airframe and quadrotor.

$$DCM^G = [i^G \ j^G \ k^G] = \begin{bmatrix} \cos(I,i) & \cos(I,j) & \cos(I,k) \\ \cos(J,i) & \cos(J,j) & \cos(J,k) \\ \cos(K,i) & \cos(K,j) & \cos(K,k) \end{bmatrix}$$

At t_0 time, zenith vector is K_0^b . At this time angular

$$\text{velocity is } \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

After a time interval dt position of zenith vector will be K_{1gyro}^b .

$$K_{1gyro}^b = K_0^b + dtV = K_0^b + dt(\omega \times K_0^b)$$

$$K_{1gyro}^b = K_0^b + (d\theta \times K_0^b)$$

This K_1^b can also be obtained through accelerometer which can be denoted as K_{1acc}^b . In practical situations these values of K_1^b obtained by accelerometer and gyroscope will be different.

Therefore

$$K_1^b \sim K_0^b + (d\theta \times K_0^b)$$

$$I_1^b \sim I_0^b + (d\theta \times I_0^b)$$

$$J_1^b \sim J_0^b + (d\theta \times J_0^b)$$

If we are using a 6 DOF inertial measurement unit which doesn't have a magnetometer then resulting orientation gets a drifting heading (i.e. IMU can't detect whether object is headed north, south, west or east). To reduce this drifting error a virtual north is introduced to the system.



Figure 4 : CAD drawing of prototype model



Figure 5: Photograph of Prototype Model

Principal axes of inertia and principal moments of inertia: (pounds * square inches)
(Taken at the center of mass)

$$\begin{aligned} I_x &= (0.72, 0.00, 0.69) & P_x &= 11.83 \\ I_y &= (0.69, 0.00, -0.72) & P_y &= 12.77 \\ I_z &= (-0.00, 1.00, 0.00) & P_z &= 24.50 \end{aligned}$$

Moments of inertia: (pounds * square inch)
(Taken at the center of mass and aligned with the output coordinate system)

$$\begin{aligned} L_{xx} &= 12.28 & L_{xy} &= 0.00 & L_{xz} &= 0.47 \\ L_{yx} &= 0.00 & L_{yy} &= 24.50 & L_{yz} &= -0.00 \\ L_{zx} &= 0.47 & L_{zy} &= -0.00 & L_{zz} &= 12.32 \end{aligned}$$

Moments of inertia: (pounds * square inch)
(Taken at the output coordinate system)

$$\begin{aligned} I_{xx} &= 12.47 & I_{xy} &= 0.11 & I_{xz} &= 0.10 \\ I_{yx} &= 0.11 & I_{yy} &= 25.49 & I_{yz} &= -0.05 \end{aligned}$$

8. Final Controller Board Pinout Diagram:

Below is the list of components which has been used in our prototype model:

1. **Power Plant:** 4 Odin 2210 brushless motors (1200 rpm/v, 810 gm thrust)
2. **Battery:** Li-Po battery 2200 mAh 20C
3. **Propellers:** 4 propellers 2 CR 2 CCR (9.0*4.5 each)
4. **Electronic Speed Controllers:** 4 TurnigyMultistar ESC's (20 Amp each)
5. **Flight Controller:** ArduinoDueminalove (Atmega 328)
6. **Sensors:** Sparkfun 6DOF (ITG3200/ADXL345)

7. Mass properties of Frame:

Output coordinate System: -- Global --

Density = 0.08 pounds per cubic inch
 Mass = 0.59 pounds
 Volume = 7.04 cubic inches
 Surface area = 289.73 inches²
 Center of mass: (inches)

X = 1.16
 Y = 0.15
 Z = -0.55

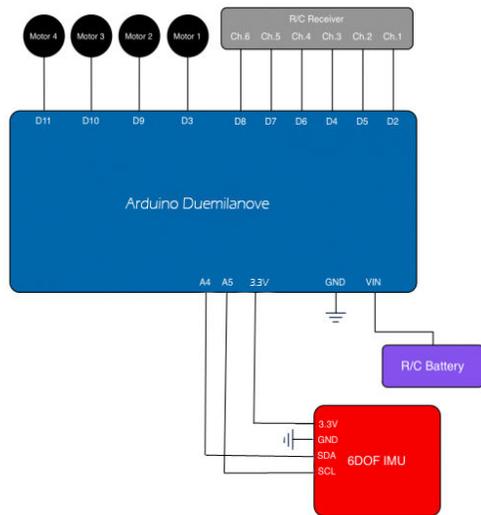


Figure 6: Final Pinout Diagram

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