Robust H_∞ guaranteed cost fault diagnosis of networked control systems with interference and data packet dropout

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Abstract: This paper focusing on a class of networked control systems with unknown interference inputs and packet dropout in sensor data, the state observer is designed to detect them when faults occur in the system. A model and the state observer are constructed for the system, and the observer error equation is kept equivalent to the discrete switched system. Based on Lyapunov stability theory and linear matrix inequality processing methods, network control system introduced is asymptotically stable, and the sufficient condition of the observer being a H_∞ state observer, and upper bound of the guaranteed cost system is given. In order to enhance the sensitivity, the threshold is selected with solving the minimization problem involving a linear matrix inequality constraint.

Keywords: Networked control, Data packet dropout, State observer, Guaranteed cost control, Lyapunov function

Introduction

Networked Control Systems are the real-time networked closed-loop control systems, through which the information transmits. Compared with the traditional point to point control system, it has some advantages like has less wiring, installation and maintenance convenient, can realize the resources sharing etc. But due to the introduction of the network, the inevitable cause delay, packet loss, data random sequence problems, in order to guarantee the system can work normally, the fault detection and diagnosis technology is quite important [1-2]. Recently, some research results has made about some NCS which existing delay, packet loss and unknown disturbance input. In the paper [3], a state observer of NCS was designed with the time-delay compensation method, and achieved the fault insulation(FDI). The design of the observer has certain conservative, and did not consider system external disturbance. For continuous systems with time delay, paper [4] has researched the guaranteed cost control of system with uncertain time-delay. And in paper [5] we have considered network control system with time-delay and data random sequence.

Based on a linear system with norm bounded and interference, based on the Lyapunov stability theory, the guaranteed cost control theory and linear matrix inequalities (LMIs) processing method. The paper presents an observer based on robust H_∞ guaranteed cost of fault diagnosis and its existing sufficiency conditions and also its upper boundary.

Problem description

The Fig.1 is the structure of NCS, we supposed the network exist in between controller and actuator, so the system only with control delay, and its upper boundary is known as time-varying long delay. We suppose that the controlled object model of the network control system is:

$$\begin{cases} \dot{x}(t) = A_f x(t) + B_f v(t) + f(t) \\ y(t) = C_f(t) \end{cases} \tag{1}$$

Where, $x(t) \in R^n$ is state vector, $v(t) \in R^m$ is input vector, $y(t) \in R^l$ is output vector, $f(t) \in R^n$ is fault vector. Normally, $f(t)$ is zero vector, when $f(t)$ is non zero vector, means the system breakdown. A_f, B_f, C_f are known as suitable dimension matrixes.
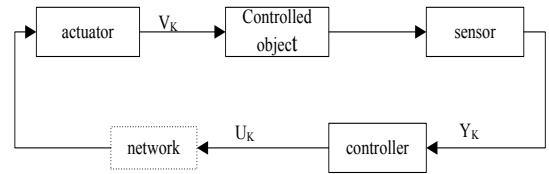

Fig.1 The structure of the network system

We do reasonable supposition to this system:
1. The node of the sensor is driven by time, which is sampled with known fixed period T. The controller and the processor are driven by event;
2. The large time-delay value of the system is known, and is great than T, and its upper boundary is $\bar{d}$;
3. By a mode of single packet transmission, and does not exist packet loss;
4. Existing time stamp in the node packet.

From (1), assume the control time-delay is τ_k at the k time, and have

$$T < \tau_k = \tau_k^{ca} = (d-1)T + T/2 + \Delta\tau \le \bar{d}T \tag{2}$$

Where, $-0.5T \le \Delta\tau \le 0.5T$ is an uncertain time-delay, and $0 \le \Delta\tau + 0.5T \le T$, $d \le \bar{d} = [\tau_k^{wc}/T]$; here $[*]$ indicates the smallest integer greater than $*$; τ_k^{wc} is known as the delay in the worst case. We can get the discrete system model include network's generalized controlled object with discrete to the controlled object by the sampling period [8].

$$\begin{cases} x_{k+1} = Ax_k + \Gamma_0(\tau_k)u_{k-d+1} + \Gamma_1(\tau_k)u_{k-d} + f_k \\ y_k = Cx_k \end{cases} \quad (3)$$

In the formula (3): $A = e^{A_f T}$,

$$\Gamma_0(\tau_k) = \int_0^{0.5T-\Delta\tau} e^{A_f s} ds B, \Gamma_1(\tau_k) = \int_{0.5T-\Delta\tau}^{T} e^{A_f s} ds B$$

We can get the network control system model according to the derivation of literature [6]:

$$\begin{cases} x_{k+1} = Ax_k + (B_0 + DF(\Delta\tau)E)u_{k-d+1} + (B_1 - DF)(\Delta\tau)E)u_{k-d} + f_k \\ y_k = Cx_k \end{cases} \quad (4)$$

Where, $B_0 = \int_0^{0.5T} e^{A_f s} ds B_f$, $B_1 = \int_{0.5T}^{T} e^{A_f s} ds B_f$,

$$D = \left\| \int_0^{0.5T} e^{A_f s} ds \right\|_2 e^{A_f(0.5T)}, E = B_f$$

$$F(\Delta\tau) = \left\| \int_0^{0.5t} e^{A_f s} ds \right\|_2^{-1} \int_0^{-\Delta\tau} e^{A_f s} ds \text{ and}$$

$F^T(\Delta\tau)F(\Delta\tau) \le I$. For convenience, make $F(\Delta\tau) = F$ and assume the system (A,C) can be observed.

Observer design

When the interference is added into Networked Control System and existing packet loss between the sensor and controller, the Fig.1 become the Fig.2. So, the system can be discrete into,

$$\begin{cases} x_{k+1} = Ax_k + (B_0 + DF(\Delta\tau)E)u_{k-d+1} + \\ (B_1 - DF(\Delta\tau)E)u_{k-d} + w_k + f_k \\ y_k = Cx_k \end{cases} \quad (5)$$

Because the system has packet loss between sensor and controller, hence, can through if there is a sensor data to controller at k time, the network control system is divided into two events. In the Fig.2, the random variables λ_k indicates if has sensor data into controller in the first k cycle, so

$$\lambda_k = \begin{cases} 0 & no\ data\ received,\ event\ 1 \\ 1 & received\ data, \quad event\ 2 \end{cases} \quad (6)$$

$\{\lambda_k; k \ge 0\}$ is a list of independent Bernoulli random variable. The data-delay between sensor and controller can be seen as the valid data has not received in this period, that happened data packet loss.
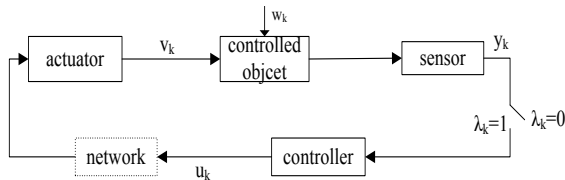

Fig.2 The structure of the network system with packet loss

Issue 1: There is no sensor data reach into controller at the k time, construct observer in the controller side

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + B_0 u_{k-d+1} + B_1 u_{k-d} \\ \hat{y} = C\hat{x}_k \end{cases} \quad (7)$$

Define observer state estimation error vector $e_k = x_k - \hat{x}_k$, then the state estimation error equation of observer system is

$$e_{k+1} = Ae_k + DFEu_{k-d+1} - DEFu_{k-d} + w_k + f_k \quad (8)$$

If the controller adopt the state feedback control law $u_k = Kx_k$, and quote the augmented state vector $z_k = \begin{bmatrix} e_k^T & x_k^T & \cdots & x_{k-d}^T \end{bmatrix}^T$, then the formula (8) can translated into

$$z_{k+1} = \begin{bmatrix} A & 0 & \cdots & DFEK & -DFEK \\ 0 & A & \cdots & (B_0 + DFE)K & (B_1 - DFE)K \\ 0 & I & \cdots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix} z_k + \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} w_k + \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} f_k \quad (11)$$

Issue 2: At the k time, the controller can received the data from sensor, we can construct the fault observer as

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + B_0 u_{k-d+1} + B_1 u_{k-d} + L(y_k - \hat{y}_k) \\ \hat{y} = C\hat{x}_k \end{cases} \quad (10)$$

Then define observer state estimation error vector

$$e_{k+1} = (A - LC)e_k + DFEu_{k-d+1} - DEFu_{k-d} + w_k + f_k \quad (11)$$

By the same theory, formula (11) can translate into

$$z_{k+1} = \begin{bmatrix} A-LC & 0 & \cdots & DFEK & -DFEK \\ 0 & A & \cdots & (B_0 + DFE)K & (B_1 - DFE)K \\ 0 & I & \cdots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix} z_k + \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} w_k + \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} f_k \quad (12)$$

Synthesize the formula (9) of issue 1 and the formula (12) of issue 2, when the system is normal the observer model is

$$z(k+1) = \Phi_i z(k) + Gf(k), i = 1.2 \quad (13)$$

$$\Phi_1 = \begin{bmatrix} A & 0 & \cdots & DFEK & -DFEK \\ 0 & A & \cdots & (B_0 + DFE)K & (B_1 - DFE)K \\ 0 & I & \cdots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} A-LC & 0 & \cdots & DFEK & -DFEK \\ 0 & A & \cdots & (B_0 + DFE)K & (B_1 - DFE)K \\ 0 & I & \cdots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \end{bmatrix}^T$$

It can be seen, for every k, observer system (13) can be described into the discrete switched system as shown fellow

$$\begin{cases} z_{k+1} = \sum_{i=1}^{2} \omega_i \Phi_i z(k) + Gw(k) \\ \varepsilon(k) = Hz(k) \end{cases} \quad (14)$$

The number of its subsystem is two, and $\omega_i \to \{0,1\}$, $\sum_{i=1}^{2} \omega_i = 1$, H is output power matrix. Make the observer output error $\varepsilon(k)$ is fault detection residuals, we can judge whether the system is out of order through the following rules,

$$\begin{cases} \|\varepsilon_k\| \leq \bar{\varepsilon} \quad normal \\ \|\varepsilon_k\| > \bar{\varepsilon} \quad fault \end{cases} \quad (15)$$

Where, $\|\varepsilon_k\| = \sqrt{\varepsilon(k)^T \varepsilon(k)}$ is European norm of vector, $\bar{\varepsilon}$ is the selected fault detection threshold.

Stability analysis
Relative definition and lemma:

Define the guaranteed cost function of observer system

$$J = \sum_{k=0}^{\infty} z^T(k)Qz(k) \quad (16)$$

Where, $Q = diag(Q_1, Q_2)$, Q_1, Q_2 is given symmetric positive defined matrix.

Define 1 For any given disturbance attenuation factors $\gamma > 0$, if there exist fault observer (10) and positive data J^*, make the observer error system (14) meet the conditions as fellows:
(1) when $w(k) = 0$, the observer error system (14) is asymptotic stable;
(2) the closed-loop system has corresponding performance index(16) and its upper boundary is J^*, and $J \leq J^*$;
(3) in the initial condition, make H_∞ performance metrics satisfied with $\|\varepsilon\|_2 < \gamma \|w\|_2$ and $\|\bullet\|_2 = \left[\sum_{k=0}^{\infty} (\bullet)^T (\bullet)\right]^{\frac{1}{2}}$, then we can call the observer system has the ability of H_∞ guaranteed cost, and J^* is upper boundary.

Lemma 1[7] For discrete switched systems like

$$x(k+1) = \sum_{i=1}^{r} \omega_i A_i x_k \quad (17)$$

where, $i = 1, \cdots, r$, $r < \infty$ is the number of subsystem, $\omega_i \to \{0,1\}$, $\sum_{i=1}^{2} \omega_i = 1$, if exist common symmetry positive-definite matrix P, make all the subsystem are meet

$$A_i^T P A_i - P < 0 \quad (18)$$

that is to say the observer system stability.

Lemma 2[8] Assume Z is a symmetric matrices, and broken into block matrix with three line and three series, if exist matrix X makes

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} + X^T \\ * & * & Z_{33} \end{bmatrix} < 0 \quad (19)$$

There is

$$\begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} Z_{11} & Z_{13} \\ * & Z_{33} \end{bmatrix} < 0 \quad (20)$$

Theorem 1 For the observer error system (13) if exist common symmetry positive-definite matrix P makes

$$\begin{bmatrix} -P + H^T H & 0 & \Phi_i^T \\ * & -\gamma^2 I & E^T \\ * & * & -P \end{bmatrix} < 0 \quad (21)$$

Where $i = 1, 2$; $*$ is the part of symmetrical matrix, then the observer has the ability of robust H_∞ guaranteed cost control and the performance of γ.

Proof: $V(k) = z(k)^T Pz(k)$ is a Lyapunov function. P is a positive definite matrix, then

$$\begin{aligned} \Delta V(k) &= z(k+1)^T Pz(k+1) - z(k)^T Pz(k) \\ &= z(k)^T \left(\Phi_i^T P\Phi_i - P\right) z(k) + z(k)^T \Phi_i^T PGw(k) \\ &\quad + w(k)^T G^T P\Phi_i z(k) + w(k)^T G^T PGw(k) \end{aligned} \quad (22)$$

First when $w(k) = 0$, the system is asymptotic stable.

$$\Delta V(k) = z(k)^T \left(\Phi_i^T P\Phi_i - P\right) z(k) \quad (23)$$

From lemma 2 make $Z_{11} = -P + H^T H$, $Z_{13} = \Phi_i^T$, $Z_{33} = -P^{-1}$ and from formula(22) we can obtain

$$\begin{bmatrix} -P + H^T H & \Phi_i^T \\ * & -P^{-1} \end{bmatrix} < 0 \quad (24)$$

according to Schur mend theorem, formula (23) is equivalent to $\Phi_i^T P\Phi_i - P < 0$, then $\Delta V(k) < 0$.

From lemma 1 we know the observer system is asymptotically stable at this moment.

Then prove has guaranteed cost upper boundary J^*:
Form formula (23)we can get

$$\Delta V(k)=\begin{bmatrix} z(k)^T & w(k)^T \end{bmatrix}\Pi_i\begin{bmatrix} z(k) \\ w(k) \end{bmatrix} \qquad (25)$$

And then obtain

$$\Pi_i=\begin{bmatrix} \Phi_i^T P\Phi_i - P & \Phi_i^T PG \\ G^T P\Phi_i & G^T PG \end{bmatrix} \qquad (26)$$

first when $i=1$,

$$\Pi_1+diag(Q,0)=\begin{bmatrix} \Phi_1^T P\Phi_1 - P+Q & \Phi_1^T PG \\ G^T P\Phi_1 & G^T PG+Q \end{bmatrix} \qquad (27)$$

according to lemma 1, there has

$$\begin{bmatrix} \Phi_i^T P\Phi_i - P+Q & \Phi_i^T PG \\ G^T P\Phi_i & G^T PG+Q \end{bmatrix}<0 \qquad (28)$$

that is

$$\Delta V^1(k)<-z(k)^T Qz(k)\le 0 \qquad (29)$$

After the similar induction with subsystem 1, can easily indicates that when the condition is satisfied in lemma 2, $\Delta V^i(k)<0$, $i=(1,2)$, then from $\Delta V_i(k)<-z_k^T Qz_k$, $i=(1,2)$, can get

$$J=\sum_{k=0}^{\infty}z_k^T Qz_k<-\sum_{k=0}^{\infty}\Delta V(k)\le z(0)^T Pz(0) \qquad (30)$$

So there is guaranteed cost upper boundary. At last let's prove the H_∞ performance indicator of the observer system:

When $w(k)\ne 0$, introduce $T(k)=\|\varepsilon(k)\|^2-\gamma^2\|w(k)\|^2+\Delta V(k)$, then

$$T(k)=\begin{bmatrix} z(k)^T & w(k)^T \end{bmatrix}\Omega\begin{bmatrix} z(k) \\ w(k) \end{bmatrix} \qquad (31)$$

and,

$$\Omega=\begin{bmatrix} \Phi_i^T P\Phi_i - P+H^T H & \Phi_i^T PG \\ G^T P\Phi_i & G^T PG-\gamma^2 I \end{bmatrix} \qquad (32)$$

then $\sum_{k=0}^{\infty}T(k)<0$ is equivalent to $\Omega<0$, that is

$$\begin{bmatrix} -P+H^T H & 0 \\ 0 & -\gamma^2 I \end{bmatrix}+\begin{bmatrix} \Phi_i^T \\ G^T \end{bmatrix}P\begin{bmatrix} \Phi_i & G \end{bmatrix}<0 \qquad (33)$$

according to Schur mend theorem, formula (33) is equivalent to formula (21). And

$$\sum_{k=0}^{\infty}T(k)=\sum_{k=0}^{\infty}(\|\varepsilon(k)\|^2-\gamma^2\|w(k)\|^2+\Delta V(k))=\|\varepsilon(k)\|_2^2-\gamma^2\|w(k)\|_2^2+V_\infty-V_0$$

From initial condition, $V_\infty-V_0\ge 0$, so that when $\sum_{k=0}^{\infty}T(k)<0$,

there has $\|\varepsilon(k)\|_2^2<\gamma^2\|w(k)\|_2^2$.

Threshold selection strategy

In the fault detection system, the selection of threshold is very important, take the interference upper boundary into consideration can strengthen the robustness and can reduce the false alarm cased by interference. Take γ as unknown, use its optimal value to improve sensitivity. So, the fault detection threshold is $\bar{\varepsilon}=\gamma^* w_m$.

Theorem 2 For the observer error system such as formula (14), when the following optimize problem has the best solution min γ_1, the limited condition is

$$\begin{bmatrix} -P+H^T H & 0 & Y \\ * & -\gamma^2 I & G^T P \\ * & * & -P \end{bmatrix}<0 \qquad (34)$$

When formula (34) is satisfied, exist γ_1^*, and make $\gamma^*=\sqrt{\gamma_1^*}$ is the optimize performance index of the H_∞ fault observer.

Summary

In this paper a kind of network control system with control delay, data packet loss and unknown diugh design a robust H_∞ fault observer, realize the robust fausturbance, throlt detection, and analyzes the robustness and sensitivity, give the selection method of threshold optimization. Use Lyapunov theory and a series of matrix inequalities have proved the system is asymptotic stability, and also use LMIs theory to deduce its guaranteed cost upper boundary.

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