

## An Application of Optical Flow Calculation Using Runway Image: A Preliminary Study

Raja Munusamy<sup>†</sup>

*Department of Aerospace Engineering, University of Petroleum & Energy Studies,  
Dehradun, Uttarakhand-248007, India<sup>#</sup>*

**Ugur Guven, Sudhir Kumar Chaturvedi, and Pavan Kumar Nanduri**

*Department of Aerospace Engineering, University of Petroleum & Energy Studies,  
Dehradun, Uttarakhand-248007, India*

### Abstract

The main contribution of this work is the determination of the optical flow of the image. It may be defined as the apparent movement of patterns of brightness in a sequence of images. Optical flow is caused by both motion of the observer and by motion of objects in the scene. The Optical flow algorithm uses images separated in time by 2 Seconds. The Focus of Expansion algorithm finds the FOE from the flow field.

*Keywords:* Optical flow method, Runway Target Images, FOE, Flow field.

### 1. Introduction

Traditionally, approximations to image motion have been used to infer information and scene structure. Towards this end, different motion and structure paradigms have been developed, sometimes using optical flow as an intermediate representation of motion, correspondences between image features, correlations or properties of intensity structures. These paradigms are generally classified into three main groups;

**Velocity:** Three dimensional motion and scene structure may be inferred from two dimensional

Velocity Fields, by relating the motion and structure parameters to optical flow. These parameters include instantaneous translation and rotation rates and possibly surface parameters or relative depth.

**Disparity:** Image disparities, either established as image features correspondences or local correlations, may be used to compute three dimensional translation vectors rotation matrices and surface attributes.

**Intensity:** Image intensities and their derivatives are sometimes used directly to obtain motion and structure parameters, thus avoiding an explicit intermediate representation of image motion such as optical flow or disparity fields. (**Beauchemin and Barron, 1995**)

## 2. Introduction to Optical Flow

Optical flow (or optic flow) may be defined as the apparent movement of patterns of brightness (features of a scene) in a sequence of images. Optical flow is caused by both motion of the observer (or camera), and by motion of objects in the scene.

For the purposes of this thesis, since the high-velocity motion of the UAV relative to the world will be much greater than the motion of faraway objects on the ground, it is safe to assume a rigid scene and use a simpler algorithm. (**Matthew R. Parks, 2006**). There are several methods for calculating the optical flow which mainly can be categorized into-

- (1) Intensity Based Differential methods.
- (2) Multiconstraint methods.
- (3) Frequency based methods.
- (4) Correlation based methods.
- (5) Multiple Motion methods.
- (6) Temporal refinement methods.

## 3. Intensity Based Differential Methods

Differential techniques compute image velocity from spatiotemporal derivatives of image intensities. The image domain is therefore assumed to be continuous (or differentiable) in space and time. The initial hypothesis in measuring image motion is that the intensity structures of local time varying image regions are approximately constant under motion for at least a short duration also the intensity of the given pixel changes only because of the motion.. Formally, if  $I(x, y, t)$  am the image intensity function, then

$$I(x, y, t) = I(x + \delta x, y + \delta y, t + \delta t) \quad (1)$$

Where  $\delta x, \delta y$  is the displacement of the local image region in  $(x, t)$  after time  $\delta t$ . Expanding the left hand side of this equation in a Taylor series yields

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t + \text{Higher Order Terms} \quad (2)$$

The higher order terms are small enough to be ignored. Using (1) we can write:

$$\frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = 0 \text{ Thus,}$$

$$I_x u + I_y v = -I_t \quad (3)$$

Where  $I_x$  and  $I_y$  are the spatial intensity gradient and  $v = (u, v)$  is the image velocity. Equation (3) is known as the optical flow constraint equation, and defines a single local constraint on image motion, see Figure .1.

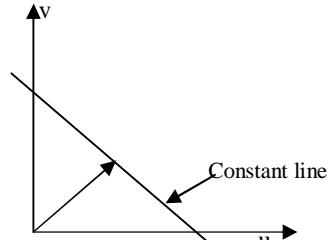


Fig.1. Image Velocity space

The optical flow constraint equation defines a line in velocity space. The normal velocity  $v_{\perp}$ is defined as the vector perpendicular to the constraint line, i.e. the velocity with the smallest magnitude of the optical flow constraint line. However, this constraint is not sufficient to compute both components of  $v$  as the optical flow constraint equation is ill posed. That is to say, only  $v_{\perp}$ ,the motion component in the direction of the local gradient of the image intensity function, may only be estimated. This phenomenon is known as the aperture problem and only at image locations where there is a sufficient intensity structure (or Gaussian curvature) can the motion be fully estimated with the use of the optical flow constraint equation, see Figure 2.

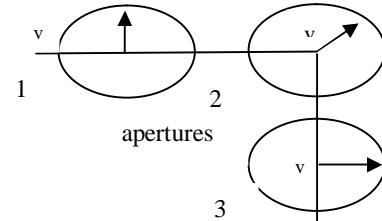


Fig.2. Aperture Problem

Through apertures 1 and 3 only normal motions of the edges forming the square can be estimated due to a lack of local structure. Inside aperture at the corner point the motion can be fully measured since there is sufficient local structure, both normal motions are visible. For example, the velocity of a surface that is homogeneous or containing texture with a single orientation cannot be recovered optically.

There are Global and local 1<sup>st</sup> and 2<sup>nd</sup> order methods based on the “optical flow constraint equation” (3) can be used to compute optical flow. The size of the neighborhood for obtaining a velocity estimate

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determines whether each individual technique is local or global.

### 4.Global Methods:

Global methods use (3) and an additional global constraint, usually a smoothness regularization term, to compute dense optical flows over large image regions.

Horn and Schunck Method alone is described here.

#### 4.1.Horn and Schunck Method

It is called global because it uses (3) and an additional global constraint, usually a smoothness regularization term, to compute dense optical flows over large image regions.

Horn and Schunck proposed to estimate  $(u, v)$  such that following error function,  $E$ , is minimized:

$$E(x, y) = (I_x u + I_y v + I_t)^2 + \lambda (u_x^2 + u_y^2 + v_x^2 + v_y^2), \quad (4)$$

Where the first term is the optical flow constraint from the equation (3), and the second term corresponds to the smoothness of optical flow. For correct optical flow the first term should be close to zero, or the square of the first term should be small. Since the motion of the most real world objects is smooth, the second term enforces the smoothness constraint. Differentiating  $E$  with respect to  $u$  and  $v$  and equating to zero, we get:

$$\frac{\partial E}{\partial u} = (I_x u + I_y v + I_t) I_x + \lambda (u_{xx} + u_{yy}) = 0, \quad (5)$$

$$\frac{\partial E}{\partial v} = (I_x u + I_y v + I_t) I_y + \lambda (v_{xx} + v_{yy}) = 0 \quad (6)$$

We can solve these two equations for  $u$  and  $v$  as:

$$u = u_{av} - I_x \frac{P}{D} \quad (7) \quad v = v_{av} - I_y \frac{P}{D} \quad (8)$$

where  $P = I_x u_{av} + I_y v_{av} + I_t$ ,  $D = \lambda + I_x^2 + I_y^2$ ,  $u_{av} = u - u_{xx} - u_{yy}$  and  $v_{av} = v - v_{xx} - v_{yy}$  (**Berthold K.P. Horn and Brian G. Shunck, 1981**).

The calculations above are iterative and the no. of iterations depends on the error and time. In the proposed system the Horn and Schunck method was found to be working satisfactorily.

#### 4.2Local Methods

Local Methods use normal velocity information in local neighborhoods to perform a least squares minimization to find the best fit for  $v$ .

#### 4.2.1Lukas –Kanade Method

This is one the most popular differential methods for calculation of optical flow. This method is based on the local Taylor series approximations of the image signal, that is: it uses partial derivatives with respect to the spatial and temporal coordinates.

$$\frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t = 0$$

Thus from (3),

$$I_x u + I_y v = - I_t$$

This is an equation in two unknowns and cannot be solved as such. This is known as the aperture problem of the optical flow algorithms. To find the optical flow another set of equations is needed, given by some additional constraint. The solution given by Lukas–Kanade is a non-iterative method which assumes a locally constant flow.

Assuming that the flow  $(u, v)$  is constant in the small window of size  $(m \times m)$  with  $m > 1$ , which is centered around a pixel  $(x, y)$  and numbering the pixels within as  $1..n$ ,  $n=m^2$ , a set of equations can be found

$$\begin{aligned} I_{x1} u + I_{y1} v &= - I_{t1} \\ I_{x2} u + I_{y2} v &= - I_{t2} \end{aligned} \quad (9)$$

$$I_{xn} u + I_{yn} v = - I_{tn}$$

With this there are more than two equations in two unknowns and thus the system is over determined and can be solved by the Least Squares method. (**Bruce D. Lucas, Takeo Kanade, 1981**)

#### 4.2.2Hierarchical Lukas Kanade Method

Traditionally optical flow was computed using only one scale of resolution, usually defined by the visual sensor leading to the problem of measuring large image motions. In this case because of low sampling

rates and aliasing effects (3) becomes inappropriate. A general way of circumventing this problem is to apply optical flow techniques in a hierarchical coarse-to-fine framework. Hierarchical frameworks allow the images to be decomposed in different scales of resolution in the form of Gaussian or Laplacian pyramids. Because of a low frequency representation at coarser resolutions the optical flow constraint equation becomes applicable in the case of large image motions at the finest resolution. In addition to handling fast motions, hierarchical processing also offers increased computational efficiency. In such frameworks, velocity or displacement estimates are cascaded through each resolution level as initial estimates subject to refinement. At the coarsest level, initial estimates are computed and then projected onto a finer level of resolution and refined once again.

## 5. Focus of Expansion

As one moves through the world of static objects, the visual world as projected on the image seems to flow past. In fact, for a given direction of translatory motion and direction of gaze, the world seems to be flowing out of one particular image point, which is called the focus of expansion (FOE). Each direction of motion and gaze induces a unique FOE, which may be a point at infinity if the motion is parallel to the retina (image) plane. (**Danna Ballard Christopher Brown, 1982**).

### 5.1 The Algorithm

Let  $V(x,y) = (v_x(x,y), v_y(x,y))$ ,  $\{0 \leq x \leq N\}, \{0 \leq y \leq N\}$  be the velocity field obtained from a sequence of two consecutive images of size  $N \times N$  using optical flow. These images can be thought of as perceived by an observer approaching a runway that is densely enough textured that there is a displacement vector computable at every position of the image. As mentioned above, the vector field contains only radial components around the focus of expansion

$C = (c_x, c_y) = (\lambda_{\text{foe}}, \mu_{\text{foe}})$ , so

$$V(x, y) = \begin{pmatrix} x - c_x \\ y - c_y \end{pmatrix}$$

The complete image can be divided into four sectors around C:

- Sector 1, "upright": for all vectors  $v_1$  it holds  $v_{1x} > 0$  and  $v_{1y} > 0$

- Sector 2, "up left": for all vectors  $v_2$  it holds  $v_{2x} < 0$  and  $v_{2y} > 0$
- Sector 3, "downright": for all vectors  $v_1$  it holds  $v_{3x} < 0$  and  $v_{3y} < 0$
- Sector 4, "down left": for all vectors  $v_1$  it holds  $v_{4x} > 0$  and  $v_{4y} < 0$

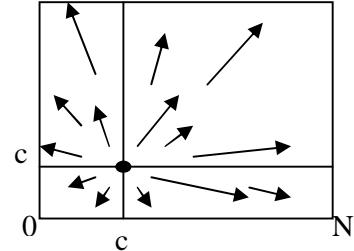


Fig.3. Partition of the vector field in four sectors

Let  $P_{hy}(y)$  be the normalized horizontal projection of the y-components of  $V$ , and similarly  $P_{hx}(y)$ ,  $P_{vy}(x)$  and  $P_{vx}(x)$  the three projections. Then,

$$P_{hy}(y) = \frac{1}{N} \sum_{x'=0}^{N-1} v_y(x', y) \quad (10)$$

$$P_{hx}(y) = \frac{1}{N} \sum_{x'=0}^{N-1} v_x(x', y) \quad (11)$$

$$P_{vy}(x) = \frac{1}{N} \sum_{y'=0}^{N-1} v_y(x, y') \quad (12)$$

$$P_{vx}(x) = \frac{1}{N} \sum_{y'=0}^{N-1} v_x(x, y') \quad (13)$$

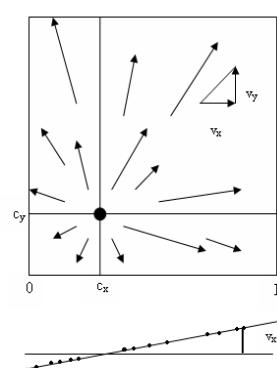


Fig.4. Horizontal and vertical projections

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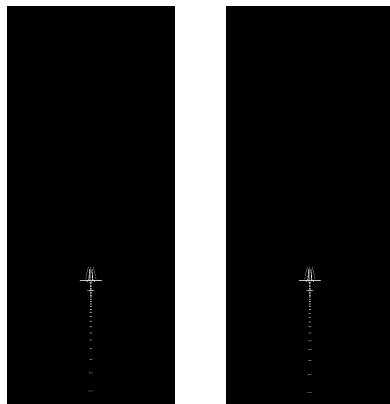


Fig.5. Runway Image

**Table .1 Results**

Method			FOE Location		Attitude Angles			
			512x512	256x256	512x512		256x256	
					Theta	Psi	Theta	Psi
Lucas-Kanade			(512, 242)	(256, 256)	-23.11	1.33	-12.04	-12.04
Hierarchical Lucas-Kanade			(512, 512)	(249, 252)	-23.11	-23.11	-11.4	-11.68
Horn and Schunck	(512, 258)	(256, 253)	-23.11	-0.19	-12.04	-11.77		

After the projection the pixel values having minimum projection magnitude  $P_{hx}$  and  $P_{vy}$  is selected as the locations of FOE  $c_y$  and  $c_x$  respectively.(Christof Born, 1993)

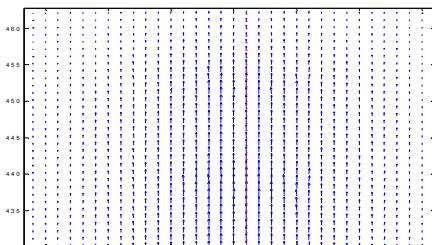


Fig.6.Lucas- Kanade Method

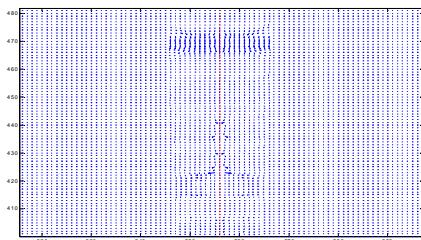


Fig.7.Hierarchical Lucas- Kanade Method

Fig.8. Horn and Schunck Method

## 6. Conclusion

This paper determines the optical flow by using intensity based different methods. The Opticalflow algorithm uses images separated in time by 2 Sec. To find the velocity flow field. The Focus of Expansion algorithm finds the FOE from the flow field. The angle of attack is determined by the separation of FOE from the center of the FOV.

## 7. Future Work

This method is mainly used to integrate the controller. The vision information also integrates with Inertial Measuring Unit (INS) for the future applications.

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