Rainfall Frequency Analysis of Sudan by Using Bayesian Markov chain Monte Carlo (MCMC) methods

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Abstract—**This paper deals with at-site rainfall frequency estimation in the case when also information on hydrological events from the past with extraordinary magnitude is available. For the joint frequency analysis of systematic observations and historical data, respectively, the Bayesian framework is chosen, which, through adequately defined likelihood functions, allows for incorporation of different sources of hydrological information, e.g., mean annual rainfall, historical events as well as measurement errors. The distribution of the parameters of the fitted distribution function and the confidence intervals of the rain quantiles are derived by means of the Markov chain Monte Carlo simulation (MCMC) technique.**

The paper presents a sensitivity analysis related to the choice of the most influential parameters of the statistical model, which are the length of the historical period h and the perception threshold X_0. These are involved in the statistical model under the assumption that except for the events termed as 'historical' ones, none of the (unknown) rains from the historical period h should have exceeded the threshold X_0. Both higher values of h and lower values of X_0 lead to narrower confidence intervals of the estimated rain quantiles; however, it is emphasized that one should be prudent of selecting those parameters, in order to avoid making inferences with wrong assumptions on the unknown hydrological events having occurred in the past.

The Bayesian MCMC methodology is presented on the example of the mean annual rains observed at Sudan in the period 1901–2002.

Keywords-frequency analysis; Markov chain Monte Carlo simulation (MCMC); mean annual rainfall; threshold; historical data

I. INTRODUCTION (HEADING 1)

The hydrologic and hydrometeorological extremes and critical thresholds derived from complex hydrological and meteorological events for engineering design are usually obtained on the basis of single site characteristics (e.g., the annual maximum daily rainfall or discharge). Therefore, hydrological and meteorological frequency analyses have also mainly focused on one characteristic value (e.g., [9]–[4]. Various methods have been proposed to reduce the uncertainties of at-site rainfall frequency analyses and produce more robust rains quantile estimates based on larger sample sizes. Two main families of approaches can be distinguished [17]:

(i) 'Spatial extension' of information on rainfall through regional rainfall frequency methods based on aggregating statistically homogeneous data to build large data samples [13], and

(ii) 'Temporal extension' of information on rainfall through at-site rainfall frequency studies on gauged streams extended by historical rainfall [22].

To overcome the problem of relatively short data series for frequency analysis the need to investigate extremes also spatially was traditionally widely acknowledged in the hydrological community. The very basic paradigm of this approach originates from the index flood method introduced by Dalrymple [10] and it is commonly used to implement a regional frequency analysis for a particular variable of interest. The aim of regional frequency analysis is to increase the information content of the analysis and to reduce the uncertainty of the design values estimates by 'trading space for time'. To address this issue, spatial (Regional) properties of extremes are studied and regional frequency analysis is typically applied. The regional approaches usually involve two major steps: the delineation of homogeneous regions (sometimes referred to as 'pooling groups') and the estimation of extreme value quantiles at the sites of interest using information from all sites in the region. Traditionally, homogeneous pooling groups were formed based on geographical position or administrative boundaries [19]-[3]-[29]. Therefore, Acreman and Wiltshire [1] suggested a pooling approach with no need of having adjacent members in groups, i.e., groups defined in a flexible way.

This concept was further developed in different ways: Burn [5] introduced the region of influence (ROI) focused pooling method, while various clustering techniques [5] and canonical correlation analysis [20] have been proposed to form homogeneous pooling groups. An important role among the various regional frequency analysis methods has the one based on the L-moments [13]. Numerous authors contributed to refinements of these directions of research [14] – [25] - [17] – [6].

A large part of our knowledge on extreme rainfall values is based on inventories of data regarding extraordinary events [28] – [24]-[18]-[7] – [27] – [2] [8]. Since such extraordinary events are important source of information on the rainfall extremes, attempts were undertaken to include these into at-site frequency analyses [12] - [26] - [21]. This is

obtained in this section by including past historical mean annual values in rainfall frequency analyses by the Bayesian Markov chain Monte Carlo (MCMC) framework [15] [22]. Let us illustrate the principles of the inclusion of historic data in at-site rainfall frequency analyses. Suppose that the information on historical hydrological consists of m extraordinary rainfall events, and the joint data sample of systematic and historical observations is stationary. In order to properly account for the historical information, the evaluation of the m historical mean annual rainfall is not sufficient. It is also important to consider the number of years h in which these events were the major rains and to evaluate the threshold X_0 which has certainly not been exceeded during this period by the other rains. In other words, the historical information consists not only in the m mean annual values but also in h − m years of nonexceedance of the threshold X_0. The choice of h and X_0 should meet the criterion of "exhaustiveness" (i.e., no other major rains should have exceeded X_0 in the period of time h), which is a necessary condition for a proper statistical inference with censored data [16]-[11]. The Bayesian MCMC procedure [22] is a flexible tool, which can handle the information both on the historical and systematic observations through adequately defined likelihood functions in a straightforward way, and more importantly, can account for uncertainties in the measurements of the hydrologic extremes, and provides estimates of confidence bounds for the estimated quantiles. It has been demonstrated that the inclusion of the historic period leads to a clear reduction of the confidence intervals [22].

This section aims at demonstrating that this method, although it may bring a significant added value in rainfall frequency analyses, is still not free from subjectiveness and both skill and care have to be exercised when applied. Several choices that have to be made are not trivial.

II. THE DATA

The data used have been collected from 15 rain gauging stations, the records of which are published by the Sudan National Meteorology Department. Altitude and latitude have been assumed as initial statistics of hydrologic homogeneity, and station selection criteria were based on these characteristics. All sites used in this procedure are located between 31°50_ and 33°25_N latitudes, and 1530 and 2300 m altitudes (MSL). Records used for the analysis have ended the same year, and there are no gaps in the records (Table I).

III. METHODOLOGY

Bayesian Markov chain Monte Carlo (MCMC) methods provide a computationally convenient way to fit frequency distributions for rainfall frequency analysis by using different sources of information as large rainfall records, historical rains, uncertainties (particularly measurement errors), regional information and other hydrologic information. They also provide an attractive and straightforward way to estimate the uncertainty in parameters and quantile metrics [23].

Since a rainfall frequency analysis with information on historical events included is a relatively complex methodology, in the next subsections the most relevant details and settings of the procedure.

A. Selecting Bayesian inference

The Bayesian approach is a branch of statistical analysis that is based on a unique philosophy: the statistical inference is drawn in the way that the initial beliefs on the subject of the interest are modified according to the observed data. Thus, the Bayesian inference combines two kinds of information: (i) the prior knowledge (belief, hypothesis) on the unknown parameters that may come from other data sets, logical intuition or the past experiences of the analyst and (ii) the information encapsulated in the observed data, which are represented by the likelihood function [26].

A Bayesian inference is based on the application of the Bayes' theorem:

$$P(\theta|D) = \frac{P(\theta|D)P_1}{P(D)} \qquad (1)$$

where P (θ) is the prior (marginal) distribution of the parameters θ (it does not take into account any information contained in the observed data D); P (θ|D) is the posterior distribution of the parameters θ, having the data D observed; P (D|θ) is the conditional probability of the data, given the parameters θ; and P (D) is the prior (marginal) distribution of the data D.

P (D) only serves as a normalization constant in order to obtain a unit area under the posterior PDF P (θ|D).

The conditional probability where the second argument is considered as a parameter is also called likelihood function: p (θ|D = d) or _ (D|θ = Θ). Bayes' theorem, using the aforementioned notation and having a continuous variable θ can be rewritten as follows:

$$P(\theta|D) = \frac{\ell(D|\theta)P(\theta)}{\int_\Omega \ell(D|\theta)P(\theta)} \qquad (2)$$

Where: integral in the denominator is computed through the entire parameter space Ω.

The mathematical formulation of the way the hypothesis of the statistical properties of the parameters θ (the existing/prior beliefs) is updated by having observed the data D (in the light of new pieces of knowledge). Naturally, the final posterior distribution may serve as a prior distribution in a further Bayesian inference.

One of the main advantages of the Bayesian inference is that it results in a full posterior probability of the parameters: one may easily derive the credible intervals (counterparts of the traditional confidence intervals) of the parameters or any of their functions. Compared to this, the traditional methods of statistical analysis that are usually based on asymptotical assumptions, look at the parameters of a distribution function as fixed (unknown) constants, and the result of a statistical analysis is usually a point estimate of the parameters [24].

TABLE I.			THE HYDROLOGICAL AND GEOGRAPHICAL CHARACTERISTIC OF THE RAINFALL SITES SELECTED IN SUDAN							
Site name	Code	Data length	Mean ARF	L-cv	L-skew.	L-kurt.	lat.	Long.	Area.	Elev.
Abu Hamad	02ABH640	58	10.6034	0.7029	0.5437	0.3206	19.533	33.333	122.1	299.6
Atbra	02ATB680	58	59.3793	0.4231	0.1992	0.1345	17.667	33.967	30.4	316.3
Port Sudan	03PSD641	15	63.8933	0.42	0.0645	0.0767	19.583	37.217	218.8	525.3
Khartuom	04KHA721	102	150.8023	0.2835	0.1752	0.168	15.6	32.55	22.1	39.5
Gadarif	05GDF752	41	603.5854	0.1053	-0.0233	0.205	14.033	35.4	75.2	363.5
Kasala	05KSL730	28	240.7354	0.198	-0.023	0.1379	15.467	36.4	36.7	522.6
Madani	06WMD751	58	305.9828	0.1762	0.0403	0.096	14.383	33.483	27.5	416.7
Elobiet	08OBT771	90	359.3592	0.171	0.086	0.1399	13.183	30.217	185.3	492.8
Elfashir	09FSH760	84	262.332	0.2223	0.193	0.1969	13.617	25.333	296.4	405.7
Elgeniena	09GEN770	30	447.322	0.1901	0.0509	0.1614	13.483	22.45	79.4	526.2
Nyala	10NYL790	43	393.3309	0.1376	0.0674	0.1113	12.05	24.883	127.3	775.8
Senar	12SNR762	60	458.7983	0.1173	-0.0342	0.1466	13.55	33.617	37.8	692
Malakal	13MLK840	58	742.5498	0.1068	0.0463	0.1436	9.55	31.65	77.7	392.4
Waw	14AWE852	28	1078.941	0.1003	0.042	0.0979	8.767	27.4	93.9	487.8
Juba	15JUB941	43	961.4919	0.1105	0.0177	0.1987	4.867	31.6	22.9	468.2

B. Markov Chain Monte Carlo simulations

In general, an analytical computation of the integral in the denominator of (2) is very hard if not impossible; thus, that is the main reason why statistical methods based on the Bayesian inference have not been used frequently in the past [22]. The rapid development of computers in the past 2–3 decades, however, has opened wide perspectives for numerical evaluation of complex mathematical problems. Markov chain Monte Carlo (MCMC) methods represent a class of such algorithms. By means of Monte Carlo simulations, the Metropolis-Hastings algorithm generates a Markov chain, which results in a sample, distribution of which converges to the posterior distribution $P(\theta|D)$. In other words, MCMC simulation draws samples from the posterior distribution of the parameters without having computed the normalization constant of (2) analytically [22]. The quality of the sample improves as a function of the number of steps of the Markov chain. The resulting data sample then serves for the estimation of marginal distributions of the joint probability distribution function, mean values, standard deviations, and confidence intervals not only for the parameters themselves but also for their arbitrary functions such as the required quantiles (design values) of the analyzed hydrological extremes [22].

C. Plotting Position Formulae

There are several plotting position formulae (PPF) that are used in frequency analysis to get a quick glance on the empirical distribution of the data sample analyzed: to check whether they follow a particular distribution, if there are some errors or outliers etc. [22].

One of the most frequently used PPFs is the one of Cunnane:

$$p_i = \frac{i - 0.4}{n + 0.2} \tag{3}$$

Where n denotes the sample size, i: is the rank of the observations in an ascending order, and pi is the cumulative probability of non-exceedance of the ith data.

In the case of a joint frequency analysis of systematic and historical data, the PPF (3) should be slightly modified according to the number of the historical events k that occurred during the whole n-year period analyzed. For the historical events, one should apply

$$p_j = \frac{k_j - 0.4}{nk + 0.2}, j = 1, ..., k \tag{4}$$

and for the systematic data

$$p_i = 1 - \frac{n-k}{n}\left(1 - \frac{1-0.4}{s-(k+0.2)}\right), i = 1, \tag{5}$$

where the notation is in accordance with the one introduced in section 3.2 as well as with the formulae presented by Bayliss and Reed (2001, p. 34). In this section, we apply (4) and (5) for the visualization of the observed rainfall data on the probability plots.

D. Probability Distribution Function

To demonstrate different aspects of joint probability modeling of the systematic and the historical data, we employ the 3-parameter log-normal distribution (LN3). The selected distribution function is one of the most frequently used statistical models in flood frequency analyses [13].

The cumulative distribution function of the LN3 distribution is

$$F(x) = \Phi(y) \tag{6}$$

Where Φ is the CDF of the standard normal distribution and

$$y = \begin{cases} -k^{-1} log [1 - k(x - \xi)], \\ (x - \xi)/\alpha, \end{cases} \qquad (7)$$

Where ξ, α, and k are the location, scale and shape parameters, respectively [13]. The special case with k= 0 yields the standard normal distribution.

E. Setting of the Frequency Model

A number of parameters to characterize various features of a hypothetical data set consisting both of systematic measurements and historical data are introduced. Herein, this notation on the records of rainfall from the 15 selected stations in Sudan is applied as follows:

- The length of the systematic observations is unequivocally given: s = 53 years.
- The 1962 rainfall event is exceptional in the light of the other rainfall peaks observed during the whole period with the systematic observations; thus, we consider this rainfall as historical one. Moreover, this is the only extraordinary event that appears in the analysis; therefore, c = 1.
- The perception threshold X_0 is unknown. Nevertheless, we suppose that X_0 lies somewhere between the absolute maximum of the systematic records (1462 mm, recorded in 1962 (Wau station)) and the secondary maximum (518 mm, recorded in 1969 (Malakal station)). Since any information on rainfall having occurred before the systematic observations unknown, the value of the perception threshold level X0 just below the magnitude of the historical event, i.e., let X_0 = 600 mm. Then a sensitivity analysis related to the choice of this value is performed.
- The length of the historical period h is also unknown. We only suppose that the extraordinary 1962 event has not been exceeded during the whole analyzed period n = h + s. In other words, we have to make an initial estimate of the return period of this extraordinary event, and assume that the magnitudes of all the unknown events that have occurred in the 'historical' period do not exceed the perception threshold X_0 (i.e., m = 0). A sensitivity analysis regarding the choice of h is also carried out in this section.
- Since the magnitude of the historical rainfall event is very well known, it is incorporated into the statistical model. Uncertainties in the magnitude of this event are not considered in the analysis herein.
- The Bayesian approach allows for choosing any prior distribution p (θ) of the parameters θ of the selected distribution function (2). Since an arbitrary choice is allowed, we choose the simplest solution: we do not put any stress on the priors of θ, i.e., we use uniform initial distribution p (θ) = const., which does not have effects on the evaluation of (2).

F. Selection of the Length of Historical Period

The way the choice of the length of the historical period h affects the quantile estimates of rainfall frequency analysis of systematic and historical rain events is presented in Fig. 3 and Table II. The panel of Fig. 1 consists of 6 subplots. From the top left corner the first plot displays the case with the historical event excluded ('No historical information' in its title), i.e., a frequency analysis only based on the records from the period 1902–2000. The other plots are related to a joint analysis of systematic and historical information with different estimates of the parameter h, namely h = 500, 1000, 1500, 2000 and 3000 years. The solid line in the middle of the plots is the fitted distribution function, while the dashed line above (below) it is the upper (lower) bound of the 5-95% confidence intervals of rain quantiles. The systematic data are represented by empty circles, while the historical information is depicted by a solid black circle. The perception threshold has been set to X_0 = 600 mm in all simulations related to Fig. 1.
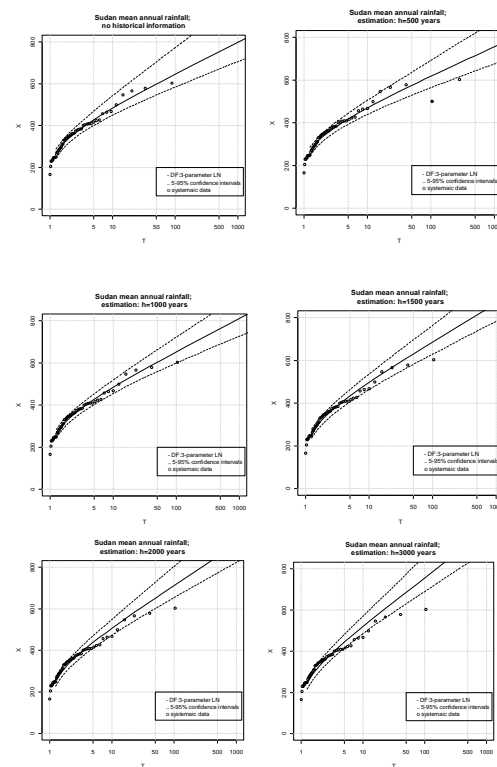

Figure 1. Three-parameter log-normal distribution function (LN3) fitted to the Sudan mean annual rainfall stations: sensitivity analysis related to the selection of the length of the historical period h. DF stands for distribution function.

TABLE II. ESTIMATION OF THE RAIN QUANTILES QT

Return period T (years)	MCMC settings	QT (mm)	CI$_{0.05}$ (mm)	CI$_{0.95}$ (mm)	ΔCI (mm)	ΔCI/ QT (%)
100	no hist. information	482.99	448.91	542.56	93.64	19.388
	h= 500 years	469.45	438.40	505.28	66.87	14.246
	h=1000 years	484.13	454.91	519.17	64.26	13.27
	h=1500 years	496.44	463.10	536.31	73.21	14.74
	h=2000 years	505.94	472.49	548.05	75.55	14.93
	h=3000 years	520.31	485.79	570.16	84.36	16.21
1000	no hist. information	645.55	583.76	772.64	188.88	29.25
	h= 500 years	618.13	564.45	693.08	128.62	20.80
	h=1000 years	652.14	599.27	722.48	123.21	18.89
	h=1500 years	685.13	628.70	762.80	134.10	19.57
	h=2000 years	711.27	654.10	803.93	149.83	21.06
	h=3000 years	755.53	689.69	864.69	175.00	23.16
10000	no hist. information	950.34	824.44	1250.24	425.80	44.80
	h= 500 years	891.97	783.22	1062.01	278.78	31.25
	h=1000 years	970.05	852.67	1129.44	276.77	28.53
	h=1500 years	1052.6	932.29	1238.83	306.53	29.12
	h=2000 years	1120.01	994.49	1356.44	361.95	32.31
	h=3000 years	1242.13	1089.66	1528.98	439.32	35.36

Table II. shows the estimation of the rain quantiles QT and their confidence intervals corresponding to the return periods T = 100, 1000 and 10000 years at the Sudan statio, with no historical information involved, and various assumptions concerning the length of the historical period h, respectively. CI$_{0.05}$ (CI$_{0.95}$) is the 5% (95%) confidence limit of the estimates QT, $\Delta CI = CI_{0.95} - CI_{0.05}$

G. Selection of Perception Threshold

According to the magnitude of the historical plod peak (1462.9 mm) and the maximum of the rest of the rain peak records is (518 mm), we selected 6 different values for the sensitivity analysis related to the threshold X$_0$:

• 1200 mm,
• 1000 mm,
• 600 mm,
• 580 mm,
• 520 mm,
• 520 mm.

On the other hand, the length of the historical period is set to a constant value, h = 1000 years. The graphical outputs of the analysis are presented through a panel of plots in Fig. 2 where the different threshold values are accentuated by a corresponding horizontal line. The characteristics of the selected quantiles and their CIs are summarized in Table III.

Table III shows the estimation of the rain quantiles QT and their confidence intervals corresponding to the return periods T = 100, 1000 and 10000 years at the Sudan rain stations, with various assumptions concerning the perception threshold X$_0$. CI$_{0.05}$ (CI$_{0.95}$) is the 5% (95%) confidence limit of the estimates QT, $\Delta CI = CI_{0.95} - CI_{0.05}$
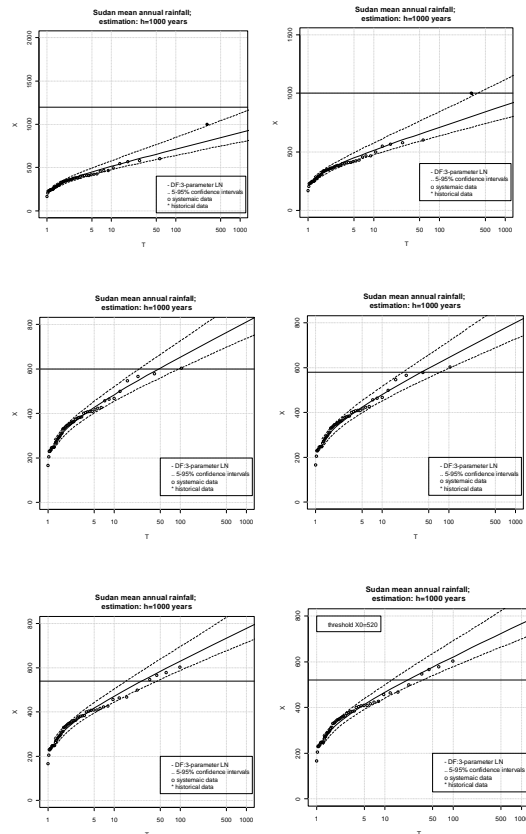

Figure 2. Three-parameter log-normal distribution function (LN3) fitted to the Sudan mean annual rain stations: sensitivity analysis related to the selection of the perception threshold X$_0$. DF stands for distribution function.

TABLE III. ESTIMATION OF THE RAIN QUANTILES QT

Return period (years)	MCMC settings	QT (mm)	CI $_{0.05}$ (mm)	CI $_{0.95}$ (mm)	ΔCI (mm)	ΔCI/QT (%)
100	X0 = 1200 mm	711.40	639.31	847.97	208.66	29.339
	X0 = 1000 mm	709.04	636.30	841.87	205.57	28.99
	X0 = 600 mm	652.57	602.26	726.26	123.99	19.00

	X0 = 580 mm	645.25	597.23	718.56	121.32	18.80
	X0 = 540 mm	629.71	587.40	696.50	109.10	17.32
	X0 = 520 mm	621.61	577.97	692.24	114.26	18.38
1000	X0 = 1200 mm	902.71	792.42	1126.34	333.92	36.99
	X0 = 1000 mm	898.58	785.13	1116.99	331.86	36.93
	X0 = 600 mm	811.96	737.53	930.03	192.49	23.70
	X0 = 580 mm	801.32	728.64	921.22	192.58	24.03
	X0 = 540 mm	778.55	714.58	884.84	170.25	21.86
	X0 = 520 mm	767.28	699.87	880.52	180.65	23.54
10000	X0 = 1200 mm	1098.22	943.08	1429.42	486.34	44.28
	X0 = 1000 mm	1092.06	932.02	1409.75	477.72	43.74
	X0 = 600 mm	971.98	868.36	1141.60	273.24	28.11
	X0 = 580 mm	957.74	855.94	1131.31	275.37	28.75
	X0 = 540 mm	927.12	839.84	1075.52	235.68	25.42
	X0 = 520 mm	912.47	821.27	1077.80	256.52	28.11

H. The Effect of the Record Length on the Result

In the report by Szolgay et al. (2008), the collective of authors raised concerns about the appropriateness of the flood peak measurements made in the last decades of the 19th century (possible inhomogeneities present in the data series; details not reported herein). Due to this fact we decided to exclude the problematic part of the data series and restrict the analysis to the supposedly homogeneous data set belonging to the period 1900–2002.In this section the analysis is restricted to the data set belonging to the period 1900-2002. The results of this analysis are presented in Fig. 3 and Table IV.
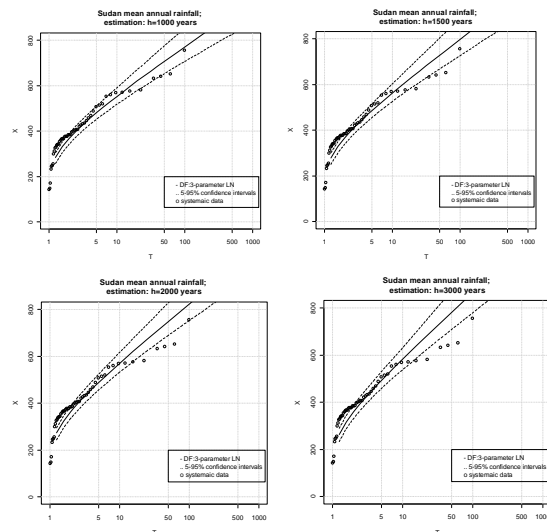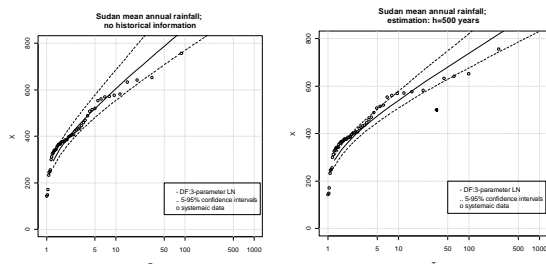

Figure 3. Three-parameter log-normal distribution function (LN3) fitted to the Sudan rainfall data: analysis based on a shorter period of systematic observations s (1900–2002). DF stands for distribution function.

TABLE IV. ESTIMATION OF THE RAIN QUANTILES QT

Return period T （years）	MCMC settings	QT (mm)	CI$_{0.05}$ (mm)	CI$_{0.95}$ (mm)	ΔCI (mm)	ΔCI/ QT (%)
100	no hist. information	863.31	769.24	1039.86	270.62	31.34
	h= 500 years	738.98	677.18	821.11	143.93	19.47
	h=1000 years	770.22	707.14	854.86	147.71	19.17
	h=1500 years	797.71	727.13	890.14	163.00	20.43
	h=2000 years	823.01	752.78	919.65	166.86	20.27
	h=3000 years	865.61	780.06	980.26	200.20	23.12
1000	no hist. information	1120.81	972.48	1416.38	443.90	39.60
	h= 500 years	929.21	831.91	1067.87	235.96	25.39
	h=1000 years	981.66	877.31	1129.69	252.37	25.70
	h=1500 years	1030.10	917.61	1194.06	276.45	26.83
	h=2000 years	1076.13	961.98	1244.15	282.16	26.22
	h=3000 years	1156.56	1016.18	1360.73	344.54	29.79
10000	no hist. information	1389.48	1178.53	1830.11	651.58	46.89
	h= 500 years	1122.01	984.82	1327.40	342.58	30.53
	h=1000 years	1198.61	1049.46	1421.61	372.14	31.04
	h=1500 years	1271.38	1105.49	1520.33	414.83	32.62
	h=2000 years	1341.92	1177.18	1599.29	422.10	31.45
	h=3000 years	1468.1	1258.12	1791.22	533.10	36.31

Table IV shows the estimation of the rain quantiles QT and their confidence intervals corresponding to the return

periods T = 100, 1000 and 10000 years at the Sudan rainfall stations, based on a shorter period of systematic observations s (1900–2002). CI0.05 (CI0.95) is the 5% (95%) confidence limit of the estimates QT, $\Delta CI = CI_{0.95} - CI_{0.05}$

IV.   DISSCUSION OF MCMC RESULTS

The main results of the frequency analysis are presented in terms of figures (probability plots) and tables, where the main focus was set to the rainfall quantiles corresponding to return periods T = 100, 1000 and 10000 years. The latter value, although it may seem unreasonably high, is justified by the fact that the rainfall quantile corresponding to the return period of T = 10000 years has been defined as the critical design value.

The sensitivity analyses is presented, which examine the effects of the choice of the most important parameters of the statistical model (the length of the historical period h, the threshold X_0 and the length of the systematic records s) on the quantile estimates and the width of their confidence intervals.

In Fig. 3 comparing the first subplot with no historical information included and the rest of the plots corresponding to different h values it is discernible that the inclusion of the historical event with an extraordinary magnitude reduces the width of the confidence bounds. This fact is underpinned by the numerical evaluation of the confidence intervals (CIs) of the representative rain quantiles: both the absolute and relative widths of the CI (ΔCI and $\Delta CI/QT$, respectively) for the near highest value of the parameter h are approximately half of the widths of the corresponding CI for the alternative with no historical information involved (Table II). This holds for all three return periods considered. From Figures 3 it is also clear that the value of h controls the slope of the fitted CDF: higher values of h are associated with less steeper CDFs.

The inclusion of historical information evidently reduces the uncertainty in the quantile estimates. Nevertheless, one of the most important questions still remains open: which of the selected h values should be preferred? In a traditional rain frequency analysis that is only based on systematic data, different goodness-of-fit tests (such as the test of $\chi 2$, test of Kolmogorov and Smirnov or the Z-test; see, e.g., Wilks, [13]) can be applied in a simple way to assess the 'closeness' of the fitted distribution to the observed data. Nevertheless, as soon as any historical information is incorporated, testing the goodness of the fit becomes a difficult business. As far as known, there are no methods in the literature to accomplish such a task. Therefore, decide is made to use a visual inspection method. We check whether 90% of the observed data visualized through the modified plotting position formulae lie or do not lie within the 90% confidence interval estimated by the MCMC simulation procedure. Although the method is subjective, it can indicate with respect to the CDF where the fit is not sufficiently good, the presence of changes of slopes (e.g., due to threshold effects) or whether another type of distribution would be more suitable. For these reasons, as for other visual techniques, our visual inspection method could be used in the engineering practice. Based on

the visual inspection, we eliminated the two highest estimates of h (2000 and 3000 years; see two bottom plots in Fig. 3 since in these two cases, a larger number of data points (which belong to the highest rainfall peaks observed during the period with systematic records, and therefore, are of enhanced importance) get outside the confidence bounds. In order to find the only acceptable h value, we also fitted another distribution functions (e.g., generalized extreme value, 3-parameter log-Pearson, 2-parameter lognormal etc.) to the same data set with the same settings of the statistical model. The results are not reported herein; however, taking into consideration the visual check of other distribution functions we conclude that the most acceptable estimate of the length of the historical period is about h = 1000–1500 years.

From Fig. 2 and table III the outcomes indicate that the gradual lowering of the perception level results in two effects: a) narrower uncertainty bounds (by setting the threshold X_0 lower and lower), and b) less steep slope of the fitted distribution function. Both effects can easily be explained.

Case a) means that by setting the threshold X_0 lower and lower, one assumes that the unknown past values were not particularly high, i.e., they varied in a narrower range. At the same time, case b) means that having a lower threshold X_0, the magnitudes of the unknown peak discharges that have occurred in the past are supposed to be lower, therefore, the distribution function is also fitted towards the lower values. While the first effect could, in principle, be beneficial for the quantile estimates by having narrower CIs, this theoretical advantage is outweighed by the second effect, which – similarly to the one discussed above – puts several data points outside the confidence bounds. Based on these considerations we conclude that the initial selection of the perception threshold (X_0 = 600 mm) is acceptable for the recent analysis. It also can be higher; however, it definitely should not be set considerably lower.

Keeping the settings of the statistical model applied so far, and having a shorter period of systematic observations, the MCMC simulations lead to lower quantile estimates compared to the results based on the whole data records (Table II). It is likely that this is only a sampling effect since the eliminated period 1950–1996 contains a number of rainfall events of a relatively high magnitude. On the other hand, the effect of the shorter period s on the uncertainty of the estimated quantiles is unclear (Table IV). Lower quantile estimates QT are accompanied with narrower absolute width of the confidence intervals ΔCI; however, these effects result in enhanced relative width of the CIs $\Delta CI/QT$ (cf. Table IV and Table II).

At the present stage of the analysis it is evident that the two parameters h and X_0 are the most dominant controls of the width of the CIs. In order to derive further information on the role of s in the statistical model, a deeper analysis is needed (possibly accompanied by further Monte Carlo simulations), and such a task is beyond the scope of the recent study.

V. CONCLUSION AND FINAL EVALUATION

As the particular results of the case study indicate, among the number of parameters of the statistical model to be chosen, the length of the historical period h and the perception threshold X_0 have the most remarkable influence on the width of the confidence intervals of the estimated rain quantiles. The higher the assumption of h (i.e., analysis reaches further back to the past) and/or the lower the assumption of X_0 (i.e., the unknown mean annual rains from the past are assumed to be generally low), the narrower are the confidence bounds of the estimated quantiles. Nevertheless, selecting high values of h and/or low values of X_0 for the model yields a great risk that the analyst may be wrong of not considering all the (unknown) extreme events that might have occurred during the period of length h in the past. It is therefore advised to be rather prudent in selecting the model parameters to have confidence intervals only moderately narrower compared to the alternative with no historical information, instead of having considerably narrower confidence intervals but making wrong inference on "unknown" historical data that are not well estimated.

A particularly satisfying result of this study was that the outcomes of the method are relatively robust when considering the uncertainties in the estimated historical rainfall and the subjective choices in the parameterizations of the method.

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