PERFORMANCE ANALYSIS OF REDUCED ORDER AIRCRAFT BANK ANGLE CONTROL SYSTEM

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Abstract

Order reduction is very important issue in control system. Analysis, simulation and response of a lower order system are simple and fast. So in this paper, a simple but very effective approach based on Routh Criterion is used to determine reduced order model of an unstable open-loop system. Aircraft bank angle control system is considered here, which is an open loops 3rd order unstable system. This 3rd order system is converted to 2nd order system and performance comparison is done for both the systems. Simulation is carried out in MATLAB® environment.

***Keywords*:** Reduced order, aircraft bank angle, V. Krishnamurthy approach, Routh Criterion.

1. Introduction

Various research papers have been published [1-11] for reducing the order of a system. Routh criterion is simple and effective method for getting reduced order control system. The first paper on reduced order modeling using Routh criterion is published in 1975[2]. This paper improves the drawbacks of Pade approximation method. A simple approach on Routh criterion is proposed by V. Krishnamurthy and V. Seshadri [3]. It is found that in this method, system can be unstable if it has both numerator and denominator coefficients. In [6], a mixed method is proposed for reduced order modeling of linear systems. It is observed that all these method are applicable only for stable systems. It is further observed that most of the practical systems are open-loop unstable systems such as aircraft bank angle control, inverted pendulum etc [12, 13]. To the best of our knowledge, these existing approaches can't be easily applicable to open loop unstable system. Therefore, in the paper, simple approach based on Krishnamurthy's results on Routh criterion [3] is applied for reducing the order of an open loop unstable system. In order to show the application of the approach, open loop unstable transfer function of bank angle control of aircraft is considered. The order of this bank angle control system is three and it has been reduced to second-order. Here performance is analyzed for both original and reduced order system using step input and for unity feed. It is shown that performance is almost same for both of the systems.

2. Reduce Order Model of Linear System Based on V. Krishnamurthy's and Seshardri Result

Let the open loops transfer function of a system is

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$$= \frac{K}{a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + \cdots a_{m-1} s} \quad (1)$$

Steps for getting reduced order of the systems are

Step I: Determine the closed loop characteristic equation of this system say $C_r(s)$

$$C(s) = a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + \cdots a_{m-1} s + K \quad (2)$$

Step II: Apply Routh criterion to above characteristic equation

Step III: By Krishnamurthy approach based on Routh criterion, reduced characteristic equation will be

$$) = a_{r0} s^n + a_{r1} s^{n-1} + a_{r2} s^{n-2} + \cdots a_{rn}, n < m \quad (3)$$

Step IV: from above equation (3), open loop reduced order system will be

$$G_r(s) = \frac{a_{rm}}{a_{r0} s^n + a_{r1} s^{n-1} + a_{r2} s^{n-2} + \cdots a_{r(n-1)} s} \quad (4)$$

3. Reduced Order Model of Aircraft Bank Angle Control System

The open loop transfer function of aircraft bank angle control system is given by [12]

$$G(s) = \frac{114K}{s^3 + 11.4s^2 + 14s} \quad (5)$$

The closed loop characteristic of above system will be

$$C(s) = s^3 + 11.4s^2 + 14s + 114K \quad (6)$$

Apply Routh criterion to equation (6)

s^3	1	14
s^2	11.4	114K
s^1	$\frac{14-114K}{11.4} = z$	0
s^0	114K	

Using Krishnamurthy's approach, reduced order closed loop characteristic equation will be

$$C_r(s) = 11.4s^2 + zs + 114K \quad (7)$$

So reduced order open loop transfer function of the system will be

$$G_r(s) \frac{114K}{11.4s^2 + zs} \quad (8)$$

This is an unstable system.

4. Simulation and results:

Let gain K=0.16 so that

Peak overshoot P.O.=20%

Peak time t_P=2.7 sec

From equation (5) original open loop transfer function of the aircraft system

$$\frac{\emptyset(s)}{\emptyset_d(s)} = \frac{18.24}{s^3 + 11.4s^2 + 14s + 18.24}$$

From equation (8) reduced open loop transfer function of the aircraft system

$$\left.\frac{\emptyset(s)}{\emptyset_d(s)}\right|_{reduced} = \frac{18.24}{11.4s^2 + 12.4s + 18.24}$$

Now the step response of above two systems with unity feed is shown in figure 1 and their time domain specifications are shown in table 1 for comparison.

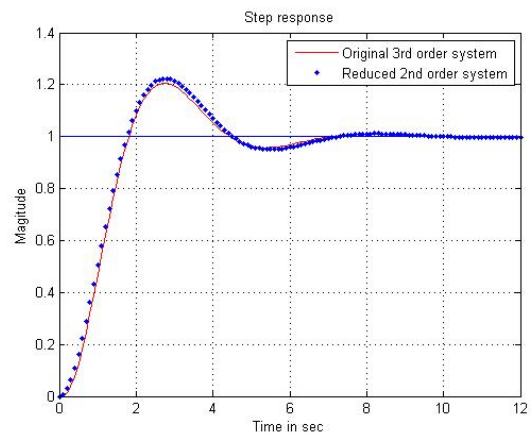

Figure 1: Step response of Original 3rd order and reduced 2nd order system

Table 1: Performance comparison of original 3rd order and reduced 2nd order system

Specifications	Original System	Reduced System
Rise time	1.1715	1.1987
Settling Time	6.3440	6.6300
Settling Min	0.9254	0.9499
Settling Max	1.2054	1.2239
Overshoot%	20.5443	22.3905
Undershoot	0	0
Peak Overshoot	1.2054	1.2239
Peak Time	2.7003	2.7278

5. Conclusion:

As we see from figure 1, there are slight variations in the time specifications of the reduced order system. The Rise time, Peak time & peak overshoot are increased by around 2%, but the settling time which has the maximum variation is increased by 4.5%. There is no change in undershoot. Further we can set the parameter of the system so that the variations in the system output are minimized. So this method of reduction[3] can be used for analysis of the higher order system and it can also be applied to other open loop unstable electrical or mechanical systems.

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