On designing state feedback controller for time-delay system of neutral type*

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Abstract - This paper makes the linear uncertain neutral system as the research object, and designs state feedback observer, achieving the observed state. And the paper designs an observer based on state feedback controller, to control the neutral system. And then we make simulation through Matlab Simulink toolbox, verifying the effectiveness of the algorithm. In addition, by using cone complementarity algorithm, the solutions to the observer gain matrices are obtained by solving a set of linear matrix inequalities (LMIs).

Index Terms - neutral system, the feedback controller, linear matrix inequality (LMI)

1. Introduction

In recent decades, the stability problems of time-delay neutral system have been brought great attention by many scholars at home and abroad, especially the analysis of feedback controlling about system. And the feedback controlling has become one of the very active researches in automatic controlling [1-3]. In the past two years, the feedback controlling delay system is used as separate chapters in controlling theory monograph by writers [4-6].

Neutral time-delay system is a special time-delay system; it can accurately reflect the laws of things' changes, revealing the nature of things. [7] discussed a problem of a varying delay neutral system of delay-dependent asymptotic stability. [8] studied a class of problem of Lurie controlling systems with time-varying asymptotic stability and H ∞ state feedback controller, by using Lyapunov function and linear matrix inequalities. This article places uncertain neutral system as the research object, and gives the designed method of the state feedback observer, then designs the controller based on the observer, controlling the neutral system and using Matlab simulation to verify its feasibility.

2. Description of the system

Establish the following Linear Uncertain Neutral Systems:

$$\begin{cases} \dot{x}(t) = (A+\Delta A)x(t) + A_h x(t-h) \\ \quad + A_d \dot{x}(t-h) + Bu(t) \\ y(t) = Cx(t) \\ x(t) = \phi(t) \end{cases} \tag{1}$$

Where $x(t) \in R^n$, and it is the state vector of the system, and $u(t) \in R^m$ is the controlling input vector, $y(t) \in R^q$ is the measure of the output vector: A , A_h , A_d , B , C are in line with the requirements dimension real constant matrix; h is a positive constant and indicates the number of time lag, $\phi(t)$ is the given real-valued differentiable function with the continuity, and $\phi(t)$ is the system initial conditions. ΔA is function of the real matrix, expressing varying parameter uncertainty.

Parameters are assumed as uncertainties, ΔA satisfies the conditions

$$\Delta A(t) = DF(t)E$$

Where D , E are appropriate dimensions constant matrixs, $F(t)$ is an unknown variable matrix, and $F^T(t)F(t) \le I$, where I is the unit matrix of the appropriate dimensions. The purpose of this article is to design a state observer for (1), like form (2).

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_h\hat{x}(t-h) + A_d\dot{\hat{x}}(t-h) + Bu(t) \\ \quad + L\left[y(t) - \hat{y}(t)\right] + L_h\left[y_h - \hat{y}_h\right] \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{2}$$

Where

$$y_h = y(t-h) \quad \hat{y}_h = \hat{y}(t-h)$$

L , L_h are observer gain matrixes.

Such that the following error dynamic system is asymptotically stable, so as to achieve the state, which can be observed.

Define the estimation error as

$$e(t) = x(t) - \hat{x}(t)$$

From the equation (1) and (2), there is the error dynamic system:

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$$\dot{e}(t) = (A + \Delta A - LC)e(t) + (A_h - L_h C)e(t-h) + A_d \dot{e}(t-h) \quad (3)$$

For the error system (3)

$$\frac{d}{dt}\{\dot{e}(t) - A_d \dot{e}(t-h)\} = \overline{A}e(t) + \overline{A_h}e(t-h) \quad (4)$$

Which $\overline{A} = A + \Delta A - LC \quad \overline{A_h} = A_h - L_h C$

3. Main conclusions and certification

Theorem 1 If there is a positive definite matrix R, Q and rectangular matrices W, so that the following LMIs are established

$$\begin{bmatrix} \Sigma & RA_h - W_h C - A^T RA_d + C^T W^T RA_d & \sqrt{2}RD \\ * & -Q & 0 \\ * & 0 & -I \end{bmatrix} < 0 \quad (5)$$

$$Q + A_d^T E^T EA_d - A_h^T RA_d + C^T W_h^T A_d - A_d^T RA_h + A_d^T W_h C > 0 \quad (6)$$

Where

$$\Sigma = Q + A_d^T E^T EA_d - A_h^T RA_d - A_d^T RA_h + C^T W_h^T A_d + A_d^T W_h C + E^T E + A^T R + RA - C^T W^T - WC$$

The error system (3) is asymptotically stable, and the observer gain matrix is the following

$$L = R^{-1}W \quad L_h = R^{-1}W_h$$

Proof: Define g function:

$$g(e_t) = e(t) - A_d e(t-h) \quad (7)$$

Take

$$V_1(e_t) = g^T(e_t) R g(e_t) \quad (8)$$

$$V_2(e_t) = \int_{t-h}^{t} e^T(s)[Q + A_d^T E^T EA_d - \overline{A}_h^T RA_d - A_d^T R\overline{A}_h] e(s)ds$$

First, in order to guarantee the positive definiteness of V requires R> 0,

$$Q + A_d^T EEA_d - \overline{A}_h^T RA_d - A_d^T R\overline{A}_h > 0 \quad (9)$$

Taking the derivative of V under the conditions of the error system formula (4),

$$\dot{V}_1 = \dot{g}^T(e_t) R g(e_t) + g^T(e_t) R \dot{g}(e_t)$$

$$= \left[\overline{A}e(t) + \overline{A_h}e(t-h)\right]^T R\left[e(t) - A_d e(t-h)\right] + \left[e(t) - A_d e(t-h)\right]^T R\left[\overline{A}e(t) + \overline{A_h}e(t-h)\right]$$

$$= e^T(t)\left(\overline{A}^T R + R\overline{A}\right)e(t) + 2e^T(t)\begin{pmatrix} R\overline{A_h} - \\ \overline{A}^T RA_d \end{pmatrix} e(t-h) - e^T(t-h)\left[\overline{A_h}^T RA_d + A_d^T R\overline{A_h}\right]e(t-h)$$

$$= e^T(t)\left[(A + \Delta A - LC)^T R + R(A + \Delta A - LC)\right]e(t) + 2e^T(t)\left[\left(R\overline{A_h} - (A + \Delta A - LC)^T RA_d\right)\right]e(t-h) - e^T(t-h)\left[\overline{A_h}^T RA_d + A_d^T R\overline{A_h}\right]e(t-h)$$

$$= e^T(t)\begin{bmatrix} \Delta A^T R + A^T R + RA + R\Delta A - \\ C^T L^T R - RLC \end{bmatrix} e(t) + 2e^T(t)\begin{bmatrix} R\overline{A_h} - \Delta A^T RA_d - A^T RA_d \\ +C^T L^T RA_d \end{bmatrix} e(t-h) - e^T(t-h)\left[\overline{A_h}^T RA_d + A_d^T R\overline{A_h}\right]e(t-h)$$

$$= 2e^T(t) R\Delta A e(t) + e^T(t)\begin{pmatrix} A^T R + RA - \\ C^T L^T R - RLC \end{pmatrix} e(t) + 2e^T(t)\left[R\overline{A_h} - A^T RA_d + C^T L^T RA_d\right]e(t-h) - 2e^T(t) R\Delta A A_d e(t-h) - e^T(t-h)\left[\overline{A_h}^T RA_d + A_d^T R\overline{A_h}\right]e(t-h)$$

$$\le e^T(t) RDF(t)F^T(t)D^T Re(t) + e^T(t)E^T Ee(t) + e^T(t)\left(A^T R + RA - C^T L^T R - RLC\right)e(t) + 2e^T(t)\left[R\overline{A_h} - A^T RA_d + C^T L^T RA_d\right]e(t-h) + e^T(t) RDF(t)F^T(t)D^T Re(t) + e^T(t-h)A_d^T E^T EA_d e(t-h) - e^T(t-h)\left[\overline{A_h}^T RA_d + A_d^T R\overline{A_h}\right]e(t-h)$$

$$\le e^T(t)\begin{pmatrix} 2RDD^T R + E^T E + A^T R \\ +RA - C^T L^T R - RLC \end{pmatrix} e(t) + e^T(t)\begin{bmatrix} R\overline{A_h} - A^T RA_d \\ +C^T L^T RA_d \end{bmatrix} Q^{-1} \begin{bmatrix} R\overline{A_h} - A^T RA_d \\ +C^T L^T RA_d \end{bmatrix}^T e(t) + e^T(t-h)(Q + A_d^T E^T EA_d - \overline{A_h}^T RA_d - A_d^T R\overline{A_h})e(t-h)$$

$$\dot{V}_2 = e^T(t)[Q + A_d^T E^T E A_d - \bar{A}_h^T R A_d - A_d^T R \bar{A}_h) e(t)$$
$$-e^T(t-h)(Q + A_d^T E^T E A_d - \bar{A}_h^T R A_d - A_d^T R \bar{A}_h) e(t-h)$$

Therefore

$$\dot{V}(e_t) \le e^T(t)\Omega e(t)$$

Where

$$\Omega = A^T R + RA + E^T E + 2RDD^T R + Q + A_d^T E^T E A_d - \bar{A}_h^T R A_d - A_d^T R \bar{A}_h - C^T L^T R - RLC + \begin{pmatrix} R\bar{A}_h - A^T R A_d \\ + C^T L^T R A_d \end{pmatrix} Q^{-1} \begin{pmatrix} R\bar{A}_h - A^T R A_d \\ + C^T L^T R A_d \end{pmatrix}^T$$

By schur complement, it is equivalent to the following form:

$$\begin{bmatrix} \Theta & R\bar{A}_h - \bar{A}^T R A_d + C^T L^T R A_d & \sqrt{2}RD \\ * & -Q & 0 \\ * & 0 & -I \end{bmatrix} < 0 \qquad (10)$$

Where

$$\Theta = A^T R + RA + E^T E + Q + A_d^T E^T E A_d - \bar{A}_h^T R A_d - A_d^T R \bar{A}_h - C^T L^T R - RLC$$

After a simple deformation matrix inequality, (10) is equivalent to the linear matrix inequalities (5).And the inequality (9) is equivalent to the following inequality

$$Q + A_d^T E E A_d - (A_h - L_h C)^T R A_d - A_d^T R (A_h - L_h C) > 0$$

That is

$$Q + A_d^T E E A_d - A_h^T R A_d A_d - A_d^T R A_h + C^T W_h^T A_d + A_d^T W_h C > 0 \qquad (11)$$

Where W = RL, theorem is proved.

4. Simulation

Base on the theorem 1 given following simulation examples, it verifies that the criterion in Theorem 1 is feasible.
The example 1 Neutral system can be selected as

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + A_1 x(t - h(t))$$

Where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad A_1 = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 0 \\ 0 & 5 \end{bmatrix}$$

Thus by the contents of the Theorem 1, we obtain

$$Q = \begin{bmatrix} 45.7416 & 1.4751 \\ 1.4751 & 51.8909 \end{bmatrix} R = \begin{bmatrix} 45.2452 & 0.3747 \\ 0.3747 & 25.6752 \end{bmatrix}$$

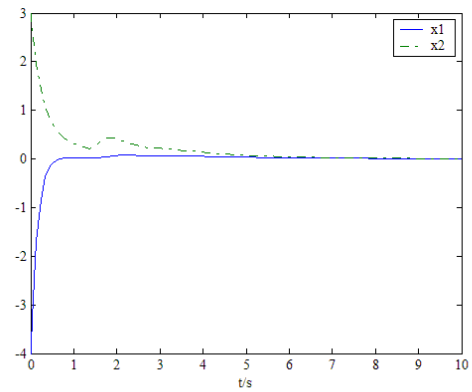

Figure 1 The simulation curves of the state of the system

Figure 1 illustrates the system state will soon be able to reach a steady state, so the criterion of theorem 1 is feasible. That is, by using the effect of observer and controller in theorem1, we can control the system, making itself from unstable to stable.

5. Summary

In this paper, we use uncertain neutral system as the research object, and give design of observer of the neutral system. And then we design controller by observer, finally control the system. All conclusions are given in the form of the LMI, and we simulate through Matlab Simulink toolbox, to verify the effectiveness of the algorithm.

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