Performance Analysis of Pre-detection and Post-detection Diversity reception schemes for Mobile Satellite Systems.

Dr.Sonika Singh [1],
Electronics & Communication Engineering Department,
Dehradun Institute of Technology, Dehradun,-248001 (U.K) .
gsonika@gmail.com

Dr. Sandip Vijay [2],
Electronics & Communication Engineering Department,
Dehradun Institute of Technology, Dehradun,-248001 (U.K) .
vijaysandip@gmail.com

Dr. Gagan Singh [3]
Electrical Engineering Department,
Dehradun Institute of Technology, Dehradun,-248001 (U.K) .
gaganus@gmail.com

Abstract

For mobile satellite systems, quality of service and service availability depend on the line-of-sight satellite availability. The satellite visibility is one of the major factors which influence satellite link availability, since the satellite channel behavior is depended on the link conditions between satellite and land user. Because of this, the mobile satellite systems are using satellite constellation in low elevation orbits, which can provide multiple satellite visibility. In this paper a closed form expression for the performance of the post-detection product detector combiner (PDC) operating on L correlated branches in Nakagami fading has been derived, considering the DPSK signaling scheme and Nakagami flat fading channel. The average bit error rate (BER) obtained with this scheme is compared to the ideal predetection maximum ratio combining (MRC), showing limited loss. The post-detection product detector combiner is shown to perform better than the selection diversity combiner (SDC) under the considered case of mobile satellite systems.

Keywords: mobile satellite systems, pre-detection diversity, post-detection diversity, differential phase-shift keying, Bit error probability, Nakagami flat fading channels, SDC, MRC.

1. Introduction

The channel characterization is a prerequisite for the analysis of QoS. To evaluate the satisfaction level of users employing LMSS, telecommunication systems Quality of Service (QoS) analysis is necessary for improved signal reception and capacity enhancement.The majority of these satellite systems are operated in L and S bands with their satellite installed in low and medium earth orbits (LEO/MEO). On the physical layer, the performance of land mobile satellite systems (LMSS) [1] is strongly affected by their channel environment and the elevation angle that governs the fading condition due to the shadowing and blockage. Multiple reflections of the radio signal cause the signal to arrive at the mobile station via multiple paths, which differ in amplitude, phase and delay time. The multipath reception in combination with the low link margins and low elevation angles is the main cause for signal outages, reduced communication quality and system capacity. A well-known method to combat the effects of multipath fading is to obtain

not just one, but several versions of a signal at the receiver. This principle is known as diversity. There are many methods for combing the signals that are received on the disparate diversity branches, and several ways of categorizing them. Among the various diversity techniques used frequency diversity is not a viable option for most of the mobile satellite systems [2] because the coherence bandwidth is quite large (from several tens of kilohertz to a few megahertz depending on the circumstances) and in any case the pressures on spectrum utilization are such that multi frequency allocations cannot be made. Two other techniques are polarization diversity and field diversity; polarization diversity relies on the scatterers to depolarize the transmitted signal, and field diversity uses the fact that the electric and magnetic components of the field at any receiving point are uncorrelated. Both these methods have their difficulties, since sufficient depolarization is not always possible along the transmission path for polarization diversity to be successful, and there are difficulties with the design of antennas suitable for field diversity. Time diversity, i.e. repeating the message after a suitable time interval, is more popularly in digital systems where storage facilities are available at the receiver. Finally, it is space the diversity (obtaining signals from two or more antennas physically separated from each other) which seems to be the most attractive and convenient method of diversity reception for mobile satellite systems.

After obtaining the necessary versions of the signal, the signals are processed to obtain the best results using a linear diversity combiner [3]. There are various possibilities, but what is `best' really amounts to deciding what method gives the optimum improvement, taking into account the complexity and cost involved. In linear diversity combining, the various signal inputs are individually weighted and then added together. If addition takes place after detection the system is called a post-detection combiner; if it takes place before detection the system is called a predetection combiner. In the predetection combiner it is necessary to provide a method of co-phasing the signals before addition. Assuming that the initial processing has been done, the output of a linear combiner consists of L branches, where $s_k t$ is the envelope of the kth signal to which a weight a_k is applied. In mobile s the received signal amplitude can be affected by fade phenomena due to multipath propagation [4]; a general model for the fading was proposed by Nakagami [5], under the name of *m*-distribution. Performance improvement can be achieved by using an L branch diversity receiver, though, in some practical mobile systems, branch correlation reduces diversity gain and must be accounted for in the system design. For independent branches, the predetection MRC is the optimum scheme and is frequently considered since its performance gives an upper bound for suboptimal combiners. In [6] the BER of binary signaling on Nakagami channels with predetection MRC and correlated branches has been presented.. In practice, the coherent detection is difficult to implement on faded channels and signaling schemes are preferred, which operate with differential modulation and detection. Moreover, predetection MRC has high implementation complexity, therefore, it is of interest to analyze the performance of simpler diversity schemes. We consider the product detector [7]-[9]. A differential product is evaluated at each branch and the decision variable is the unweighted sum of the outputs of the L differential products. This combiner will be referred to as post-detection product detector combiner while in[6] it is named post-detection equal gain combiner (EGC) and in [9] post-detection MRC. In this paper , the performance of 2-DPSK signals on *m*-distributed fading channels has been analyzed using an L branch post-detection PDC with generic branch correlation. The analysis of combiners is carried out in terms of CNR or SNR, with the following assumptions :

a) The noise in each branch is independent of the signal and is additive.
b) The signals are locally coherent, implying that although their amplitudes change due to fading, the fading rate is much slower than the lowest modulation frequency present in the signal.
c) The noise components are locally incoherent and have zero means, with a constant local mean square (constant noise power).
d) The local mean square values of the signals are statistically independent.

2. Mathematical Model

The received signal is corrupted by an AWGN which is assumed statistically independent on each branch. In the complex baseband model, the received signal at the *i*th bit interval at the input of the *k*th branch is [10]:

$$r_k(t) = R_k e^{j\phi_k} s_l(t) + n_k(t), \quad k = 1,\ldots,L; l = 1,2 \tag{1}$$

where $s_l(t)$ represents the i th transmitted signal. For binary PSK, $s_1(t) = g(t - iT)e^{j\phi_0}$ and $s_2(t) = g(t - iT)e^{j(\phi_0 + \pi)}$, being *g(t)* an unit energy pulse. $n_k(t)$ is AWGN with one sided power spectral density N_0 in watts per hertz (W/Hz) units, ϕ_k is the fading phase shift, and R_k is the fading amplitude which follows the Nakagami-*m* pdf [11]

, $R_k \geq 0$

$$p(r) = \frac{2m^m r^{m-1}}{\Gamma(m)\Omega^m} exp\left(\frac{-mr^2}{\Omega}\right) \tag{2}$$

where $\Gamma(.)$ is the Gamma function, $\Omega_k = \langle R_k^2 \rangle$ the mean square value of R_k and $m \geq 0.5$ the fading severity.

We consider nonselective and slow fading so that ϕ_k and R_k remain constant over at least two consecutive bit intervals. After matched filtering the signal on each branch, the post-detection PDC takes the unweighted sum of the L differential detectors outputs, and its decision variable, which is tested for being positive or negative is:

$$Z = \mathrm{Re}\left[\sum_{k=1}^{L}(R_k e^{j\phi_k} + N_{k_2})(R_k e^{-j\phi_k} + N_{k_1}^*)\right] \tag{3}$$

Where N_{k_1} and N_{k_2} are the AWGN components at the output of the matched filters in two consecutive bit intervals.

The instantaneous SNR per bit γ, at the output of the combiner, is equal to the sum of the signal-to-noise ratios (SNR's) γ_k [12], where γ_k is the SNR at the input of the detector in the *k*th branch:

$$\gamma = \sum_{k=1}^{L}\gamma_k \tag{4}$$

Moreover, the input instantaneous SNR, $\gamma_k = (R_k^2 T)/(2N_0)$, follows the Gamma pdf:

$$p_{\gamma_k}(\gamma_k) = \frac{(m/G_k)^m}{\Gamma(m)}\gamma_k^{m-1}\exp\left(-\frac{m\gamma_k}{G_k}\right), \quad \gamma_k \geq 0 \tag{5}$$

where the input average SNR for the *k*th branch, G_k, is expressed by $G_k = \Omega_k T/2N_0$.

In the case of space diversity, if the antennas are closely spaced, with respect to the carrier wavelength, the fading at the branch inputs are not independent. The branch correlation is encoded by the matrix M_Γ, whose elements $M_\Gamma(k,h)$, are given by the power correlation coefficients between branches *k* and *h*.

3. Error Probability Analysis

The conditional error probability $P_e(\gamma)$, given γ, is [8]

$$P_e(\gamma) = \exp(-\gamma)\sum_{k=0}^{L-1}\frac{c_k}{k!}\gamma^k \tag{6}$$

Where the coefficients c_k are given by:

$$c_k = \frac{1}{2^{2L-1}}\sum_{n=0}^{L-1-k}\binom{2L-1}{n}. \tag{7}$$

As fading is independent of noise, the BER, P_e, is evaluated by averaging (6) over the p.d.f. of the fading variable γ

$$P_e = \int_0^\infty P_e(\gamma)p_\gamma(\gamma)d\gamma. \tag{8}$$

By defining the moment generating function (MGF) $C_\gamma(s)$ of the random variable γ as:

$$C_\gamma(s) = \int_0^\infty p_\gamma(\gamma)\exp(-s\gamma)d\gamma \tag{9}$$

and substituting (6) into (8), the BER can be rewritten as a function of the derivatives of the MGF

$$P_e = \sum_{k=0}^{L-1}(-1)^k\frac{c_k}{k!}\left[\frac{d^k}{ds^k}C_\gamma(s)\right]_{s=1}. \tag{10}$$

The MGF of γ, given by (4), can be expressed as [4]

$$C_\gamma(s) = \left|I + \frac{s}{m}D_G M_X\right|^{-1} \tag{11}$$

where $D_G = diag\{G_1, G_2, ..., G_L\}$, ***I*** is the $L \times L$ identity matrix, M_X is an $L \times L$ matrix the elements of which are given by $M_X(k,h) = \sqrt{M_\Gamma(k,h)}$ for *k,h=1,...,L* and $|A|$ denotes the determinant of te matrix A. For balanced branches $G_k = G_0$ for any k; then, expanding the MGF in terms of the eigenvalues λ_i of M_X, we have:

$$C_\gamma(s)=\prod_{i=1}^{L}(1+\frac{s}{m}G_0\lambda_i)^{-m}=\prod_{i=1}^{L}f_i(s)\cdot \tag{12}$$

To evaluate the derivatives in (10) it is observed that:

$$\frac{d^k}{ds^k}C_\gamma(s)=\frac{d^k}{ds^k}\left[\prod_{i=1}^{L}f_i(s)\right]=$$
$$=\sum_{n_1=0}^{k}\binom{k}{n_1}f_1^{(k-n_1)}(s)\sum_{n_2=0}^{n_1}\binom{n_1}{n_2}f_2^{(n_1-n_2)}(s)\cdot$$
$$\cdot\sum_{n_3=0}^{n_2}\binom{n_2}{n_3}f_3^{(n_2-n_3)}(s)\cdots\sum_{n_{L-1}=0}^{n_{L-2}}\binom{n_{L-2}}{n_{L-1}}\cdot$$
$$\cdot f_{L-1}^{(n_{L-2}-n_{L-1})}(s)\cdot f_L^{n_{L-1}}(s) \tag{13}$$

where, with some tedious but simple algebra, it results

$$\frac{d^n}{ds^n}f_i(s)=f_i^{(n)}(s)=(-1)^n\frac{(m-n-1)!}{(m-1)!}\eta_i^n(s)f_i(s) \tag{14}$$

Where,

$$\eta_i(s)=\left(\frac{G_0}{m}\lambda_i\right)\Big/\left(1+\frac{s}{m}G_0\lambda_i\right). \tag{15}$$

Finally, substituting (12)-(15) into (10), and noting that for predetection MRC and 2-DPSK signals the BER is [6]

$$[P_e]_{MRC}=0.5\left[C_\gamma(s)\right]_{s=1} \tag{16}$$

It is obtained,

$$P_e=2[P_e]_{MRC}\cdot$$
$$\cdot\left[\sum_{k=0}^{L-1}c_k\sum_{n_1=0}^{k}H_m(k-n_1)\eta_1^{(k-n_1)}(s)\cdot\right.$$
$$\cdot\sum_{n_2=0}^{n_1}H_m(n_1-n_2)\eta_2^{(n_1-n_2)}(s)\cdot$$
$$\cdot\sum_{n_3=0}^{n_2}H_m(n_2-n_3)\eta_3^{(n_2-n_3)}(s)\cdots$$
$$\cdot\sum_{n_{L-1}=0}^{n_{L-2}}H_m(n_{L-2}-n_{L-1})H_m(n_{L-1})\cdot$$
$$\left.\cdot\eta_{L-1}^{(n_{L-2}-n_{L-1})}(s)\eta_L^{(n_{L-1})}(s)\right]_{s=1} \tag{17}$$

with the coefficients $H_l(h)=\Gamma(l+h)/(\Gamma(l)\Gamma(h+1))$.

Alternatively, (17) can be written as,

$$P_e=2[P_e]_{MRC}\sum_{k=0}^{L-1}c_kY_L(k) \tag{18}$$

Where the polynomials $Y_p(q)$ are given by,

$$Y_p(q)=\sum_{r=0}^{q}H_m(q-r)\left[\eta_p^{q-r}(s)\right]_{s=1}Y_{p-1}(r) \tag{19}$$

and they can be recursively evaluated starting from $Y_1(q)=H_m(q)\left[\eta_1^q(s)\right]_{s=1}$.

(20)

In the simple case of Rayleigh fading (*m=1*) and balanced independent branches ($\lambda_i=1$ for any *i*), we find the result given in $[6, eq.(7.4.26)]$. In fact $[\eta_i(s)]_{s=1}=\eta_0=G_0/(1+G_0)$ for any *i*, and:

$$Y_L(k)=\binom{L-1+k}{k}\eta_0^k\cdot \tag{21}$$

Then, the BER, given by (18), becomes

$$P_e=2[P_e]_{MRC}\sum_{k=0}^{L-1}\binom{L-1+k}{k}c_k\left(\frac{G_0}{1+G_0}\right)^k. \tag{22}$$

Noting that $c_0=1/2$, (18) can be rewritten as:

$$P_e=[P_e]_{MRC}+2[P_e]_{MRC}\sum_{k=1}^{L-1}c_kY_L(k)=[P_e]_{MRC}+\Delta P_e \tag{23}$$

Since the sum in equation (23) is positive, post-detection PDC performs worse than predetection MRC. When $G_0\to 0, Y_L(k)\to 0$, for any *k*; then $\Delta P_e\to 0$, and the post-detection PDC gives the optimum performance of the predetection MRC at very low SNR.

On the contrary, for high SNR $[\eta_i(s)]_{s=1}\to 1$ and ΔP_e can be expressed as a polynomial of degree (*L-1*) in *m*. Fig.1 and Fig 2 shows the BER for DPSK modulation with SDC and MRC respectively. This shows that the post-detection PDC performs only 1-3 dB worse than the predetection MRC, depending on *m, L*, and branch separation *d*. For example, as $G_0\to\infty, \Delta P_e=4[P_e]_{MRC}mc_1$, for *L=2*, and $\Delta P_e=[P_e]_{MRC}[6mc_1+mc_2(9m+3)]$ for *L=3*. Hence, the higher is the fading severity, the smaller is the loss of post-detection PDC with respect to predetection MRC.

4. Results

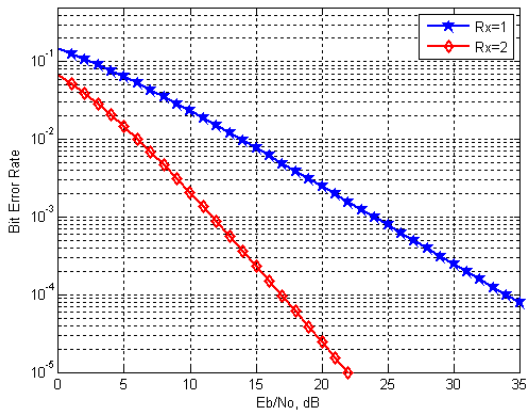

Fig1. BER for DPSK modulation with SDC

Fig1. shows the BER plot of SDC for DPSK modulation. It can be observed from Table 1 that diversity improvement is better with two satellite antenna receiving paths with the bit energy to noise density ratio values as follows for different values of BER:

Table 1. BER performance with SDC

BER (Rx=1)	Eb/No(dB)	BER(Rx=2)	Eb/No(dB)
10^{-2}	13	10^{-2}	6
10^{-3}	24	10^{-3}	11
10^{-4}	32	10^{-4}	16
10^{-5}	35	10^{-5}	21

Fig2. shows the BER plot of MRC for LMSS for DPSK modulation . It can be observed from Table 2. that diversity improvement is better with two satellite antenna receiving paths, with the bit energy to noise density ratio values as follows for different values of BER.

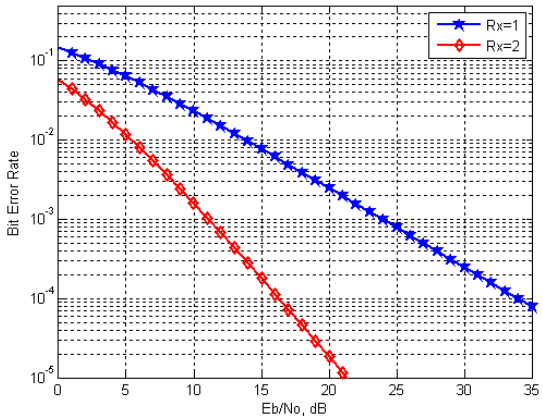

Fig2. BER for DPSK modulation with MRC

Table 2. BER performance with MRC

BER(Rx=1)	Eb/No(dB)	BER(Rx=2)	Eb/No(dB)
10^{-2}	13	10^{-2}	6
10^{-3}	23	10^{-3}	10.2
10^{-4}	31.6	10^{-4}	15.5
10^{-5}	34.8	10^{-5}	20.8

The average SNR improvement of SDC is typically about 1 dB worse than with MRC, but still much better than without diversity.

5. Conclusion

The present work provides a closed form expression for the performance of L branch post-detection PDC diversity in correlated Nakagami flat fading and AWGN, for 2-DPSK signaling for mobile satellite systems. The BER has been derived in terms of polynomials which can be recursively evaluated. The PDC diversity has a simpler structure than the MRC and is easier to implement. The performance comparison between post-detection PDC and predetection MRC has been provided. It is concluded that the post-detection PDC performs only 1-3 dB worse than the predetection MRC, depending on *m, L*, and branch separation *d*. Furthermore, separation between two adjacent antennas greater then 0.2λ is enough to obtain most of the diversity gain. For L=2 also a comparison has been presented with SDC, which shows that post-detection product detector combiner performs better than SDC.

Thus, though MRC is the most optimal linear combining technique, it is seldom implemented in mobile satellite systems because the receiver complexity for MRC is directly proportional to the number of resolvable paths (i.e. branch signals) available at the receiver. Alternatively, SDC has been widely adopted in practice due to its good performance and ease of implementation. Because SDC does not require estimation of the channel (path) fading amplitudes, it is often used in practice as a reduced complexity alternative to the optimum MRC scheme.

References

[1]. Barts R. M. and Stutzman W. L, "Modeling and simulation of mobile satellite propagation," IEEE Trans. Antennas Propagat., vol. 40, pp. 375-382, 1992.

[2] Chini Paolo, Giovanni GiambeneA ,y and Sastri Kota,"A survey on mobile satellite systems", International Journal Of Satellite Communications, Int. J. Comm. Syst. Network; 28:29–57, 2010.

[3] J.D.Parsons, “The mobile radio propagation channel”, John Wiley & Sons Ltd., ISBN 0-471-98857-X, 2nd ed..pp. 307-333,2000.
[4]. G.J. Foschini, M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Commun., vol. 6, no. 3, pp. 311-335, 1998.
[5] Yacoub M.D, Bautista, and Guedes L.G,“On Higher Order Statistics of the Nakagami-m Distribution,” IEEE Transactions on Vehicular Technology, vol. 48, No. 3, 1999.
[6] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation model," IEEE Trans. Comm., vol 47, pp. 44-52, Jan. 1999.
[7] P. Y. Kam, "Bit error probability of MDPSK over the nonselective Rayleigh fading channel with diversity reception, "IEEE Trans. Comm., vol. 39, pp. 220-224, Feb. 1991.
[8] A. A. Abu-Dayya and N. C. Beaulieu, "Comparison of diversity with simple block coding on correlated frequency-selective fading channels," IEEE Trans. Comm., vol. 43, pp. 2704-2713, Nov 1995.
[9] F. Adachi, "Post detection optimal diversity combiner for DPSK differential detection, "IEEE Trans. Veh. Technol., vol. 42, pp. 326-336, Aug. 1993.
[10]. Goran Tomovic, Bojan Stankovic, Elektronski fakultet,“ Post-Detection diversity in Nakagami fading channels with correlated branches“ , Infoteh Jahorina, vol3, RefB II ,PP61-63, March2003.
[11] Zhang Q.T, ‘A Decomposition technique for Efficient Generation of Correlated Nakagami Fading Channels,” IEEE Journal on Selected Areas in Communications., vol.18, no. 11, pp. 2385 – 23, 2000.
[12] G. Ropokis, A. A Rontogiannis, P. T. Mathiopoulos and K. Berberidis, “An exact performance analysis of MRC / OSTBC over generalized fading channels”, IEEE Transactions on Communications, pp. 2486 – 2492, September 2010.
[13]. R.U. Nabar, H. Bolcskei, V. Erceg, D. Gesbert, A.J. Paulraj, "Performance of multiantenna signaling techniques in the presence of polarization diversity," IEEE Trans. Signal Process., vol. 50, no. 10, pp. 2253-2262, 2002
[14] G. Fedele, L. Izzo, and M. Tanda, "Dual diversity reception of M-ary DPSK signals over Nakagami fading channels," in Proc. IEEE PIMRC Toronto, Canada, Sept. 26-29, , pp. 1195-1201, 1995.
[15] G. C. Alexandropoulos and P. T. Mathiopoulos, “Selection diversity receivers over arbitrarily correlated generalized Gamma fading channels”, IET – Communications, pp.1253 – 1265, July 2010.
[16] G. C. Alexandropoulos, P. T. Mathiopoulos and N. C. Sagias, “Switch-and-Examine diversity over arbitrary correlated Nakagami-m fading channels,” IEEE Transactions on Vehicular Technology, pp. 2080 – 2087, May 2010.