Dynamic Performance Analysis Of Three Phase Induction Motor With Single Phasing

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Abstract

Induction motors are the most widely used among the all available drives. Particularly squirrel cage motor are rugged, cheaper , lighter , smaller, more efficient requiring lower maintenance and can operate in explosive environment. Under normal operating condition machines are used with three phase balance supply but sometimes single phasing due to some fault or due to particular requirement occur which may result in the operation a three phase induction motor on one phase. In this paper we have worked out a method by the help of which not only unbalancing is reduced but a three phase motor can be made to operate on single phase without causing any damage to it and its performance analysis has been carried out.

Keywords: induction motor, single phase, three phase

1. Introduction

In the case of remote hilly areas the cost of bringing a three phase power at the location of operation of the motor is high because of the construction cost of required length of three phase extension. Due to the fault this situation may also arise for short duration even if three phase supply is being made available at the site of operation of the motor.

When a three phase induction motor is operated on single phase supply there will be no starting torque. In order to start and run the motor some auxiliary device is required. The running performance of the unbalanced induction motor (running on single phase) can be improved by using phase converter. The motor can be started by the help of single phase converter under this condition. The best starting performance can be obtained by using single capacitor type phase converter by choosing proper component values.

2. Need For Studying Performance Of Three Phase Induction Motor With Single Phasing

The induction motor is a singly excited electromechanical energy conversion device with salient features such as physical ruggedness, easy & maintenance free operation and low cost. In the areas where there is three phase supply and only single wire earth return supply is available, single phase as well as three phase induction motor can be used.

The standard practice is to use three phase induction motor fitted with capacitor or inductor phase converter[1]. An increasing attention has been paid during the last two decades to the study of the transient and steady state behaviour. The

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steady state study of electrical machine requires the mathematical model relying on various differential equations relating the electromechanical energy conversion process of the machine based on circuit analysis approach which avoids detailed reference to the electromagnetic phenomenon of the machine.

In countries where irrigation pumps form the major part of the agricultural drives, the induction motors are subjected to wide voltage fluctuations in rural and remote areas. Such load and voltage fluctuations are considered to be the main cause of overheating and frequent burning of the drive motor. Equally important feature of the rural electrical supply system is the availability of three phase supply source. Due to non feasibility of supplying three phase balanced supply in remote villages and hilly areas many a times pump motors are often required to operate from single phase supply system. However customers generally prefer using three phase motor with suitable converter even when only single phase is available for reason of economy. Further the same motor can be used if three phase supply is made available. Thus it becomes essential to study the steady state performance of three phase induction motor under various adverse conditions generally encountered in practice.

3. Methods For Study Of Static Performance Of Three Phase Induction Motor

The various methods which are available for the study of static performance of three phase induction motor can be categorized as under [2]:

- **(i)** Simulation of adverse operating conditions at laboratory level and study of static performance of three phase induction motor.
- **(ii)** Simulation of motor and adverse operating conditions on computer and study of its static behaviour.
- **(iii)** Induction motor simulation under adverse operating conditions on a digital computer and experimental verification of its performance in lab.

The first method has the disadvantage of being time consuming and it may also result in deterioration of motor insulation. In the case of second method accuracy largely depends upon how accurately the simulated conditions are able to simulate the actual conditions which we encounter in practice.

In this paper we have gone for the third method where first a mathematical model of the induction motor was developed and then its simulation was carried out under varying operating conditions using C programming and the results obtained were verified experimentally in the lab.

In this paper an attempt has been made to obtain the steady state analysis of three phase induction motor under the following conditions of supply:

- **(i)** Balanced three phase supply
- **(ii)** Sudden single phasing in steady state condition of motor
- **(iii)** Operation of three phase induction motor by single phase supply with the help of capacitor phase converter.

The first case is normal operating situation of three phase induction motor while the second case may occur with fair probability. The third case may be practiced in the remote and rural areas where three phase supply lines are not available. An attempt has been made in this paper to optimize the capacitor used as phase converter. The optimized value of the capacitor for various speeds was determined by using a computer programme written in C language.

4. Mathematical Model Of Induction Motor

For single phase operation first the suitable value of capacitor has to be predicted which requires the equivalent circuit parameters of the induction motor. The equivalent circuit parameters were obtained by the help of blocked rotor test, no load test and stator dc resistance test.

A mathematical model to study the dynamic performance of three phase induction motor has been developed. Induction motor volt – ampere equations were developed to study dynamic and transient performance. Finally equivalent circuits are developed for three cases of operation i.e. balanced three – phase, single phasing and operation with single phase supply using a capacitor phase converter.

The three phase induction motor having winding sa, sb, sc and rotor winding ra, rb, rc. The ra winding is at an electrical angle θ with respect to sa winding at time t. The voltage equations can be written in matrix form as [3]:-

$$[V]=[R][i]+[\alpha]\,p\,[i]+[G][i]+[G][i]\,p\theta \quad (1)$$

Where

$$[V] = [V_{sa}\ V_{sb}\ V_{sc}\ V_{ra}\ V_{rb}\ V_{rc}]^T$$
$$[i] = [i_{sa}\ i_{sb}\ i_{sc}\ i_{ra}\ i_{rb}\ i_{rc}]^T$$
$$[G] = d/d\theta\ [\alpha] \quad and \quad p = d/dt$$

The developed electromagnetic torque is

$$T = 1/2\,[P/2][i]^T\,[G][i] \qquad (2)$$

Where P is the number of poles of the machine

4.1 Operational equivalent circuit

The deviation of the general motor performance equations using instantaneous symmetrical component leads to operational positive and negative circuits which involve operational sequence impedances such that:

$$V_S^+ = Z_S^+ * I_S^+$$
$$V_S^- = Z_S^- * I_S^-$$

The original volt ampere equation can therefore be transformed to obtain the equations with respect to $V_{sa} V_{sb} V_{sc} V_{ra} V_{rb} V_{rc}$ by using d-q transformation. The resulting performance equations in terms of sequence quantities are

$$V_S^+ = (R_S + X_{1S}\,p)\,i_S^+ + X_m\,p(i_S^+ + i_r^+)$$
$$O = V_r^+ / (1 - j\,v/p) = (R_r/(1 - j\,v/p) + X_{1r}\,p)ir^+ + X_m\,p(i_s^+ + r_r^+) \qquad (3)$$

$$V_S^- = (R_S + X_{1S}\,p)\,i_S^- + X_m\,p(i_S^- + i_r^-)$$
$$O = V_r^- / (1 + j\,v/p) = (R_r/(1 + j\,v/p) + X_{1r}\,p)ir^- + X_m\,p(i_s^- + r_r^-) \qquad (4)$$

where

R_S, X_{1S} are stator resistance and leakage reactance.

R_r, X_{1r} are rotor resistance and leakage reactance to stator.

X_m is magnetizing reactance, all on per phase basis

v is the per unit speed

p is the per unit differential operator (d/dt).

Equation (3) corresponds to the positive sequence operational equivalent circuit as shown in fig 2. The operational positive sequence impedance Z_S^+ is a function of machine parameters, p.u. speed and operator p. Similarly equation (4) indicates the negative sequence operational equivalent circuit which is obtained directly by replacing j by –j in fig.2. Z_S^- is the complex conjugate of Z_S^+.

The developed electromagnetic torque in terms of sequence quantities may be expressed as:-

$$T_e = jX_m / w_r\,[i_r^+ - i_S^- - i_r^- - i_S^+]$$

where w_r is the mechanical synchronous speed of machine in rad/sec.

The instantaneous symmetrical component theory when coupled with counter rotating field theory extends the use of equivalent circuit concept for any type of unbalance and unsymmetrical operation. Therefore the equations are written in terms of their real and imaginary parts. For digital simulation they are dealt as real variables. Let subscript x and y denote the real and imaginary parts of the instantaneous symmetric voltage and current respectively.

$$\left.\begin{array}{lll} V_S^+ = V_{SX} + jV_{SY} & & i_S^+ = i_{SX} + ji_{SY} \\ V_S^- = V_{SX} - jV_{SY} & \& & i_S^- = i_{SX} - ji_{SY} \\ V_r^+ = V_{rX} + jV_{rY} & & i_r^+ = i_{rX} + ji_{rY} \\ V_r^- = V_{rX} + jV_{rY} & & i_r^- = i_{rX} + ji_{rY} \end{array}\right\} \quad (5)$$

The negative sequence component is the conjugate of positive sequence component. Hence positive sequence component is sufficient for the dynamic analysis of the induction motor. The instantaneous symmetrical component mathematical model of induction motor as developed is arranged for different operating condition. i.e. balanced and unbalanced.

Case I: With normal three phase supply voltage

The voltage and torque equations of the induction motor with normal three phase supply which is assumed to be balanced are given by

$$V_{SX} = R_S\,i_{SX} + X_S\,d/dt\,(i_{SX}) + X_m\,d/dt\,(i_{rX})$$
$$V_{SY} = R_S\,i_{SY} + X_S\,d/dt\,(i_{SY}) + X_m\,d/dt\,(i_{rY})$$
$$V_{rX} = R_r\,i_{rX} + X_r\,d/dt\,(i_{rX}) + X_m\,d/dt\,(i_{SX}) + V\,(X_m\,i_{SY} + X_r\,i_{rY})$$
$$V_{rY} = R_r\,i_{rY} + X_r\,d/dt\,(i_{rY}) + X_m\,d/dt\,(i_{SY}) + V\,(X_m\,i_{SX} + X_r\,i_{rX})$$

where

$X_S = X_m + X_{1S}$

$X_r = X_m + X_{1r}$

$T_{e1} = 2\,X_m\,(i_{rX}\,i_{SY} - i_{SX}\,i_{RY})$ (6)

The actual winding current can be computed from the symmetrical components as given by equations below:

$$i_{sa} = 2/\sqrt{3}i_{SX}$$
$$i_{sb} = 1/\sqrt{3}(-I_{SX} + \sqrt{3}i_{SY}) \qquad (7)$$
$$i_{sc} = 1/\sqrt{3}(-I_{SX} + \sqrt{3}i_{SY})$$

Case II: Sudden single phasing

The disconnection of one supply line from a three phase source during operation of three phase induction motor is known as single phasing. The common causes of single phasing are burning of fuse wire in any one phase under imbalances, faulty contact of the star delta or auto transformer starters. Single phasing may even may even stall a motor in full load condition and it is one of the major cause of burning of stator winding. In this case let us take the phase winding sc is disconnected from the supply during the operation of three phase induction motor when it has achieved the rated speed in steady state. The current and voltage relation in this case are given by

Vab = Vsa – Vsb ; isc = - isa ; isc = 0

$$V_{sa} = 1/\sqrt{3}\left(V_S^+ + V_S^-\right)$$

$$V_{sb} = 1/\sqrt{3}\left(V_S^+ + V_S^-\right)$$

$$i_S^+ = 1/\sqrt{3}(1-a)i_{sa}$$

$$i_S^- = 1/\sqrt{3}\left(1-a^2\right)i_{sa}$$

$$V_{ab} = \left[Z_S^+ + Z_S^-\right]i^-$$

The above have been derived from the equivalent circuit shown in fig. 6 where i_{r1}^+ and i_{r1}^- are fictitious current in positive and negative sequence rotor loop. They are related to the current i_{sa} in the stator. The current i_{r1}^+ and i_{r1}^- are written as:-

$$i_{r1}^+ = i_{r1x} + \text{j}\, i_{r1y} \;\&\; i_{r1}^- = i_{r1x} - \text{j}\, i_{r1} \qquad (7)$$

From fig 6 the performance equations can be written as

$$V_{ab} = 2R_S i_{sa} + 2X_S p i_{sa} + X_m p\left(i_{r1}^+ + i_{r1}^-\right)$$

$$R_r/(1 - jv/p)i_{r1}^+ + X_r p i_{r1}^+ + X_m p i_{sa} = 0 \quad (8)$$

$$R_r/(1 + jv/p)i_{r1}^- + X_r p i_{r1}^- + X_m p i_{sa} = 0$$

The equation (7) gets modified as

$$V_{ab} = 2R_S i_{sa} + 2X_S p i_{sa} + \sqrt{3}X_m p i_{rx} - X_m p i_{ry}$$

For rotor loop equations involving sequence components considering only real part, the equation of the first loop and of the second loop is given by equation (9) and (10) respectively

$$\sqrt{3}/2\, R_x i_{rx} - 1/2\, R_r i_{ry} + \sqrt{3}/2\, X_r p i_{rx} - 1/2\, X_r p i_{ry} + X_m p i_{sa} i_{sa} + v_{rx}\left[1/2\, i_{rx} + \sqrt{3}/2\, i_{ry}\right] = 0 \qquad (9)$$

$$1/2\, R_r i_{rx} + \sqrt{3}/2 R_r i_{rx} + 1/2\, X_r p i_{rx} - \sqrt{3}/2\, X_r p i_{ry} - \sqrt{3}/2 vXp_r i_{rx} + 1/2\, vX_r i_{ry} - vX_m i_{sa} = 0 \qquad (10)$$

The expression for the electromagnetic torque is given by

$$T_{e2} = X_m i_{sa}\left(i_{rx} + \sqrt{3}i_{ty}\right) \qquad (11)$$

The dependent variables of the system under this case are taken as i_{as}, i_{rs}, i_{ry}, and v (p.u. speed)

Case III: Operation of three phase induction motor with single phase supply using capacitor phase converter

The overall rating of three phase motor operating with single phase supply with proper choice of starting and running capacitors as phase converter be as high as 70% of balanced rating. Therefore it is now important to modify the mathematical model developed in case I under this adverse condition of supply. A capacitor phase converter used with single phase supply to carry out the operation of three phase induction motor either it is delta connected or star connected as shown in fig. 7(a) and 7(b). In this situation the induction motor is operated with asymmetrical connections.

The volt ampere equation in terms of positive and negative sequence impedance is given for delta connected induction motor are:

$$V = \frac{1}{\sqrt{3}}\begin{bmatrix}(R_S + X_S p)i_S^+ + X_m p i_r^+ + \\ (R_S + X_S p)i_S^- + X_m p i_r^-\end{bmatrix} \quad (12)$$

$$\frac{1}{\sqrt{3}}\begin{bmatrix} a\left\{(R_S + X_S p)i_S^+ + X_m p i_r^+\right\} + \\ a^2\left\{(R_S + X_S p)i_S^- + X_m p i_r^-\right\} \\ + jX_C/p\left(i_S^+ - i_S^-\right)\end{bmatrix} = 0 \quad (13)$$

On substituting from equation (5) in equation (12) and taking the real part the performance equation becomes

$$V = 2/\sqrt{3}R_S i_{sx} + 2/\sqrt{3}X_S p i_{sx} = 2/\sqrt{3}X_m p i_{rx} \quad (13)$$

$$\frac{1}{\sqrt{3}}R_S i_{sx} + R_S i_{sy} + \frac{1}{\sqrt{3}}X_S p i_{sx} + X_S p i_{sy} + \frac{1}{\sqrt{3}}X_m p i_{rx} + X_m p i_{ry} + 2\frac{X_C}{p}i_{sy} = 0 \qquad (14)$$

The developed torque in per unit is expressed as

$$Te_3 = 2X_m\left(i_{rx} i_{sy} - i_{sx} i_{ry}\right) \qquad (15)$$

The voltage across the capacitor during transient period must be acknowledged for proper selection of capacitor to be used as phase converter. The voltage across the capacitor is given by

$$V_C = -2X_C q_{sy} \text{ where } q_{sy} = {i_{sy}}/{p}$$

5. Optimization Of The Capacitor

When an unbalanced supply voltage is considered the evaluation of machine behaviour becomes more complex due to emergence of negative sequence component of stator current. The positive sequence current produces a constant amplitude traveling wave rotating at synchronous speed in forward direction and is responsible for the effective driving torque. The presence of negative sequence current component in stator and rotor winding is also undesirable as it leads to a noticeable increase in ohmic loss, reduces efficiency and produces more noise & vibration resulting in derating of motor.

The value of the capacitance required for proper operation of three phase induction motor on single phase was derived by minimizing the unbalance

$U = |V_2/V_1|$ which has been considered as an unbalanced factor where V_1 and V_2 are positive and negative sequence voltages.

For minimum unbalance in case of delta connected machine we have

$$V_1 = V\left(a^2 - \sqrt{3}X_C Y_2\right) / \left(-\sqrt{3}j - \sqrt{3}\right) X_C \left(Y_1 + Y_2\right)$$

$$V_2 = -V\left(a - \sqrt{3}X_C Y_2\right) / \left(-\sqrt{3}j - \sqrt{3}\right) X_C \left(Y_1 + Y_2\right)$$

$$U = \left[\left(1 + 3X_c^2 Y_1^2 - X_c Y_1 K_1\right) / \left(1 + 3X_c^2 Y_2^2 + X_c Y_2 K_2\right)\right]^{1/2}$$

where

$$K_1 = \sqrt{3}\cos(\phi_1) - 3\sin(\phi_1)$$

$$K_2 = \sqrt{3}\cos(\phi_2) + 3\sin(\phi_2)$$

and φ_1 and φ_2 are impedance angles of positive and negative sequence impedances. Y_1 and Y_2 are positive and negative sequence admittances.

For minimum unbalance

$dU/dX_C = 0$ after differentiating gives

$$C_1 X_C^2 + C_2 X_C + C_3 = 0$$

$$\left.\begin{aligned} \text{where}\quad & C_1 = 3Y_1Y_2\left(Y_1K_2 + Y_2K_1\right) \\ & C_2 = 6\left(Y_1^2 - Y_2^2\right) \\ \& \; & C_3 = -\left(Y_1K_1 + Y_2K_2\right) \end{aligned}\right\} \qquad (16)$$

The above equations were solved using computer programming. The value of X_C which minimized U has been computed for all rotor speeds.

6. Result Analysis of Performance of Three Phase Induction Motor on Single Phase

Delta Connected motor was used for analyzing its performance in the laboratory and the following parameters on per phase basis were obtained from no–load test, blocked rotor test and d.c. resistance test:

Effective stator resistance r_s =3.44 ohm

Stator leakage reactance X_{1S} =10.473 ohm

Rotor resistance (referred to stator)

r_r =6.673 ohm

Rotor leakage reactance (referred to stator)

X_{1r} = 10.473 ohm

Magnetizing reactance X_m = 173.834 ohm

Table 1. Details regarding all parameters calculated from mathematical model derived when the machine is operating at three phase normal supply.

TABLE 1 For three Phase operation

Torque = tension * 9.81 * radius of pulley (Nm)

O/P power = Torque * 2 * 3.14 * N / 60 (W)

S.No.	Voltage (Volt)	Current (Amp)	Speed (rpm)	I/P (W)	Tension (kg)	Torque (Nm)	O/P (W)	Efficiency (%)	Power Factor
1	415	3.9	1465	160					
2	415	4.9	1450	1980	12.0	11.1834	1697.26	85.72	0.654
3	415	5.1	1440	2100	13.5	12.5813	1896.25	90.29	0.659
4	415	5.4	1440	2400	15.3	14.2588	2149.09	89.54	0.666
5	415	6.1	1430	3040	18.0	16.7751	2510.79	82.59	0.668
6	415	6.6	1430	3600	19.8	18.4526	2761.86	76.71	0.686
7	415	7.2	1427	4160	23.5	21.9008	3271.09	78.63	0.6676
8	415	7.6	1410	4480	25.0	23.2987	3438.42	76.75	0.6692
9	415	8	1410	4720	25.5	23.7647	3507.19	74.31	0.6752
10	415	8.5	1410	4820	27.2	25.349	3741.01	77.61	0.6724

Table 2. Details regarding all parameters calculated from mathematical model derived when the same machine is operating on single phase using capacitor converter.

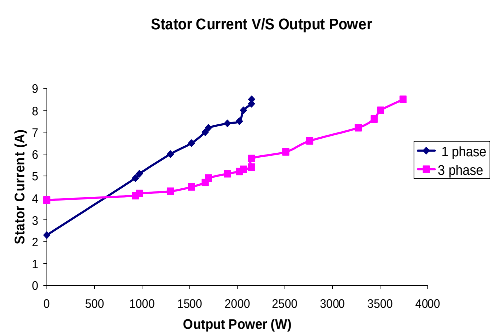
TABLE 1 (for single phase operation)

Torque = tension * 9.81 * radius of pulley (Nm)

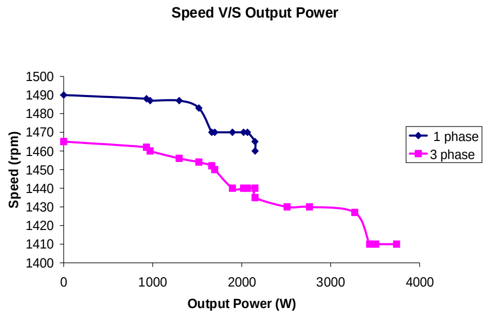
O/P power = Torque * 2 * 3.14 * N / 60 (W)

S.No.	Voltage (Volt)	Current (Amp)	Speed (rpm)	I/P (W)	Tension (kg)	Torque (Nm)	O/P (W)	Efficiency (%)	Power Factor
1	415	2.3	1490	600					
2	415	4.9	1488	1700	6.4	5.9644	930.18	54.71	0.7040
3	415	5.1	1487	1840	6.7	6.2440	971.82	52.81	0.7382
4	415	6.0	1487	2280	8.95	8.3409	1298.17	56.93	0.7036
5	415	6.5	1483	2480	10.5	9.7854	1518.89	61.24	0.6892
6	415	7.0	1470	2720	11.6	10.8106	1663.31	61.15	0.6946
7	415	7.5	1470	2960	14.1	13.1404	2021.78	68.30	0.6840
8	415	8.0	1470	3040	14.4	13.4200	2064.80	67.92	0.6873
9	415	8.5	1460	3040	15.1	14.0724	2150.45	70.73	0.6924

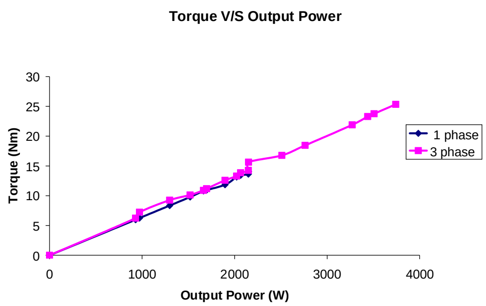
Graph No. 1 Stator current v/s O/P power



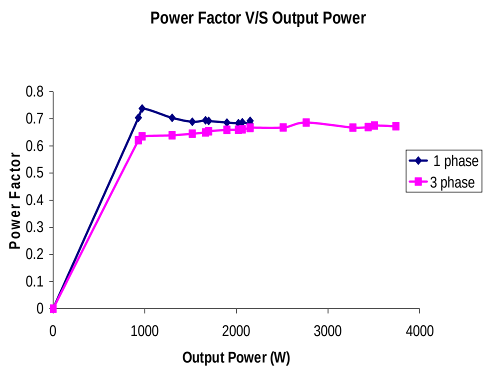
Graph No. 2 Speed v/s O/P Power



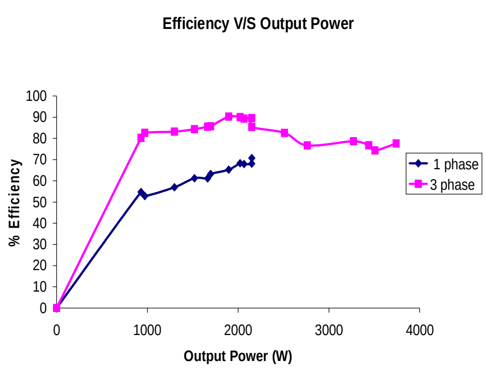
Graph No. 3 Torque v/s O/P power



Graph No. 4 Power factor v/s O/P power



Graph No. 5 Efficiency v/s O/P power



Conclusion: The change in power output of 3 phase motor from no load to full load is 56.7% when operated on single phase and 54.7% on three phase. Similarly change in speed when operated on single phase supply is 2% and 3.75% when operated on three phase supply. Similarly the change in efficiency from no load to full load when operating on single phase is 2.26% whereas on three phase supply is 14.6%. However the efficiency of motor when operating on single phase reduces by just 7% as compared to operation on three phase for maximum load. The lowest value of p.f. obtained when operating on single phase supply was 0.6924 whereas on three phase supply the highest p.f. obtained was 0.686

We conclude that when a three phase induction motor is made to operate on single phase supply then the motor runs at constant speed with comparable efficiency at better power factor.

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