Absorption of Boundary Reflection on Simulation of Elastic Waves in Heterogeneous Medium

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***Abstract*—Suppression of boundary reflections is a critical issue in numerical simulation of elastic waves. For this purpose, many processing approaches of boundary are worked out. One of ideal methods is PML, which has been used widely. This paper uses PML absorbing boundary condition in the simulation of full seismic wavefields. Numerical experiments showed that the thickness of PML for qSV and qSH waves is larger than P wave, 13 grid points is appropriate.**

Keywords-numerical simulation; elastic wave; absorbing boundary; anisotropy; snapshots

I. INTRODUCTION

Numerical simulation of seismic wavefields is an important approach for exploring and identifying various waves [1,2]. It plays significant role in seismic data processing [3]. The simulation of waves by finite-difference or finite-element methods in unbounded domains requires a special treatment for the boundaries of the necessarily truncated computational domain [4]. For this purpose, many processing approaches of boundary are worked out. For example, absorbing boundary condition (Clayton and Engquist, 1977) [5], transparent boundary (Reynolds, 1978) [6], nonreflecting boundary condition (Cerjan et al., 1985) [7], absorbing boundary condition with perfectly matched layer(PML) (Berenger, 1994, Rappaport, 1995, Chew and Liu, 1996) [8-10, 4], and so on. PML was introduced when Berenger studied the propagation of electromagnetic waves. PML has found successful applications in simulation of acoustic and elastic wave. It works for anisotropy heterogeneous media. Following the way of doing used by Collino F in [4], this paper presented high-order staggered-grid finite difference of PML with $o(\Delta t^2, \Delta x^{2N})$ accuracy, and application of it to the simulation of seismic waves in anisotropy heterogeneous media. Numerical experiments show that the thickness of PML for qSV and qSH waves is larger than qP waves. The thickness corresponding to thirteen grid points could meet requirements.

II. SPLIT PML BOUNDARY CONDITIONS FOR FIRST-ORDER VELOCITY-STRESS WAVE EQUATIONS

The basic idea of split PML boundary condition is to decompose unknown variables into two components in PML. One component is vertical to the boundary, and another is parallel to the boundary. In time domain, variables of wavefields are decomposed, therefore, first-order velocity-stress wave equations are split to system of equations for PML absorbing boundary conditions with damping factors. Control equations of PML absorbing boundary conditions on V_x and σ_{xx} are:

$$\frac{\partial V_x^x}{\partial t} + d_x V_x^x = \frac{1}{\rho}\frac{\partial \sigma_{xx}}{\partial x};$$

$$\frac{\partial V_x^z}{\partial t} + d_z V_x^z = \frac{1}{\rho}\frac{\partial \sigma_{xz}}{\partial z}. \quad (1)$$

$$\frac{\partial \sigma_{xx}^x}{\partial t} + d_x \sigma_{xx}^x = C11\frac{\partial V_x}{\partial x};$$

$$\frac{\partial \sigma_{xx}^z}{\partial t} + d_z \sigma_{xx}^z = C13\frac{\partial V_z}{\partial z}. \quad (2)$$

The rest equations are similar to them.

Where d_x and d_z are damping factors in x and z direction respectively. $d_x = -\frac{V_{\max}\ln\alpha}{L}[a\frac{x_i}{L} + b(\frac{x_i}{L})^2]$, $d_z = -\frac{V_{\max}\ln\alpha}{L}[a\frac{z_i}{L} + b(\frac{z_i}{L})^2]$, x_i and z_i is distances from the points within PML to boundary, $V_{\max}$ is the maximum value of primary wave, L is the thickness of PML, a=0.27, b=0.75.

PML can be used in wave equations of anisotropy media more easily, and no need special treatment. This is an advantage of the approach over other methods.

III. STAGGERED-GRID FINITE DIFFERENCE FORMULAE OF PML BOUNDARY

Suppose U 、W 、V 、R 、T 、H 、S 、Q denote discrete values of V_x 、V_y 、V_z 、σ_{xx} 、σ_{zz} 、σ_{xz} 、σ_{yz} 、σ_{xy} . Difference formulae of U 、R are as follows:

$$Ux_{i,j}^{k+1/2} = \frac{1-d_i dt/2}{1+d_i dt/2} Ux_{i,j}^{k-1/2} + \frac{dt}{\rho_{i,j} dx(1+d_i dt/2)} \cdot \sum_{n=1}^{N} C_n^{(N)} [R_{i+(2n-1)/2,j}^k - R_{i-(2n-1)/2,j}^k]. \quad (3)$$

$$Uz_{i,j}^{k+1/2} = \frac{1-d_j dt/2}{1+d_j dt/2} Uz_{i,j}^{k-1/2} + \frac{dt}{\rho_{i,j} dz(1+d_j dt/2)} \cdot \sum_{n=1}^{N} C_n^{(N)} [H_{i,j+(2n-1)/2}^k - H_{i,j-(2n-1)/2}^k]. \quad (4)$$

$$U_{i,j}^{k+1/2} = Ux_{i,j}^{k+1/2} + Uz_{i,j}^{k+1/2}. \quad (5)$$

$$Rx_{i+1/2,j}^{k+1} = \frac{1-d_{i+1/2} dt/2}{1+d_{i+1/2} dt/2} Rx_{i+1/2,j}^{k} + \frac{C11dt}{dx(1+d_{i+1/2} dt/2)} \cdot \sum_{n=1}^{N} C_n^{(N)} [U_{i+n,j}^{k+1/2} - U_{i-(n-1),j}^{k+1/2}]. \quad (6)$$

$$Rz_{i+1/2,j}^{k+1} = \frac{1-d_j dt/2}{1+d_j dt/2} Rz_{i+1/2,j}^{k} + \frac{C13dt}{dz(1+d_j dt/2)} \cdot \sum_{n=1}^{N} C_n^{(N)} [V_{i+1/2,j+(2n-1)/2}^{k+1/2} - V_{i+1/2,j-(2n-1)/2}^{k+1/2}]. \quad (7)$$

$$R_{i+1/2,j}^{k+1} = Rx_{i+1/2,j}^{k+1} + Rz_{i+1/2,j}^{k+1}. \quad (8)$$

Based on the same way, difference formulae of other equations can also be available.

IV. NUMERICAL EXPERIMENTS

A. *Absorption of boundary reflections in inheterogeneous medium*

Figure 1 is a model of in-heterogeneous medium. Source is Ricker wavelet, located in point (0,50), and its main frequency is 60Hz. $\Delta t = 0.1$ ms, $dx = 2$ m, $dz = 2$ m. Snapshots of wavefields at different times are shown in figure 2.

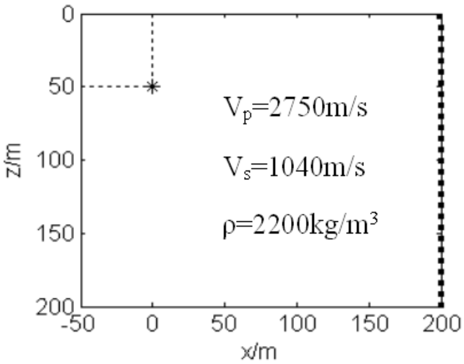
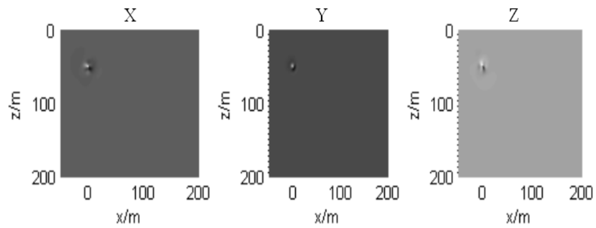
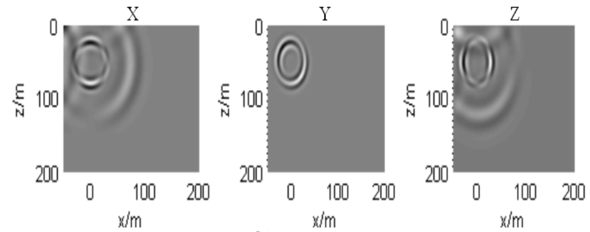


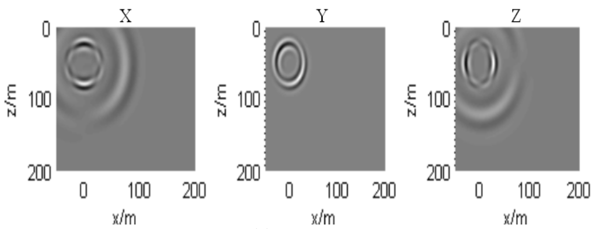
Figure 1. A model of in-heterogeneous medium



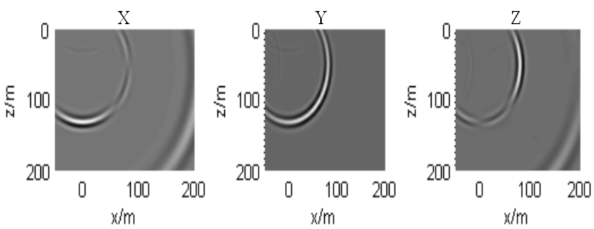
(a) t=0ms, *L*=0



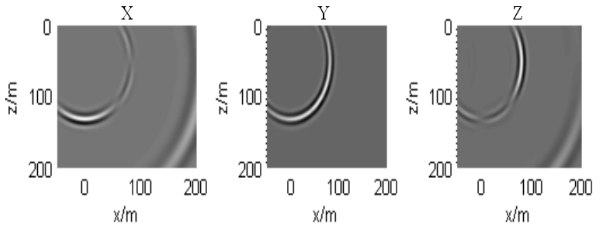
(b) t=30ms, L=0



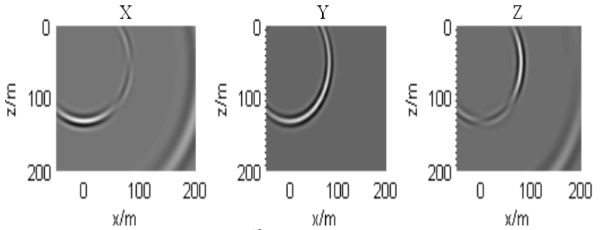
(c) t=30ms, L=5



(d) t=80ms, L=5



(e) t=80ms, L=10



(f) t=80ms, L=13

Figure 2. snapshots of wavefields at different times

The results show that L=5 can meet requirement for pressure wave. But for qSV and qSH waves, L should be larger, L=13 is appropriate.

B. *Seismograms simulated in VTI medium*

To simulate various waves that could be recorded in anisotropy medium, we use model 2 (figure 3). Source is also a Ricker wavelet located in point (0,50), but main frequency is 600Hz. $\Delta t = 0.04$ ms, $dx = 0.5$ m, $dz = 0.5$ m. Figure 4 shows simulated three components seismograms.

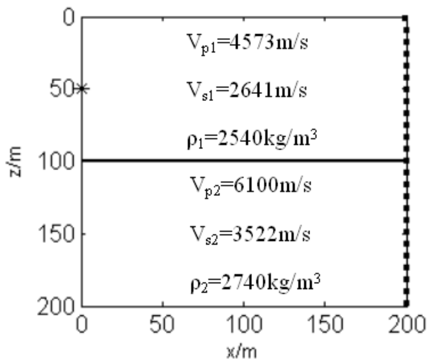
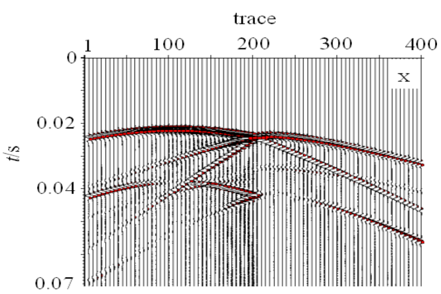


Figure 3. VTI model



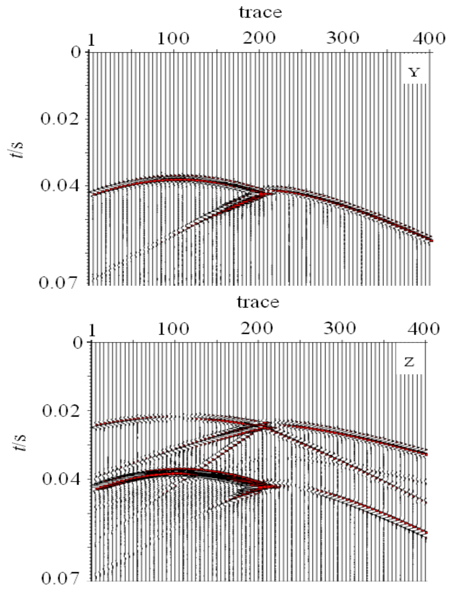


Figure 4. three components seismograms

V. Conclusions

PML boundary condition is an effective method for absorption of boundary reflections. If the source includes shear source, the thickness of PML should be about three times of only primary wave.

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