Simulation and Qualitative Analysis of a Single Mode Semiconductor Laser Model

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Abstract - In this paper the dynamics of a single-mode semiconductor laser model is investigated. The nonlinear rate equations describe the dynamic evolution of semiconductor laser. We study transients occurring in the system using numerical simulation. A fourth order Runge-Kutta method is used to calculate two coupled first-order differential equations. Changes in the pumping parameter of the system can lead to drastic changes of the character of the solutions. Qualitative analysis of the system is used for investigating the typical bifurcations and allowed conditions for the existence of damped oscillations in the system.

Index Terms – simulation, model, rate equation, dynamic systems, stability.

1. Introduction

Over the past decade instabilities in laser have become a widely discussed topic in quantum optics [1,2]. To understand the origin of such behaviour is an effort that continues today. Semiconductor lasers were developed in 1962 [3]. For the last 30 years they are widely used for transmission, recording and reading information, optical pumping. Their main distinctive features are high efficiency, small size, the possibility of modulating the output of the optical characteristics by an injection current change, compactness, relatively low cost, wide choice of wavelength, high efficiency, the ability to achieve high-speed data transmission. A significant progress in the development of semiconductor lasers came in the late 60's early 70's of the twentieth century, when the idea of using semiconductor heterostructures as an active region of the laser was used to achieve low-threshold lasing at room temperature[4]. Because of these properties semiconductor lasers based on gallium arsenide and other compounds (AlGaAs, InP, InGaAsP) nowadays are widely used in various fields of technology, such as optical fibre communication systems.

Semiconductor laser is an excellent example of nonlinear dynamic systems. To construct the different electronic systems, including a semiconductor laser as a component element, it is necessary to analyse its dynamics and transients. To get an overview of possible behaviour we can resort to numerical calculations. Another possible method to predict the nonlinear dynamic behaviour of laser is the use of qualitative analysis. In the present paper we consider typical single-mode laser rate equations. The coupled rate equations are nonlinear, so that they may under certain conditions exhibit spiking and relaxation oscillations as the laser approaches its steady state. A single mode balanced model provides certain qualitative and quantitative information on the number of practically important characteristics of the radiation, which is very important for the preliminary design of optical systems.

2. The Model

The dynamical behaviour of semiconductor lasers can be described by a set of coupled first order differential equations [5], relating the carrier concentration N and photon density S in the active region:

$$\frac{dN}{dt} = P - G(N - N_{th}) - \frac{N}{t_e} \qquad (1)$$

$$\frac{dS}{dt} = G(N - N_{th}) - \frac{S}{t_p} + b\frac{N}{t_e}, \qquad (2)$$

where t_e and t_p are the carrier and photon life times, P is the pumping rate , which is equal to $\frac{I}{qV}$ (I is the injection current, q is the electron charge, V is the volume of active region), N_{th} is the carrier density for transparency (the electron density above which the lasing gain becomes positive), $G(N - N_{th})$ is the amplification rate due to the stimulated emission, b is the spontaneous emission factor.

Equation (1) represents the rate of increase in the concentration of the carriers. It is enhanced by term pumping rate and decreased by term of the rate of losses of carriers during the spontaneous transition and losses due to stimulated transition. The rate of increase in the density of photons (2) is enhanced by the loss of photons in the resonator and the rate of spontaneous emission of photons in the laser mode and decreased by term of the rate of production of photons due to stimulated emission.

3. Discussion and Results

A. Numerical Solution

To analyze the dynamic behaviour of a single-mode semiconductor laser we study the temporal evolution of the system by integrating the rate equations (1,2). It is not generally possible to know analytic solutions for nonlinear systems. For solution of coupled differential equations, formulated in this paper, there are many methods. The main of them are based on the Taylor series expansion, and are usually limited to the algorithms of the Runge-Kutta method. Without describing variety of methods, we note that the most common and acceptable accuracy for a given class of problems is the Runge-Kutta fourth order method. This method achieves higher accuracy than the methods of the lowest order, but enough not to use the methods of higher order [6,7].

The general features can be understood with the help of Fig. 1 that shows response of the rate equations for semiconductor laser. The typical values of the different parameters used in the calculations are given in Table I.

Initially, the carrier density N raises steadily, after reaching its critical value (threshold), a sharp drop is observed, which in turn corresponds to the jump in the photon density S (see Fig. 1). From that moment, a coherent light emission starts. Then, the carrier density N starts to grow up again whereas s drops respectively. In this case critical value of N is not so high, and S is not reduced so low. These relaxation oscillations occur as long as N and S do not reach a steady state. The laser spikes are steep and narrow because of the rapid rates of rise and fall of the photon number in the cavity.

TABLE I

PARAMETERS USED IN NUMERICAL SIMULATION

Parameter	Meaning	Value	Units
N_{th}	trasparency carrier density	10^{24}	m^{-3}
t_e	carriers lifetime	$3 \cdot 10^{-9}$	s
t_p	photons lifetime	$1 \cdot 10^{-12}$	s
b	spontaneous emission factor	10^{-5}	
V	Modal volume	$3.36 \cdot 10^{-17}$	m^3

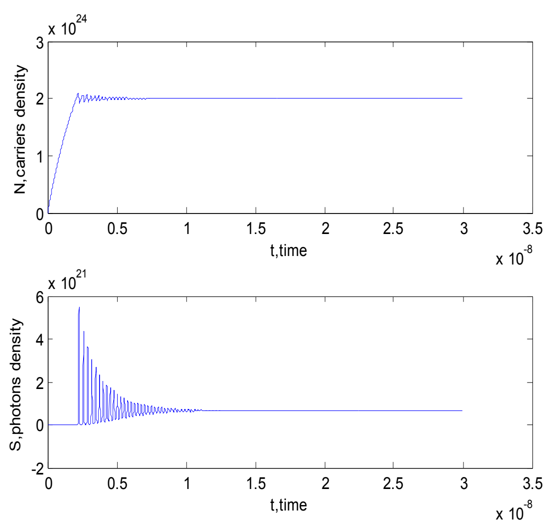

Fig. 1 Time response behavior of laser rate equations

Large and rapid changes of amplitude of photon numbers can be also observed in the phase portrait (see Fig. 2). In this plane we can follow the oscillatory trajectory that circles to convergence steady state point.

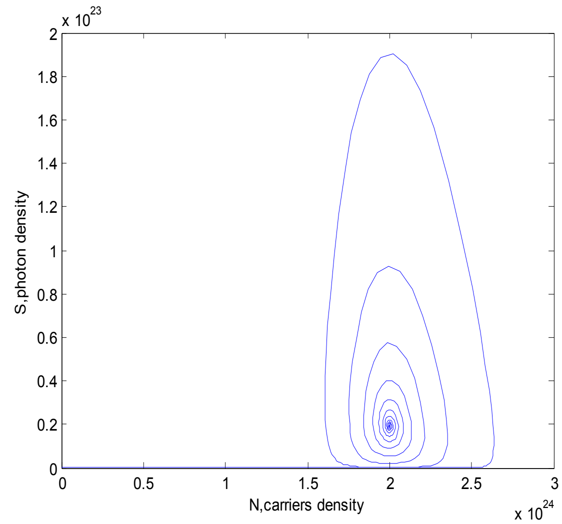

Fig. 2 Phase plane describing the laser spiking

Let’s examine how the change in the pumping parameter affects the system. The trajectory of the system after reducing the value of the pump current to $0.03 \cdot Ith$ is shown in Fig. 3. In this case N does not achieve the value, sufficient to start the lasing with current value of the pump (below the threshold). Hence, the corresponding increase in the number of photons (population inversion) is not occurred. Photon density is equal to zero.

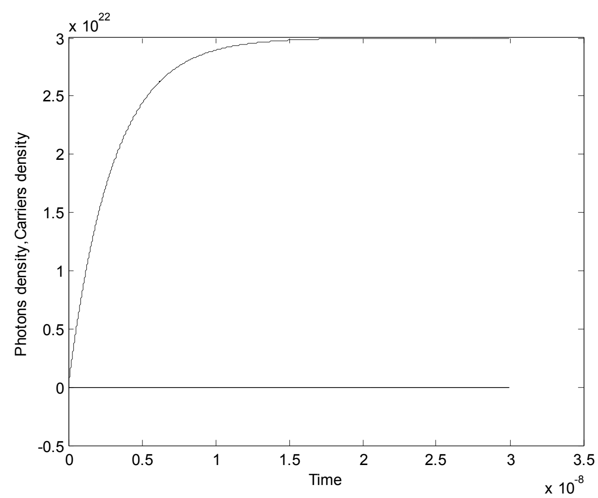

Fig. 3 Change in photons and carriers densities at the value of the pump current $I = 0.03It_th$.

It can also be established that increasing the pump current in the model increases the frequency of the relaxation oscillations and reduces the turn on delay time. The system responses for different values of the pump current are shown in Fig. 4 and Fig. 5.

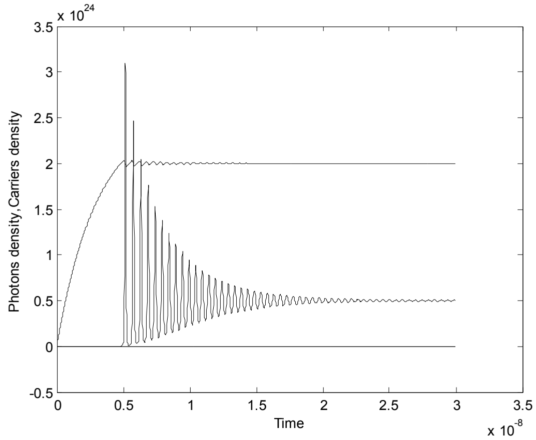

Fig. 4 Temporal evolution of carrier and photon density at $I = 2.5 \cdot Ith$.

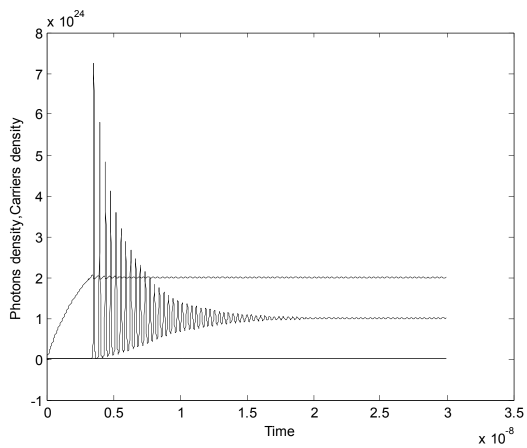

Fig. 5 Temporal evolution of carrier and photon density at $I = 3 \cdot I_t h$.

B. Bifurcation Analysis

Let us now examine the stability of the system using qualitative analysis.

Solutions of nonlinear differential equations depend on their parameters, like solutions of linear differential equations. There may be critical parameter values, at which the character of the solution changes completely. For this reason, it is important to know the qualitative behaviour of the solutions and examine stability boundaries of equilibria.

In usual semiconductor lasers, such as for a laser based on GaAs with emission wavelength near 850 nm, or InGaAsP with emission wavelength near 1310 nm in devices with cavity length L=300 μm the spontaneous emission factor β is on the order of 10^{-4}, 10^{-5} [8], so that in first approximation we can neglect corresponding term in the equation and consider following equations:

$$\frac{dN}{dt} = P - G(N - N_{th}) - \frac{N}{t_e} \tag{3}$$

$$\frac{dS}{dt} = G(N - N_{th}) - \frac{S}{t_p}. \tag{4}$$

To simplify the system of equations (3,4) we introduce the following dimensionless variables:

$$s = Sgt_e, n = Gt_p(N - N_{th}) - 1, \ p = PGt_e t_p - 1.$$

Substituting the new variables in the equations, we obtain the following equations in dimensionless form. (Dots over the variables denote derivatives over time t)

$$\dot{s} = \frac{ns}{t_p} \tag{5}$$

$$\dot{n} = \frac{p - (n+1)s - n}{t_e}. \tag{6}$$

System has two trivial steady- state solutions:

$$s_1^* = 0, \ n_1^* = p \tag{7}$$

$$s_2^* = p, \ n_2^* = 0. \tag{8}$$

To investigate the dependence of the parameter p, it is necessary to linearize the system about the steady points. To classify them, we compute the jacobian matrix:

$$\mathrm{J} = \begin{bmatrix} \frac{\partial \dot{s}}{\partial s} & \frac{\partial \dot{s}}{\partial n} \\ \frac{\partial \dot{n}}{\partial s} & \frac{\partial \dot{n}}{\partial n} \end{bmatrix} = \begin{bmatrix} \frac{n}{t_p} & \frac{s}{t_p} \\ \frac{-(n+1)}{t_e} & \frac{-(s+1)}{t_e} \end{bmatrix}.$$

Steady state (7) is a saddle since eigenvalues of the jacobian matrix J of equations (5,6), being the roots of the characteristic polynomial $\det(\mathrm{J} - \lambda I) = \lambda^2 - \lambda tr\mathrm{J} - \det \mathrm{J}$, have opposite signs:

$$\lambda_1 = -\frac{1}{t_e}, \ \lambda_2 = \frac{p}{t_p}.$$

Steady state (8) is a stable focus because

$$\lambda_{1,2} = \frac{-p - 1 \pm \sqrt{(p+1)^2 - 4pt_e / t_p}}{2t_e}, \quad \det \mathrm{J}>0.$$

The angular frequency of relaxation oscillations is defined as:

$$\omega = \mathrm{Im}\, \lambda = \sqrt{\frac{4pt_e / t_p - (p+1)^2}{2t_e}}.$$

Thus, the stability of steady states changes when p passes through 0, so that the system undergoes a transcritical bifurcation.

For steady state (8) to be a focus (corresponding to damped oscillations), the discriminant of the quadratic characteristic equation has to be negative:

$$\Delta = (tr\mathrm{J})^2 - 4\det \mathrm{J} = \sqrt{b^2 - 4ac}$$

$$= \left[\frac{-(p+1)}{t_e}\right]^2 - 4\frac{p}{t_e t_p} =$$

$$= (p^2 - 2p(2t_e / t_p - 1) + 1) / t_e^{\,2} < 0.$$

This inequality is equivalent to following conditions:

$$\begin{cases} \frac{2}{a} - 1 - \frac{2}{a}\sqrt{1-a} < p < \frac{2}{a} - 1 + \frac{2}{a}\sqrt{1-a}, \\ a < 1 \end{cases}$$

here $a = \frac{t_p}{t_e}$.

Hence, the existence of oscillations requires the rate of spontaneous recombination to be slower than the photon loss rate. It also imposes restriction on the intensity of pump. The allowed range of p is wide when $a \ll 1$: $0 < p < 4/a$ .It shrinks to $1 - 2\sqrt{1-a} < p < 1 + 2\sqrt{1-a}$ as a gets closer to 1.

4. Conclusion

A single-mode semiconductor laser simulator based on coupled differential equations was considered. A model, which represents the rate equations, was built using a fourth order Runge - Kutta method. The temporal evolutions of photon and carrier densities of the rate equations show the transients occurring in the system. The inversely proportional relation between the turn on delay time and the injected current over threshold are demonstrated, so that the higher applied current leads to the earlier starting of lasing. As a result of the simulation was also point out that as the current pump increases, the duration of the transition process decreases. The stability of the system and the conditions for the existence of damped oscillations was described using qualitative analysis; the phase plane obtained for current system verifies the oscillatory behaviour of a single-mode semiconductor laser.

5. References

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