Design of PD Observers For SPROTT Chaotic Systems

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Abstract

This paper presents a scheme of designing proportional-derivative (PD) observer for chaotic systems. The proposed design is based on contraction theory approach. For this purpose, co-ordinate transformation of variables are used to avoid derivatives of output in observer. Results are numerically verified for Sprott chaotic system. Results show the effectiveness of proposed strategy.

Keywords: Contraction theory, PD observer, Exponential convergence, Sprott chaotic system.

1. Introduction

Study of chaotic system is the one area which have been explored by many researchers in recent years.[1-5] There is a vast application of chaos synchronization in different fields. Sprott found 19 algebraically simple chaotic systems by exhaustive computer searching.[6] One of the chaotic system among them is used here.

Recently contraction theory based results are explored to design observers by many researchers.[7-9] Designing of proportional-derivative (PD) observer for Sprott chaotic system is based on the contraction theory. Contraction theory is a tool used to design and analyze the nonlinear system trajectories with respect to one another, and belongs to the class of incremental stability methods.[10-12] As it is known that incremental stability is better form of stability than stability with respect to origin. It handles synchronization problems effectively. Co-ordinate transformation are used in the observer design. Since the derivative of output is not available for the measurement. So it helps to avoid the derivatives of output in the designed system dynamics.

This paper is organized as follows: In section-1, preliminaries to contraction theory and PD observer are presented. In section-2, problem is formulated for Sprott chaotic system. In section-3, simulation results are presented. Finally, in section-4, some conclusion is presented based on the results.

2. Theoretical Preliminaries

Consider a nonlinear system

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x},t). \tag{1}$$

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where $\mathbf{x} \in \Re^m$ and $\mathbf{f}(\mathbf{x},t)$ is continuous differentiable function such that, $\mathbf{f}:(m\times 1)$ vector function. The main definition and lemma of contraction are taken from[13] and represented here same for the clarity purpose.

Definition 1. A region of the state space is called a contraction region with respect to a uniformly positive definite metric $M(\mathbf{x},t)=\Theta^T(\mathbf{x},t)\Theta(\mathbf{x},t)$ where Θ stands for a differential coordinate transformation matrix, if equivalently

$$\mathbf{F}=\left(\dot{\Theta}+\Theta\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Theta^{-1} \tag{2}$$

is uniformly negative definite (UND). This leads to the following convergence result.

Lemma 1. *For system in "Eq. (1)", system trajectory for any initial condition which starts in a ball of constant radius centered about a given trajectory and contained at all a time in a contraction region, remains in that ball and converges exponentially to given trajectory.*

Global exponential convergence to given trajectory is guaranteed if whole state space region is contracting.

2.1 *Proportional-derivative observer*

Co-ordinate transformation are used to simplify the observer design problem based on contraction analysis. Consider a system

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x},\mathbf{t}). \tag{3}$$

and output is given by

$$\mathbf{y}=\mathbf{h}(\mathbf{x},\mathbf{t}). \tag{4}$$

where $\mathbf{x}$ is the state vector and $\mathbf{y}$ is the measurement vector. Now defining the PD observer for the above system

$$\left.\begin{aligned}\dot{\bar{\mathbf{x}}}&=\mathbf{f}(\hat{\mathbf{x}},\mathbf{t})-\mathbf{k}_P(\hat{\mathbf{y}}-\mathbf{y})-\mathbf{k}_D\dot{\hat{\mathbf{y}}}\\ \hat{\mathbf{x}}&=\bar{\mathbf{x}}+\mathbf{k}_D\mathbf{y}\end{aligned}\right\} \tag{5}$$

where $\hat{\mathbf{y}}=\mathbf{h}(\hat{\mathbf{x}},\mathbf{t})$ and $\dot{\hat{\mathbf{y}}}=\frac{\partial \hat{\mathbf{y}}}{\partial \hat{\mathbf{x}}}\mathbf{f}(\hat{\mathbf{x}},\mathbf{t})+\frac{\partial \mathbf{h}}{\partial t}$

Differentiation of "Eq. (5)", leads to the following observer dynamics

$$\dot{\hat{\mathbf{x}}}=\mathbf{f}(\hat{\mathbf{x}},\mathbf{t})-\mathbf{k}_P(\hat{\mathbf{y}}-\mathbf{y})-\mathbf{k}_D(\dot{\hat{\mathbf{y}}}-\dot{\mathbf{y}}) \tag{6}$$

Since "Eq. (6)" contain $\dot{\mathbf{y}}$ and $\dot{\mathbf{y}}$ is generally not available in the output. So "Eq. (5)" is used to obtain $\hat{\mathbf{x}}$ and $\dot{\mathbf{y}}$ is not explicitly used.

3. Problem Formulation

Consider a Sprott Chaotic system

$$\left.\begin{aligned}\dot{x}_1&=2x_3\\ \dot{x}_2&=-2x_2+x_3\\ \dot{x}_3&=-x_1+x_2+x_2^{\,2}\end{aligned}\right\} \tag{7}$$

and the output is given by

$$\mathbf{y}=\mathbf{cx}=[1\ 0\ 0]\mathbf{x}. \tag{8}$$

where $\mathbf{x}$ is the state vector and $\mathbf{y}$ is the measurement vector.

The open loop response of the system with simulation period of 100 seconds and initial conditions, $[2\ \ 1\ \ 3]^T$ is shown in Fig.1. Three dimensional phase portrait is shown in Fig.1 (a). The open loop response of the system to initial condition is shown in Figs.1 (b)-(d).

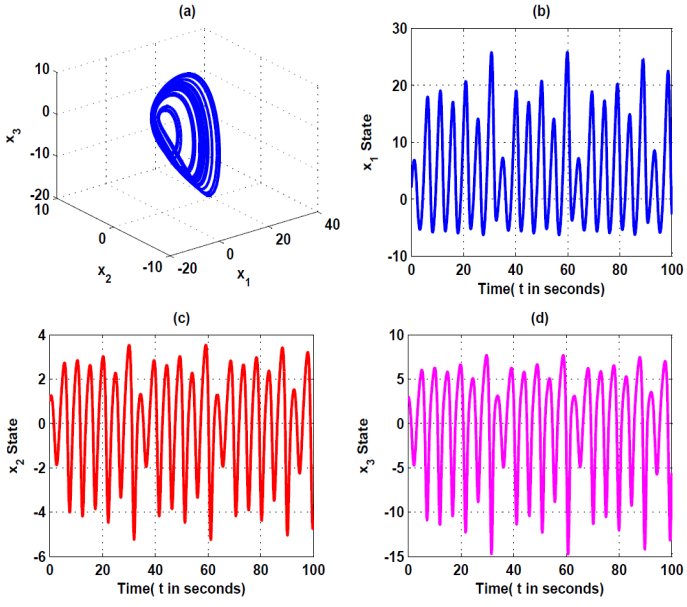

Fig. 1. System without controller: (a) Phase portrait in three dimensions and (b) – (d) time variation of state Trajectories

3.1 *Proportional-derivative observer for sprott chaotic system*

Now defining the PD observer for the above system dynamics,

$$\left.\begin{aligned}\dot{\hat{x}}_1 &= 2\hat{x}_3 - k_{P1}(\hat{x}_1 - x_1) - 2k_{D1}\hat{x}_3 \\ \dot{\hat{x}}_2 &= -2\hat{x}_2 + \hat{x}_3 - k_{P2}(\hat{x}_1 - x_1) - 2k_{D2}\hat{x}_3 \\ \dot{\hat{x}}_3 &= -\hat{x}_1 + \hat{x}_2 + \hat{x}_2{}^2 - k_{P3}(\hat{x}_1 - x_1) - 2k_{D3}\hat{x}_3\end{aligned}\right\} \quad (9)$$

and output is given by

$$\hat{\mathbf{y}} = \mathbf{c}\hat{\mathbf{x}} = [1\ 0\ 0]\hat{\mathbf{x}}. \quad (10)$$

Now using co-ordinate transformation

$$\left.\begin{aligned}\hat{x}_1 &= \bar{x}_1 + k_{D1}x_1 \\ \hat{x}_2 &= \bar{x}_2 + k_{D2}x_1 \\ \hat{x}_3 &= \bar{x}_3 + k_{D3}x_1\end{aligned}\right\} \quad (11)$$

where $\hat{\mathbf{x}}$ represents the observer states and k_{Pi} & k_{Di}, i=1,2,3 represents PD observer gains.

Now computing the virtual displacement

$$\begin{bmatrix}\delta\dot{\bar{x}}_1 \\ \delta\dot{\bar{x}}_2 \\ \delta\dot{\bar{x}}_3\end{bmatrix} = \begin{bmatrix}-k_{P1} & 0 & 2-2k_{D1} \\ -k_{P2} & -2 & 1-2k_{D2} \\ -1-k_{P3} & 1+2\bar{x}_2+2k_{D2}x_1 & -2k_{D3}\end{bmatrix}\begin{bmatrix}\delta x_1 \\ \delta x_2 \\ \delta x_3\end{bmatrix}$$

The symmetric part of the Jacobian is $\mathbf{J}_s = \frac{1}{2}\left(\mathbf{J} + \mathbf{J}^T\right)$

$$\mathbf{J}_s = \begin{bmatrix}-k_{P1} & \frac{(-k_{P2})}{2} & \frac{(1-2k_{D1}-k_{P3})}{2} \\ \frac{(-k_{P2})}{2} & -2 & \frac{(2-2k_{D2}+2\bar{x}_2+2k_{D2}x_1)}{2} \\ \frac{(1-2k_{D1}-k_{P3})}{2} & \frac{(2-2k_{D2}+2\bar{x}_2+2k_{D2}x_1)}{2} & -2k_{D3}\end{bmatrix}$$

To ensure $\mathbf{J}_s$ to be UND, the following conditions should be satisfied.

(I) $k_{P1} > 0$

(II) $8k_{P1} > k_{P2}{}^2$

(III) $$\frac{k_{P1}(2-2k_{D2}+2\bar{x}_2+2k_{D2}x_1)^2}{4} + \frac{k_{P2}{}^2 k_{D3}}{2} + \frac{(1-2k_{D1}-k_{P3})^2}{2} < 4k_{P1}k_{D3} + k_{P2}\frac{(1-2k_{D1}-k_{P3})}{2}\frac{(2-2k_{D2}+2\bar{x}_2+2k_{D2}x_1)}{2}$$

4. Simulations

Simulations is run for 100 seconds with initial conditions $[2\ 1\ 3\ 0\ 0\ 0]^T$. The observer gains are kept zero for time period t=15 sec. and after that gains are switched on. The values of the observer gain are

$k_{D1} = 5;\ \ k_{D2} = 3;\ \ k_{D3} = 20$

$k_{P1} = 2;\ \ k_{P2} = 1;\ \ k_{P3} = 3$.

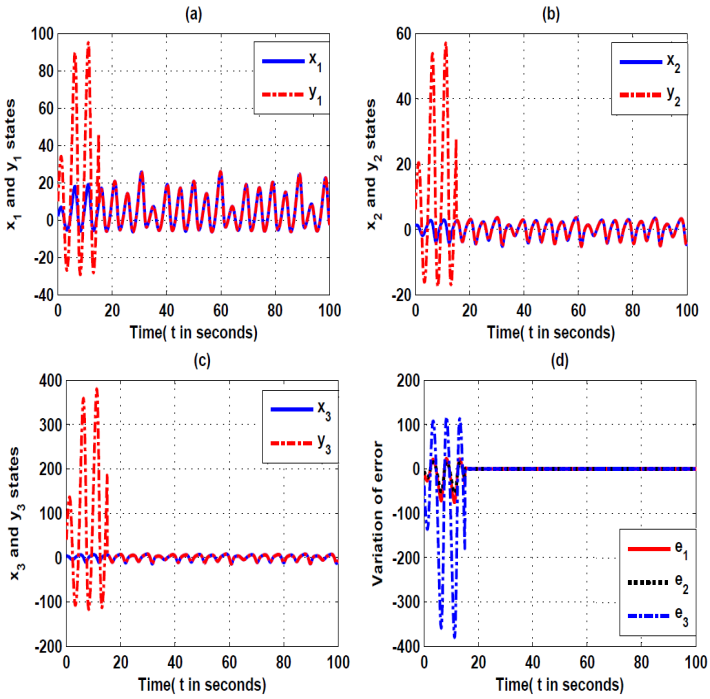

Fig. 2. Convergence behavior & variation of error

5. Conclusion

Co-ordinate transformation are used to design the observer that helps to avoid the derivative of output in the observer dynamics. Results are numerically verified for Sprott chaotic system. Simulations are given to validate the proposed approach.

References

1. Jayaram A., Tadi M., Synchronization of chaotic systems based on SDRE method, *Chaos Solitons Fractals* vol.28, Nr.3, pp.707- 715 (2006).
2. Carroll T. L. and Pecora L. M., Synchronizing chaotic circuits, *IEEE Trans*. vol.38, pp. 453-456 (1991).
3. Carroll T. L. and Pecora L. M., Cascading synchronized chaotic systems, *Physica* D 67, 126-140 (1993).
4. Carroll T. L. and Pecora L. M., Synchronizing non-autonomous chaotic circuits, *IEEE Trans. Circuits Syst.* 40, 646 (1995).
5. Z. M. Ge and W. Y. Leu, Chaos synchronization and parameter identification for loudspeaker systems, *Chaos, Solitons and Fractals*, vol. 21, pp. 1231-1247, (2004).

6. Lu. J. Chen, G and Cheng, D 2004, A new chaotic system and beyond: The generalized Lorenz-like system, *International Journal of Bifurcation and Chaos*, vol. 14, no. 5, pp. 1507-1537.
7. B. B. Sharma and I. N. kar, Observer-based synchronization scheme for a class of chaotic systems using contraction theory, *Nonlinear Dynamics*, vol.63, No.3, pp. 429-445 (2011).
8. Liao T. L., Huang N. S., An observer based approach for chaotic synchronization and secure communication, *IEEE Trans. Circuits Syst.* I 46(9), 1144-1149 (1999).
9. Manish Kumar Vaishnav, B. B. Sharma, Simple Observer Based Synchronization of Lorenz System with Parametric Uncertainty, *IOSR-JEEE* ISSN: 2278-1676, Volume 3, Issue 6, pp. 14-20, (Nov. - Dec. 2012).
10. B. B. Sharma and I. N. Kar, Design of asymptotically convergent frequency estimator using contraction theory, *IEEE Trans. On Automatic Control*, vol.53, No 8, pp.1932-1937, (2008).
11. Jouffroy J. and Slotine J. J. E, Methodological remarks on contraction theory, *IEEE Conference on Decision and Control Atlantis, Paradise Island, Bahamas,* pp. 2537-2543, 2004.
12. B. B. Sharma and I. N. Kar, Contraction theory based recursive design of stabilizing controller for a class of nonlinear systems, *IET control theory & applications*, pp.1005-1018, (2010).
13. Lohmiller W. and Slotine J. J. E., On contraction analysis for nonlinear systems, *Automatica,* vol.34, No.6, pp. 683–696, (1998).