Stylized Facts in Internal Return Rates of Trend Speculators on Stock Indices

Lukas Pichl[1], Hitomi Aoki[1], Takuya Yamano[2], Taisei Kaizoji[2]

[1]Division of Natural Sciences
[2]Division of Social Sciences

International Christian University, Osawa 3-10-2, Mitaka, Tokyo, 181-8585 Japan

Abstract

Stylized facts in market indicator data have been reported, up to date, especially on the session-to-session basis, such as the fat tail law in the distribution of $\log(P_i/P_{i-1})$ returns on various time scales. Since even this log-normalized return indicator quantifies the profit/loss resulting from one-session asset engagement, it is worthwhile studying the stylized facts associated with more complex strategies. This work explains the possible benefits of such an approach, and uses the floating Dead Cross / Golden Cross trend strategy to illustrate such an extended approach to the stylized facts in economic series.

Keywords: universal features, stylized facts, returns on strategy, trend investment, technical analysis, chartists, moving averages

1. Introduction

The problem of optimal investment is central to financial economics. Among the fundamental theorems, Kelly criterion [1] postulates the optimal portfolio as the one with the maximal expectation value of the logarithm of the returns. This is, in fact, equivalent to requiring a full sustainability of the exponential growth of the asset (constant growth ratio), given infinitely divisible currency and portfolio shares [2]. In case of the single portfolio asset, the optimal investment ratio would be p-(1-p)/b, where b is the (constant) profit and p its probability.

Claude Shannon influenced John Larry Kelly in generalizing his formulation of an originally technical optimization criterion towards the logarithmic utility function, based on Shannon's work on information content and statistical entropy. In case of stationary systems, fully randomized sampling, or asymptotic market behavior, the generalized Kelly criterion has been verified both empirically and theoretically. It is therefore important to establish stationary relations between market yields $p_i(t)$ and investment strategies, which survive in the long-term run, and give rise to the stylized facts in financial markets [3-7].

To this aim, this paper analyzes stylized facts that generalize the distribution of normalized returns in a minimalist approach. In particular, market tradings are defined on the basis of moving averages in two time scales, which translate into the buy and sell signals arising from trend curve crossing points. The session-to-session basis used in the original definition of normalized returns is replaced by the signal-to-signal period of investment. The session-to-session ratio used in the original definition of normalized returns is replaced by the inner return rate within the framework of the continuous time model.

The generalization of classical stylized facts in financial markets to the stylized facts derived from (or filtered through) investment strategies provides a useful tool to assess the viability of investment strategies in the given market. Strategies, which do not sample the universal features of the original time series are likely to extinct in the long term run. The pronounced characteristic features in such stylized facts (strategy fingerprints) could also be used as precursors of market crashes or possibly for identification of specific market players. Finally, an application of the continuous time formalism for stylized facts observed in internal return rates can possibly yield a method, which would allow us to estimate the depth of investment opportunities in financial markets with fewer limitations by particular time scales [2].

The paper is organized as follows. Section 2 explains the theoretical rationale of the generalized stylized features. The single-encounter investment in the continuous time model is complemented by an analysis of market Golden Cross and Dead Cross states in the parametrical representation of moving averages. The results for stylized facts are shown and discussed in Section 3, with an emphasis on comparison of various stock exchange market portfolios. Concluding remarks in Section 4 close this paper.

2. Stylized Facts in Strategies

As mentioned above, the stylized facts in indicator time series [3-7] are usually studied by means of log-normalized returns, $\log(P_i/P_{i-1})$. Let us consider a stock index investment (one of the fundamental types of portfolios) of variable duration of n sessions with a minimal required profit rate r. In other words, we impose a threshold on the internal rate of return R (IRR) defined as

$$0 = \sum_{t=1}^{n} \frac{C_t}{(1+R)^t}$$

with an unknown period of n sessions, cash flow C_0 equal to the buying price, and cash flow C_n equal to the selling price. The termination condition for the investment requires

$$P_T \geq P_0(1+r)^T$$

at the smallest possible time T at which the above inequality to hold (single encounter of the required IRR in the market; this corresponds to a fundamentalist strategy). By using the compound discounting across market session,
and the standard identity

$$\lim_{n\to\infty}(1+\frac{a}{n})^n = e^a \,,$$

the termination condition is equivalent to

$$P_t \geq P_0 e^{\rho T} \,,$$

where ρ is the IRR density in the continuous time model. This is the crucial independent variable in what follows. Let us note that in case of T=1, the IRR approximately reduces to the value of the log-normalized return.

For any given fixed value of ρ, there is a certain probability $p(\rho)$ that this IRR will be realized in the market, which can be computed for instance by Monte Carlo simulation. For $\rho \to 0$, $p \to 1$ and most actual market data, larger values of ρ imply a rapid decay of $p(\rho)$, because the exponential explosion in compound interest function that cannot match to the limited rates of returns achievable in real markets. The quantity $\rho p(\rho)$ is the expected IRR, and must have a non-trivial local maximum at some point ρ^*. Since in general $T \neq 1$, the stylized facts in $R(\rho)=\rho p(\rho)$ are different from the stylized facts in normalized returns. The fundamental distribution $R(\rho)$ is a reference case for the trend investment strategy explained hereafter.

Next, we proceed to redefining $R(\rho)$ in terms of

$$\mathrm{MA}(t) = \frac{1}{N}\sum_{n=0}^{N-1} P_{t-n}$$

an investment strategy, built on the comparison of short-term and long-term moving averages (MAs), The short term trend is taken as an arithmetic average of market prices over five sessions. The short term trend is compared to the long term trend (MAs over 25, 75 and 200 market sessions). A strong bullish market trend is signaled by an intersection of the rising short and long term curves (signal to buy). A strong bearish market trend is signaled by an intersection of the falling short and long term market curves (signal to sell). These two turning points are routinely referred to as the Golden Cross and Dead Cross, usually associated with the recommendations to buy and sell, respectively.

Provided these signals originate on the opposite sides of a local maximum in the market, an investment between the two turning points may turn profitable. Since both turning points are based on relative criteria, there is no guarantee that the resulting IRR density ρ from such a transaction will in fact be positive, in contrast to the previous single encounter case. The trend investment policy defined by the two adjacent turning points is compared to the fundamental case in

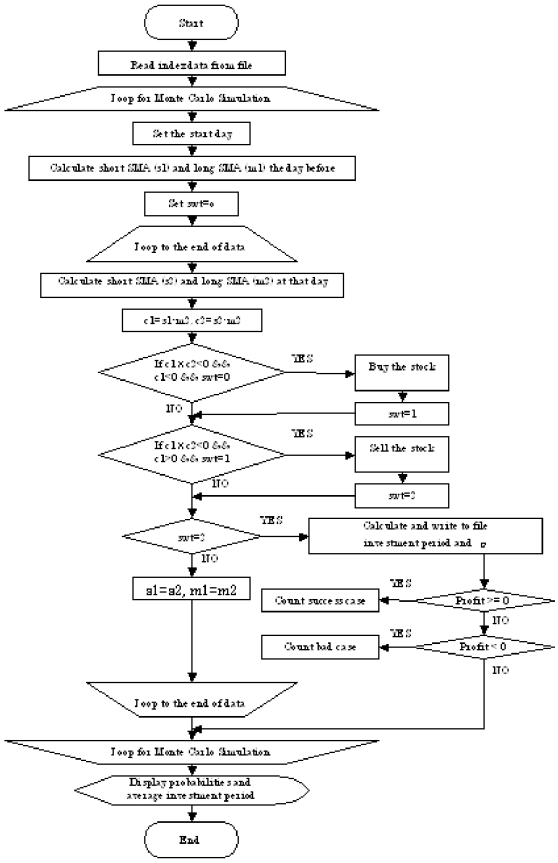

Fig. 1: Flow chart of the moving-average trend investment Monte Carlo simulation.

the next section. Figure 1 shows the flow chart of the Monte Carlo simulation of $R(\rho)$.

Let us denote the daily MA time series of short-term averages and long-term averages as θ_i and λ_i, and their difference $\Delta_i = \theta_i - \lambda_i$. The investment policy is then determined by the buy and sells signals arising in the market as follows:

- Buy: $\Delta_i \times \Delta_{i-1} < 0$ (cross), $\lambda_i - \lambda_{i-1} > 0$ (bullish long-term trend), $\theta_i - \theta_{i-1} > 0$ (bullish short-term trend), and $\theta_i + \lambda_{i-1} - (\theta_{i-1} + \lambda_i) > 0$ (acceleration of the long-term bullish trend).

- Sell: $\Delta_i \times \Delta_{i-1} < 0$ (cross), $\lambda_i - \lambda_{i-1} < 0$ (bearish long-term trend), $\theta_i - \theta_{i-1} < 0$ (bearish short-term trend), and $\theta_i + \lambda_{i-1} - (\theta_{i-1} + \lambda_i) < 0$ (acceleration of the long-term bearish trend).

By using the Monte Carlo computer simulation method, the initial day of investment i_0 is decided at random between the values $i_b = 1$ and i_e, the size of the data set. Then the data are scanned for $i_0 < i \le i_e$, until the buy signal is found at some $i = i_b$. The day i_b corresponds to the start of the investment, and the data are scanned again $i_b < i \le i_e$, now for the investment termination day, i.e. the sell signal, at some i_s. The termination condition is summarized by

$$I_{i_s}/I_{ib} \ge \exp(\rho(i_b - i_a)), \quad i_s < i_0 < i_a < i_b < i_a,$$
$$|i_b - i_a| \to \min.$$

3. Results and Discussions

The two R(ρ) distributions for the fundamental and trend investment case were obtained by Monte Carlo simulation for three stock market indices, in particular the daily time series of TOPIX, S&P 500 and FTSE 100 time series. Figure 2 shows the input data employed in all three simulations.

The stock market indices were selected as a simple portfolio. Stock indices are also often used as an underlying asset in financial derivatives instruments, hence being more representative than a particular stock title. Starting dates were selected at random; in case of the turning point strategy, the next nearest Golden Cross point is searched. All simulations were performed with N=10,000 runs.

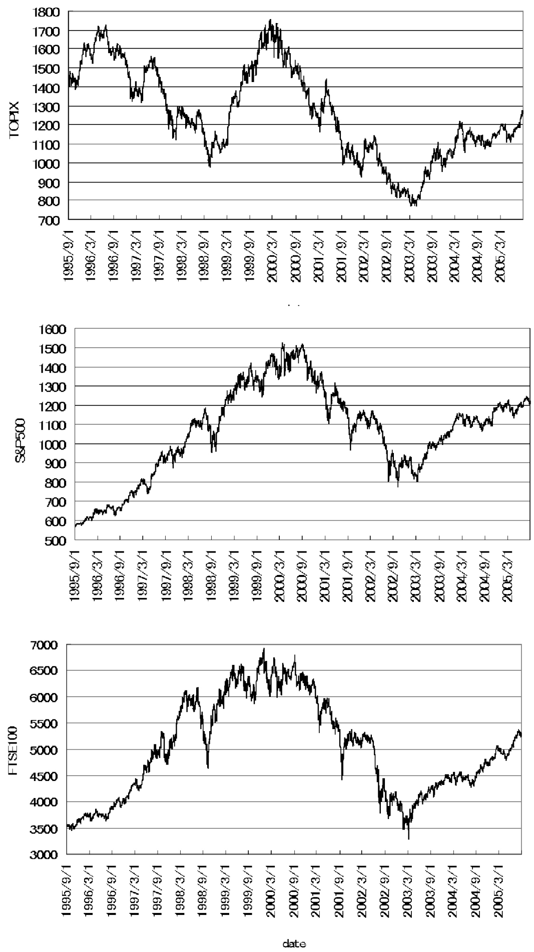

Fig. 2: Time Series of TOPIX, S&P500 and FTSE100.

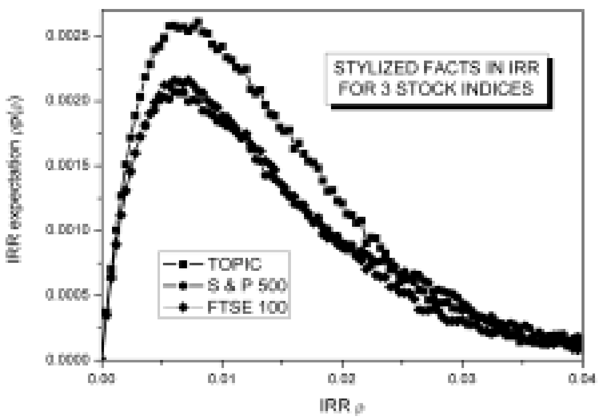

Fig. 3: Stylized facts in inner return rates of the 3 stock indices: note the marked shape similarity between S & P 500 & FTSE 100 and TOPIX.

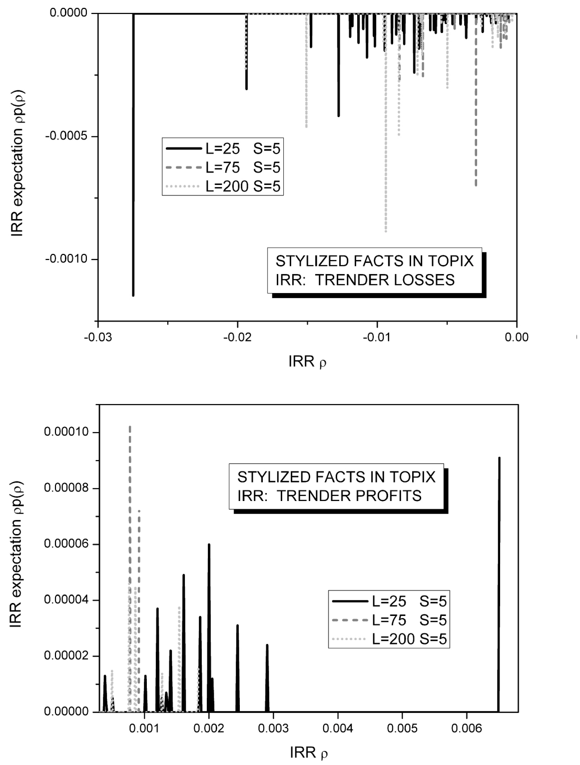

Fig. 4: Stylized facts in IRR on trend investment for TOPIX with various long-term moving averages.

The resulting distribution of R(ρ) for the fundamentalist single-encounter strategy is shown in Fig. 3. The stylized facts manifest themselves by virtually same location of the distribution peak and identical asymptotic behavior of the distribution tail, which is quite remarkable.

The higher degree of similarity between FTSE100 (UK) and S&P 500 (US) is reasonable, given the close ties of the US and UK stock market indices. The three distributions are continuous, which follows from the tight relation between normalized returns and the single-encounter policy. Let us note that about 30% of the distribution in Fig. 3 results on average from the contributions of T=2 and higher. Let us note that the Monte-Carlo (MC) technique is used only for the sake of numerical evaluation of probability density functions in Fig. 3, and not for future-trend prediction. Although a full-extent sampling could be used instead, the MC method is faster while keeping the variance of the present results less than 3%; it can also be easily extended to larger data samples, where the exhaustive search would be impractical.

In sharp contrast, the stylized facts for trend speculator's IRRs exhibit a discrete spectrum. The results in Fig. 4 are shown only for the TOPIX index. Three different values for the long-term moving averages were used, each yielding somewhat different distribution of the market turning points. Correspondingly, the overlapping peaks correspond to real turning points in the market while those seen especially in case of smaller averaging periods include also policy artifacts. It is noteworthy that most of the distribution accumulates in the negative region of ρ, which confirms the limited applicability of the turning point analysis. More interestingly, the discrete character in the stylized facts of the distribution reveals the non-equilibrium character of such an investment policy, as it is inconsistent with the long-term data time series.

4. Concluding Remarks

By relating market stylized facts to the single-encounter element of investment policy, we have revealed an interesting coincidence of the optimal IRR across several indices, which represents a new stylized fact. The similarity of the end-tail distribution is a direct result of the stylized facts in normalized returns, since large ρ values strongly correlate with T=1. Trend speculators correlate with a discrete IRR distribution; the location of the peaks is determined both by the economic fundamentals and the length of the time horizon; it is also a fingerprint of each strategy. The present work may be useful for the analysis of fluctuations in returns on market investment, and possibly contribute to a departure from Kelly's logarithmic utility function in agent-based market simulations.

5. References

[1] Jr. J. L. Kelly, "A new interpretation of the Information Rate", Bell Syst. Tech. J. **35** p. 917 (1956).

[2] F. Slanina, "On the possibility of optimal investment," Physica A **269**, p.554-563, 1999.

[3] T. Kaizoji and M. Kaizoji, "Exponential laws of a stock price index and a stochastic model," Advances in Complex Systems **6** (3) p. 1-10, 2003.

[4] A. Carbone, G. Castelli, and H. E. Stanley, "Time-Dependent Hurst Exponent in Financial Time Series," Physica A **344** p. 267-27, 2004.

[5] F. Wang, et al., "Scaling and Memory of Intraday Volatility Return Intervals in Stock Market," Phys. Rev. E 73, 026117 (2005).

[6] S.-H. Chen, T. Lux, and M. Marchesi, "Testing for non-linear structure in an artificial financial market," Journal of Economic Behavior & Organization 46(3), p. 327-342, 2001.