Similarity Knowledge Mass and Multidimensional Approximate Reasoning

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Abstract – In this paper, we introduce the type V matching scheme and the corresponding type V algorithm of multidimensional approximate reasoning with given multidimensional input and multidimensional knowledge bases on strong *Q*-logic CQ.

Index Terms – Multidimensional approximate reasoning, Multidimensional formula mass, Multidimensional knowledge mass, Strong *Q*-logic.

1. Strong *Q*-logic CQ

Define a similarity relation Q in the formula set $F(S)$ of classic propositional logic C = (C^, C*) so that for any formula $A \to B \in F(S)$ it satisfies

$$<A> \to <B> = <A \to B>,$$

$$<A> \wedge <B> = <A \wedge B>,$$

where

$$<A> \to <B> = \{A^* \to B^* \mid A^* \in <A>, B^* \in <B>\},$$

$$<A> \wedge <B> = \{A^* \wedge B^* \mid A^* \in <A>, B^* \in <B>\}.$$

($F(S)$, Q) is called strong Q-space and CQ = (C^, C*, Q) is called strong Q-logic.

2. Type V Multidimensional Approximate Reasoning

Theorem 2.1 In strong Q-logic CQ, for any natural number m and any formula group $A_1, A_2, ..., A_m \in F(S)$,

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_m > = < A_1 \wedge A_2 \wedge \cdots \wedge A_m > .$$

Proof According to the definition of strong Q-logic, for all formulas $A_1, A_2 \in F(S)$, we have

$$< A_1 > \wedge < A_2 > = < A_1 \wedge A_2 > .$$

For natural number k and any formula group

$$A_1, A_2, \cdots, A_k \in F(S),$$

suppose

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_k > = < A_1 \wedge A_2 \wedge \cdots \wedge A_k > .$$

Then we will prove that for natural number k+1 and any formula group $A_1, A_2, ..., A_k, A_{k+1} \in F(S)$,

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_k > \wedge < A_{k+1} > \\ = < A_1 \wedge A_2 \wedge \cdots \wedge A_k \wedge A_{k+1} > .$$

In fact,

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_k > \wedge < A_{k+1} > \\ = < A_1 \wedge A_2 \wedge \cdots \wedge A_k > \wedge < A_{k+1} > \\ = < A_1 \wedge A_2 \wedge \cdots \wedge A_k \wedge A_{k+1} > .$$

According to mathematic induction, for each natural number m and any formula group $A_1, A_2, \cdots, A_m \in F(S)$,

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_m > = < A_1 \wedge A_2 \wedge \cdots \wedge A_m > .$$

Obviously, the following converse theorem also exists.

Theorem 2.2 CQ = (C^, C*, Q) is strong Q-logic if regular Q-logic CQ satisfies

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_m > = < A_1 \wedge A_2 \wedge \cdots \wedge A_m >$$

for each natural number m and any formula group

$$A_1, A_2, \cdots, A_m \in F(S).$$

Consider multidimensional approximate reasoning model in strong Q-logic CQ,

$$\begin{array}{l} A_1, A_2, \cdots, A_m \to B \\ A_1^*, A_2^*, \cdots, A_m^* \\ \hline \qquad\qquad B^* \end{array}$$

that is

$$\begin{array}{l} A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B \\ A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \\ \hline \qquad\qquad B^* \end{array}$$

where

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B \in \Phi^{\perp}$$

is called *multidimensional knowledge*;

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in F(S)$$

is called *multidimensional input*; $B^* \in F(S)$ is called *output* or *approximate reasoning conclusion*.

Suppose

$$A_1^* \in < A_1 >, A_2^* \in < A_2 >, \cdots, A_m^* \in < A_m >,$$

that is

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in <A_1> \wedge <A_2> \wedge \cdots \wedge <A_m>.$$

Then we say that multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

can activate multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B.$$

Take

$$B^* \in <B>$$

as approximate reasoning conclusion, which is called multidimensional reasoning type V strong Q-solution with regard to multidimensional input $A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$ under multidimensional knowledge $A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B.$

If there exists an $i \in \{1,2,\cdots,m\}$ satisfying

$$A_i^* \notin <A_i>,$$

that is

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \notin <A_1> \wedge <A_2> \wedge \cdots \wedge <A_m>.$$

It implies that multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

can not activate multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B.$$

Hence, we do not care multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*.$$

It demonstrates there is no type V strong Q-solution to multidimensional approximate reasoning with regard to multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

under multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B.$$

This algorithm is called multidimensional approximate reasoning *type V strong Q-algorithm*.

Theorem 2.3 In strong Q-logic CQ, there exists a type V strong Q-solution of it with regard to multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

under multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B,$$

if and only if

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in <A_1 \wedge A_2 \wedge \cdots \wedge A_m>.$$

Proof According to the construction of strong Q-logic CQ, we know

$$<A_1> \wedge <A_2> \wedge \cdots \wedge <A_m> = <A_1 \wedge A_2 \wedge \cdots \wedge A_m>,$$

and because

$$A_1^* \in <A_1>, A_2^* \in <A_2>, \cdots, A_m^* \in <A_m>$$

if and only if

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in <A_1> \wedge <A_2> \wedge \cdots \wedge <A_m>,$$

so, there exists a type V strong Q-solution of multidimensional approximate reasoning with regards to multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

under knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B$$

if and only if

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in <A_1 \wedge A_2 \wedge \cdots \wedge A_m>.$$

Theorem 2.4 In strong Q-logic CQ, if there exists a type V strong Q-solution B^* of multidimensional approximate reasoning with regard to multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

under multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B,$$

we have

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \to B^* \in <A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B>,$$

which implies the multidimensional knowledge

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \to B^*$$

is Q-similar to multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B.$$

Proof Because B^* is the type V strong Q-solution of multidimensional approximate reasoning with regard to multidimensional input

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^*$$

under multidimensional knowledge

$$A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B,$$

we have

$$A_1^* \in <A_1>, A_2^* \in <A_2>, \cdots, A_m^* \in <A_m>, B^* \in <B>,$$

that is

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in < A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_m >$$

and $B^* \in < B >$. But, because of the construction of strong *Q*-logic CQ, we know

$$< A_1 > \wedge < A_2 > \wedge \cdots \wedge < A_m > = < A_1 \wedge A_2 \wedge \cdots \wedge A_m >.$$

So

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \in < A_1 \wedge A_2 \wedge \cdots \wedge A_m >, B^* \in < B >.$$

Therefore

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \to B^* \in < A_1 \wedge A_2 \wedge \cdots \wedge A_m > \to < B >.$$

And because of the construction of strong Q-logic CQ, we know

$$< A_1 \wedge A_2 \wedge \cdots \wedge A_m > \to < B > = < A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B >.$$

Therefore,

$$A_1^* \wedge A_2^* \wedge \cdots \wedge A_m^* \to B^* \in < A_1 \wedge A_2 \wedge \cdots \wedge A_m \to B >$$

3. References

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