Application of Fuzzy Linear Programming to Transportation Planning Decision Problems with Multiple Fuzzy Goals

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Abstract

This work develops a fuzzy linear programming (FLP) method for solving the transportation planning decision (TPD) problems with fuzzy goals, available supply and forecast demand. The proposed method attempts to minimize the total production and transportation costs and the total delivery time with reference to available supply and machine capacities at each source, as well as forecast demand and warehouse space constraints at each destination. An industrial case is used to demonstrate the feasibility of applying the FLP method to a real TPD problem.

Keywords: Transportation planning decisions; Fuzzy linear programming; Fuzzy set theory

1. Introduction

In real-world transportation planning decision (TPD) problems, input data and related parameters, such as available supply and forecast demand, are often imprecise/fuzzy because some information is incomplete or unobtainable. Also, the decision maker (DM) must simultaneously handle conflicting goals that govern the use of the constrained resources within organizations. Particularly, these conflicting goals are required to be optimized simultaneously by the DM, frequently in the framework of fuzzy aspiration levels (Li and Lai, 2000; Abd El-Wahed, 2006). Obviously, conventional LP method and existing solution algorithms cannot solve all TPD programming problems in uncertain environments.

Zimmermann (1976) first introduced fuzzy set theory into the ordinary LP and the multi-objective linear programming (MOLP) problems with fuzzy goals and constraints. Subsequently, Zimmermann's fuzzy linear programming (FLP) has developed into several fuzzy optimization methods to solve TPD problems. Chanas *et al.* (1984) presented an FLP model for solving the TPD problems with crisp cost coefficients and fuzzy supply and demand. Moreover, Chanas and Kuchta (1996) proposed the concept of the optimal solution of the TPD problem with fuzzy coefficients expressed as L-R fuzzy numbers, and developed an algorithm for determining the solution. Furthermore, Li and Lai (2000) designed a fuzzy compromise programming method to obtain a non-dominated compromise solution for fuzzy TPD problems with multiple objectives in which various objectives were synthetically considered with the marginal evaluation for individual objectives and the global evaluation for all objectives. More recently, Abd El-Wahed (2006) designed a fuzzy programming approach to determine the optimal compromise solution of a multi-objective TPD problem by measuring the degree of closeness of the compromise solution to the ideal solution using a family of distance functions.

This work develops an interactive FLP method for solving multi-objective TPD problems with fuzzy goals, available supply and forecast demand. The proposed method attempts simultaneously to minimize the total production and transportation costs and the total delivery

2. Problem formulation

2.1. Problem description and notation

Assume that a logistics center seeks to determine the transportation plan of a homogeneous commodity from m sources to n destinations. Each source has an available supply of the commodity to distribute to various destinations, and each destination has a forecast demand of the commodity to be received from various sources. This work focuses on developing an FLP method for optimizing the transportation plan in fuzzy environments.

The following notation is used.

- *Index sets*
  - i index for source, for all $i = 1, 2, \ldots, m$
  - j index for destination, for all $j = 1, 2, \ldots, n$
  - g index for objectives, for all $g = 1, 2, \ldots, k$
- *Decision variables*
  - Q_{ij} units transported from source i to destination j (units)
- *Objective functions*
  - z_1 total production and transportation costs ($)
  - z_2 total delivery time (hours)
- *Parameters*
  - p_{ij} production cost per unit delivered from source i to destination j ($/unit)

c_{ij}	transportation cost per unit delivered from source i to destination j ($/unit)
t_{ij}	transportation time per unit delivered from source i to destination j ($/unit)
$\widetilde{S}_i$	total available supply for each source i (units)
$\widetilde{D}_j$	total forecast demand of each destination j (units)
a_{ij}	hours of machine usage per unit produced by each source i (machine-hour/unit)
$M_{i\max}$	maximum machine capacities available for each source i (machine-hour)
B	total budget ($)
b_{ij}	warehouse space per unit delivered from source i to destination j (ft^2 /unit)
$W_{i\max}$	maximum warehouse space available for each destination j (ft^2)

2.2. Fuzzy multi-objective linear programming (FMOLP) model

2.2.1. Objective functions

- *Minimize total production and transportation costs*

$$Min\ z_1 \cong \sum_{i=1}^{m}\sum_{j=1}^{n}(p_{ij} + c_{ij})Q_{ij} \tag{1}$$

- *Minimize total delivery time*

$$Min\ z_2 \cong \sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij}Q_{ij} \tag{2}$$

The symbol ‘ $\cong$ ’ is the fuzzified version of ‘ $=$ ’ and refers to the fuzzification of the aspiration levels.

2.2.2. Constraints

- Constraints on total available supply for each source i

$$\sum_{j=1}^{n} Q_{ij} \le \widetilde{S}_i \qquad \forall i \tag{3}$$

- Constraints on total forecast demand for each destination j

$$\sum_{i=1}^{m} Q_{ij} \ge \widetilde{D}_j \qquad \forall j \tag{4}$$

- Constraints on total budget

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(p_{ij} + c_{ij})Q_{ij} \le B \tag{5}$$

- Constraints on machine capacities for each source i

$$\sum_{j=1}^{n} a_{ij}Q_{ij} \le M_{i\max} \qquad \forall i \tag{6}$$

- Constraints on warehouse space for each destination j

$$\sum_{i=1}^{m} b_{ij}Q_{ij} \le W_{j\max} \qquad \forall j \tag{7}$$

- Non-negativity constraints on decision variables

$$Q_{ij} \ge 0 \qquad \forall i,\ \forall j \tag{8}$$

3. Model development

3.1. Membership functions

The FLP approach developed herein exhibits greater computational efficiency and flexibility of the fuzzy arithmetic operations by employing the linear membership functions to represent fuzzy numbers for solving the multi-objective TPD problem in a fuzzy environment. The corresponding non-increasing continuous linear membership functions for all fuzzy objective functions can be formulated as follows.

$$f_g(z_g) = \begin{cases} 1 & z_g \le z_g^l \\ \dfrac{z_g^u - z_g}{z_g^u - z_g^l} & z_g^l < z_g < z_g^u \qquad g = 1, 2, \ldots, k \\ 0 & z_g \ge z_g^u \end{cases} \tag{9}$$

where z_g^l and z_g^u, $g = 1, 2, \ldots, k$, are the lower and upper bounds, respectively, of the gth objective function z_g.

Moreover, the corresponding non-increasing continuous linear membership functions for the fuzzy constraint (3) can be defined as follows.

$$f_i(H_i) = \begin{cases} 1 & H_i \le S_i^l \\ \dfrac{S_i^u - H_i}{S_i^u - S_i^l} & S_i^l < H_i < S_i^u \qquad i = 1, 2, \ldots, m \\ 0 & H_i \ge S_i^u \end{cases} \tag{10}$$

where $H_i = \sum_{j=1}^{n} Q_{ij}$, $i = 1, 2, \ldots, m$; S_i^l and S_i^u , $i = 1, 2, \ldots, m$, are the lower and upper bounds of the fuzzy resources, respectively, of the ith fuzzy inequality constraint.

Similarly, the corresponding non-decreasing continuous linear membership functions for the fuzzy constraint (4) can be defined as follows.

$$f_j(V_j) = \begin{cases} 1 & V_j \geq D_j^u \\ \dfrac{V_j - D_j^l}{D_j^u - D_j^l} & D_j^l < V_j < D_j^u \quad j = 1, 2, \ldots, n \\ 0 & V_j \leq D_j^l \end{cases} \tag{11}$$

where $V_j = \sum_{i=1}^{m} Q_{ij}$, $j = 1, 2, \ldots, n$; D_j^u and D_j^l , $j = 1, 2, \ldots, n$, are the upper and lower bounds of the fuzzy resources, respectively, of the jth fuzzy inequality constraint. The pattern of the non-increasing continuous linear membership functions $f_j(V_j)$ is similar to $f_g(z_g)$.

3.2. Solving the fuzzy MOLP problem

The minimum operator is used to aggregate all fuzzy sets. Introducing the auxiliary variable L enables the original FMOLP problem to be converted into an equivalent ordinary LP form. Consequently, the complete equivalent LP model for solving the fuzzy TPD problem can be formulated as follows.

$$\begin{aligned} & Max\ L \\ \text{s.t.}\quad & L \leq f_g(z_g) \quad \forall g \\ & L \leq f_i(H_i) \quad \forall i \\ & L \leq f_j(V_j) \quad \forall j \\ & \text{Eqs. (5) to (8)} \end{aligned}$$

3.3. Solution procedure

Step 1. Formulate the original FMOLP model for the TPD problems according to Eqs. (1) to (8).

Step 2. Specify the corresponding linear membership functions for all of the fuzzy objective functions and the fuzzy inequality constraints using Eqs. (9) and (11).

Step 3. Introduce the auxiliary variable L, and then transform the original FMOLP problem into an equivalent ordinary LP form using the minimum operator to aggregate all fuzzy sets.

Step 4. Solve the ordinary LP problem and obtain the initial compromise solution. If the DM is dissatisfied with the initial solution, the model should be modified until a satisfactory solution is obtained.

4. Implementation

4.1. Data description

Dali Company was used as a case study demonstrating the practicality of the proposed methodology. Table 1 lists the basic transportation data for the Dali case for the coming season. Related production data for all factories are listed in Table 2. The total budget is $300,000. The maximum warehouse space for four distribution centers is as follows: 4,000 ft^2 (Taichung), 1,700 ft^2 (Haulien), 5,000 ft^2 (Kaohsiung), and 5,800 ft^2 (Taipei).

4.2. Solution procedure for the Dali case

The corresponding non-increasing continuous linear membership functions for the objective functions can be defined using Eq. (9), as below.

$$f_1(z_1) = \begin{cases} 1 & z_1 \leq 240{,}000 \\ \dfrac{800{,}000 - z_1}{560{,}000} & 240{,}000 < z_1 < 800{,}000 \\ 0 & z_1 \geq 800{,}000 \end{cases} \tag{18}$$

$$f_2(z_2) = \begin{cases} 1 & z_2 \leq 750{,}000 \\ \dfrac{2{,}250{,}000 - z_2}{1{,}500{,}000} & 750{,}000 < z_2 < 2{,}250{,}000 \\ 0 & z_2 \geq 2{,}250{,}000 \end{cases} \tag{19}$$

LINDO computer software is used to run the equivalent ordinary LP model. The results are $z_1 = \$264{,}334$, $z_2 = 847{,}771$ hours, and the overall DM satisfaction is 0.9255. Table 3 lists the optimal TPD plan for the Dali case.

5. Computational analysis

Several significant finding regarding the practical application of the proposed FLP method are as follows. First, the interactive FLP method developed

Table 1. Summarized transportation data in the Dali case

Source	Destination 1. Taichung	2. Haulien	3. Kaohsiung	4. Taipei	Available supply (dozen bottles)
1. Changhua	0.8*/6**	3.0/40	2.1/12	1.8/16	[18,000, 26,000]
2. Touliu	1.3/10	3.6/32	1.6/15	2.5/22	[24,000, 32,000]
3. Hsinchu	1.8/12	3.5/30	2.4/18	1.0/10	[13,000, 18,000]
Forecast demand (dozen bottles)	[8,000, 12,500]	[3,000, 6,500]	[9,500, 16,500]	[12,000, 21,000]	

Note: '*' denotes transportation cost per unit (in U.S. dollar); '**' denotes delivery time per unit (hours).

Table 2. Summarized production data in the Dali case

Factory	p_{ij} ($/unit)	a_{ij} (machine-hour/unit)	$M_{i\max}$ (machine-hours)	b_{ij} (ft^2 /unit)
Changhua	3.0	0.21	3800	0.32
Touliu	2.7	0.16	3900	0.28
Hsinchu	3.6	0.12	1600	0.30

Table 3. Optimal transportation plan for the Dali case

Item	Solutions
Q_{ij} (in dozen bottles)	Q_{11} = 12,165, Q_{12} = 0, Q_{13} = 4,580, Q_{14} = 0, Q_{21} = 0, Q_{22} = 0, Q_{23} = 11,399, Q_{24} = 13,197, Q_{31} = 0, Q_{32} = 6,239, Q_{33} = 0, Q_{34} = 7,133.
L value	L = 0.9255
Objective values	z_1 = \$264,334 z_2 = 847,771 hours

Table 4. Comparisons of three main TPD methods

Factor	Chanas and Kuchta (1996)	Li and Lai (2000)	The proposed FLP method
Objective function	Single/Fuzzy	Multiple/Fuzzy	Multiple/Fuzzy
Constraints property	Crisp	Crisp	Fuzzy
Membership function	L-R (Trapezoid)	Linear	Linear
Budget	Not considered	Not considered	Considered
Machine capacity	Not considered	Not considered	Considered
Warehouse space	Not considered	Not considered	Considered
Computational efficiency	Medium	Low	High

in this work yields an efficient compromise solution and presents the overall DM satisfaction levels with the determined goal values in a multi-objective TPD problem. For instance, the overall DM satisfaction with the determined goal values, z_1 = \$264,334 and z_2 = 847,771 hours, in the Dali case initially was generated as 0.9255. Moreover, if the DM did not accept the initial overall degree of this satisfaction value, then the L value was adjusted by changing the fuzzy data and parameters to seek a better compromise solution.

Moreover, the DM generally faces a planning problem with multiple fuzzy objectives, when making a transportation decision. The proposed FLP method can satisfy the requirement for the practical application for solving TPD problems since it attempts simultaneously to minimize the total production and transportation costs and the total delivery time in a fuzzy environment.

Finally, the proposed FLP approach exhibits greater computational efficiency by employing the linear membership functions to represent fuzzy numbers. The main advantages of the linear membership function are the simplicity and flexibility of the fuzzy arithmetic operations (Zimmermann, 1978, 1996). Table 4 compares the FLP method proposed in this work to the Chanas and Kuchta (1996) and Li and Lai (2000) methods.

6. Conclusions

This work develops an interactive FLP method for solving the TPD problems with fuzzy goals, available supply and forecast demand. The proposed FLP method attempts simultaneously to minimize the total production and transportation costs and total delivery time. An industrial case is used to demonstrate the feasibility of applying the proposed FLP method to real-world TPD problems. Consequently, the interactive method developed in this work yields an efficient compromise solution and overall DM satisfaction with the given goal values.

References

[1] W. F. Abd El-Wahed, "Interactive fuzzy goal programming for multi-objective transportation problems," *Omega*, 34, pp. 158-166, 2006.

[2] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Management Science*, l7, pp. 141-164, 1970.

[3] S, Chanas, W. Kolodziejczyk, and A. Machaj, "A fuzzy approach to the transportation problem," *Fuzzy Sets and Systems*, 13, pp. 211-222, 1984.

[4] S. Chanas and D. Kuchta, "A concept of the optimal solution of the transportation problem with fuzzy cost coefficients," *Fuzzy Sets and Systems*, 82, pp. 299-305, 1996.

[5] D. Dubois and H. Prade, "Systems of linear fuzzy constraints," *Fuzzy Sets and Systems*, 3, pp. 37-48, 1980.

[6] L. Li and K. K. Lai, "A fuzzy approach to the multiobjective transportation problem," *Computers and Operations Research*, 27, pp. 43-57, 2000.

[7] H.-J. Zimmermann, "Description and optimization of fuzzy systems," *International Journal of General Systems*, 2, pp. 209-215, 1976.

[8] H.-J. Zimmermann, "Fuzzy programming and linear programming with several objective functions," *Fuzzy Sets and Systems*, 1, pp. 45-56, 1978.

[9] H.-J. Zimmermann, *Fuzzy Set Theory and Its Application*. Boston: Kluwer, 1996.