High Resolution DOA Estimation Algorithm based on Nonuniform L-shaped Array

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***Abstract*—A parameter estimation algorithm of direction of arrival (DOA), based on a nonuniform L-shaped array, is presented for plane-wave signals. According to relation of the two x-axis and y-axis subarrays, a set of rough but unambiguous direction cosines estimates in x and y direction are obtained. By disambiguating the phase ambiguities another set of precise and unambiguous direction cosines estimates in x and y direction are obtained. Without spectral peak searching, this method has advantages of the small amount of calculation and high precision. Simulation results verify the effectiveness of the algorithm.**

Keywords-high resolution; array signal processing; parameter estimation; subspace method; phase ambiguity

I. INTRODUCTION

Array signal processing technique has played a primary role in many applications involving radar, sonar and communications. However, with the development of array signal processing approaches in practical applications, the investigations on high resolution DOA estimation methods have received great interest [1-4].

Signal subspace based algorithms such as ESPRIT are known to have high resolution and be computationally efficient. Because of the good performance of L-shaped array, scholars have studied the ESPRIT algorithm based on uniform L-shaped array [5-17]. In order to ensure that each column vector of direction matrix is linearly independent, inter-element spacing of L-shaped array is not greater than half wavelength. But in practice, the inter-element spacing of array in some given conditions is often demanded to great larger than half wavelength. That means the array will have a large array aperture, and result in a DOA ambiguity [18-21].

In this paper, the nonuniform L-shape array whose inter-element spacing is greater than half a wavelength is studied. Extending the inner-sensor spacing beyond a half-wavelength can offer enhanced array resolution and direction-finding precision but will lead to a set of cyclically ambiguous angle estimates.

We propose a subspace-based DF approach which overcomes the ambiguity of DOA estimation and improves the resolution capability of L-shaped array by extending aperture size. The main feature of our proposed method is that it combines the low computation and super-resolution capability. Numerical examples show that the proposed DF method achieves a better performance.

II. SIGNAL MODEL

Fig.1 shows the one sparse L-shaped array configuration which uses the *x-y* plane consists of $4M-1$ elements. Each linear array consists of $2M-1$ elements. The element placed at the origin is common for referencing purpose. Let S1 and S2 be the two sub-arrays respectively composed of the odd and even array elements in the *x* axis, and let S3 and S4 be the two sub-arrays respectively composed of the odd and even array elements in the *y* axis. Each sub-array consists of M elements. The inter-element spacing of sub-arrays S1 (or S2, S3, S4) is d, which is great larger than half wavelength and the spacing between sub-array S1 and S2 (or S3 and S4) is D, which is less than half wavelength.

Assume that $\mathrm{K}\left(\mathrm{K}\le M-1\right)$ uncorrelated narrowband sources impinge on an L-shaped array from far-field, where the *k*th source has an elevation angle θ_k and an azimuth angle ϕ_k, with $\theta_k(0\le\theta_k\le\frac{\pi}{2})$, $\phi_k(0\le\phi_k\le 2\pi)$, $k=1,2,\ldots,K$.

The received data of L-shaped array can be written as:

$$\mathbf{X}(t)=\mathbf{AS}+\mathbf{N}(t)\,. \tag{1}$$

Where

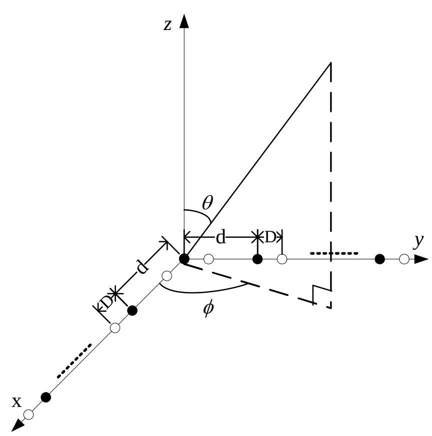

Figure 1. The geometrical configuration of the L-shaped array

$$\mathbf{A}=\left[\mathbf{a}(\theta_1,\phi_1),\mathbf{a}(\theta_2,\phi_2),\ldots,\mathbf{a}(\theta_K,\phi_K)\right], \quad (2)$$

$$\mathbf{a}(\theta_k,\phi_k)=\left[1,\mathbf{q}_x\left(\theta_k,\phi_k\right),\mathbf{q}_y\left(\theta_k,\phi_k\right)\right]^T, \quad k=1,2,\ldots,K \quad (3)$$

$$\mathbf{q}_x(\theta_k,\phi_k)=\begin{bmatrix} e^{j\frac{2\pi}{\lambda}D\sin\theta_k\cos\phi_k} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}\left[\frac{1-(-1)^m}{2}D+\frac{(-1)^m d+(2m-1)d}{4}\right]\sin\theta_k\cos\phi_k} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}\left[\frac{1-(-1)^{2M-1}}{2}D+\frac{(-1)^{2M-1}d+(2(2M-1)-1)d}{4}\right]\sin\theta_k\cos\phi_k} \end{bmatrix}, \quad (4)$$

$$\mathbf{q}_y(\theta_k,\phi_k)=\begin{bmatrix} e^{j\frac{2\pi}{\lambda}D\sin\theta_k\sin\phi_k} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}\left[\frac{1-(-1)^m}{2}D+\frac{(-1)^m d+(2m-1)d}{4}\right]\sin\theta_k\sin\phi_k} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}\left[\frac{1-(-1)^{2M-1}}{2}D+\frac{(-1)^{2M-1}d+(2(2M-1)-1)d}{4}\right]\sin\theta_k\sin\phi_k} \end{bmatrix} \quad (5)$$

and $\mathbf{S}=\left[s_1,s_2,\cdots,s_K\right]^T$ is a $K\times1$ signal vector for the K source signals. $\mathbf{N}(t)$ is the additive white Gaussian noise vector whose elements have mean zero and variance σ^2.

The auto-correlation matrix of overall data $\mathbf{X}(t)$ is

$$\mathbf{R}_{xx}=E\left[\mathbf{X}(t)\mathbf{X}(t)^H\right]=\mathbf{A}\mathbf{R}_s\mathbf{A}^H+\sigma^2\mathbf{I}_m. \quad (6)$$

Where $\mathbf{R}_s=E\left[\mathbf{S}(t)\mathbf{S}^H(t)\right]$ is signal covariance matrix, $\mathbf{I}_m$ denotes $m\times m$ $(m=4M-1)$ identity matrix. Because the sources $s_i(t)$ are uncorrelated to each other, so $\mathbf{R}_s$ is a diagonal matrix, each diagonal element is the power of the sources, $\mathbf{R}_{xx}$ is $m\times m$ $(m=4M-1)$ matrix.

III. PROPOSED ALGORITHM

Let $\mathbf{E}_s$ be the matrix composed of the K eigenvectors corresponding to the K largest eigenvalues of $\mathbf{R}_{xx}$ and let $\mathbf{E}_n$ denote the matrix composed of the remaining eigenvectors of $\mathbf{R}_{xx}$. According to the subspace theory, the signal subspace can be expressed explicitly as

$$\mathbf{E}_s=\mathbf{A}\mathbf{T}. \quad (7)$$

Let
$$\mathbf{E}_{sx}=\mathbf{E}_s\left(1:2M,:\right), \quad (8)$$

$$\mathbf{E}_{sy}=\begin{bmatrix}\mathbf{E}_s\left(1,:\right)\\ \mathbf{E}_s\left(2M+1:4M\text{-}1,:\right)\end{bmatrix} \quad (9)$$

and
$$\mathbf{E}_{sx}=\begin{bmatrix}\mathbf{E}_{sx1}\\ \mathbf{E}_{sx2}\end{bmatrix}=\begin{bmatrix}\mathbf{A}_{x1}\\ \mathbf{A}_{x2}\end{bmatrix}\mathbf{T}, \quad (10)$$

$$\mathbf{E}_{sy}=\begin{bmatrix}\mathbf{E}_{sy1}\\ \mathbf{E}_{sy2}\end{bmatrix}=\begin{bmatrix}\mathbf{A}_{y1}\\ \mathbf{A}_{y2}\end{bmatrix}\mathbf{T}. \quad (11)$$

Where
$$\mathbf{A}_{x1}=\left[A_{x1}(1),\cdots,A_{x1}(K)\right], \quad (12)$$

$$A_{x1}(k)=\begin{bmatrix}1\\ e^{j\frac{2\pi}{\lambda}d\sin\theta_k\cos\phi_k}\\ \vdots\\ e^{j\frac{2\pi}{\lambda}(M-1)d\sin\theta_k\cos\phi_k}\end{bmatrix}\left(k=1,\cdots,K\right)$$ is an $M\times1$ column vector.

Where
$$\mathbf{A}_{x2}=\left[A_{x2}(1),\cdots,A_{x2}(K)\right], \quad (13)$$

$$A_{x2}\left(k\right)=\begin{bmatrix} e^{j\frac{2\pi}{\lambda}D\sin\theta_k\cos\phi_k}\\ e^{j\frac{2\pi}{\lambda}(D+d)\sin\theta_k\cos\phi_k}\\ \vdots\\ e^{j\frac{2\pi}{\lambda}(D+(M-1)d)\sin\theta_k\cos\phi_k}\end{bmatrix}\left(k=1,\cdots,K\right)$$ is an $M\times1$ column vector.

$\mathbf{E}_{sx1}$ and $\mathbf{E}_{sx2}$ are the matrix composed of odd rows and even rows of the $\mathbf{E}_{sx}$ respectively, and meet

$$\mathbf{E}_{sx1}=\mathbf{A}_{x1}\mathbf{T},\mathbf{E}_{sx2}=\mathbf{A}_{x2}\mathbf{T}=\mathbf{A}_{x1}\mathbf{\Phi}_x\mathbf{T}, \quad (14)$$

$$\mathbf{E}_{sy1}=\mathbf{A}_{y1}\mathbf{T},\mathbf{E}_{sy2}=\mathbf{A}_{y2}\mathbf{T}=\mathbf{A}_{y1}\mathbf{\Phi}_{y}\mathbf{T}. \quad (15)$$

Where

$$\mathbf{\Phi}_x = diag\left[e^{j\frac{2\pi}{\lambda}D\sin\theta_1\cos\phi_1},\cdots,e^{j\frac{2\pi}{\lambda}D\sin\theta_K\cos\phi_K}\right]^T, \quad (16)$$

$$\mathbf{\Phi}_y = diag\left[e^{j\frac{2\pi}{\lambda}D\sin\theta_1\sin\phi_1},\cdots,e^{j\frac{2\pi}{\lambda}D\sin\theta_K\sin\phi_K}\right]^T. \quad (17)$$

Equation (14) and (16) imply that:

$$\left(\mathbf{E}_{sr1}^{\mathrm{H}}\mathbf{E}_{sr1}\right)^{-1}\mathbf{E}_{sr1}^{\mathrm{H}}\mathbf{E}_{sr2}\mathbf{T}^{-1}=\mathbf{T}^{-1}\mathbf{\Phi}_r, r=x \; or \; y. \quad (18)$$

Let $\mathbf{\Omega}=\left(\mathbf{E}_{sr1}^{\mathrm{H}}\mathbf{E}_{sr1}\right)^{-1}\mathbf{E}_{sr1}^{\mathrm{H}}\mathbf{E}_{sr2}$ ($\mathbf{\Omega}=\mathbf{E}_{sr1}^{\#}\mathbf{E}_{sr2}$), then

$$\mathbf{\Omega}\mathbf{T}^{-1}=\mathbf{T}^{-1}\mathbf{\Phi}_r. \quad (19)$$

Equation (19) implies that $\mathbf{\Phi}_r$ is a matrix whose diagonal elements are composed of the K eigenvalues of matrix $\mathbf{\Omega}$ and the full-rank matrix $\mathbf{T}^{-1}$ is composed of the K eigenvectors of matrix $\mathbf{\Omega}$. The estimation of $\mathbf{A}_{r1}$ can be written as

$$\hat{\mathbf{A}}_{r1}=\mathbf{E}_{sr1}\mathbf{T}^{-1}. \quad (20)$$

Where $r=x$ $\hat{\mathbf{A}}_{r1}=\hat{\mathbf{A}}_{x1}$; $r=y$ $\hat{\mathbf{A}}_{r1}=\hat{\mathbf{A}}_{y1}$, the column of $\hat{\mathbf{A}}_{x1}$ may not be accompanied with that of $\hat{\mathbf{A}}_{y1}$, so pairing operation is essential. Since the inter-element spacing of sub-array S1 and S2, which is labeled as d,is much larger than half wavelength, so the estimates of DOA based on sub-array S1 and S2 is ambiguous. However the spacing between sub-array S1 and S2, namely D, is less than half wavelength, and the estimates of DOA is unambiguous. The discussion for sub-array S3 and S4 is similar to S1 and S2. The proposed ambiguity resolving algorithm is illustrated by the flow as below:

Let

$$\mathbf{E}_r=\left[\Phi_r(1,1),\Phi_r(2,2),\cdots,\Phi_r(K,K)\right]^T, \quad (21)$$

$$\mathbf{F}_r=\left[\hat{\mathbf{A}}_{r1}(2,1:K)\right]^T. \quad (22)$$

From (12) and (16), $\mathbf{E}_r$ and $\mathbf{F}_r$ can be expressed as

$$\mathbf{E}_r=\left[e^{j\frac{2\pi}{\lambda}DV_{r1}},e^{j\frac{2\pi}{\lambda}DV_{r2}},\ldots,e^{j\frac{2\pi}{\lambda}DV_{rK}}\right]^T, \quad (23)$$

$$\mathbf{F}_r=\left[e^{j\frac{2\pi}{\lambda}dV_{r1}},e^{j\frac{2\pi}{\lambda}dV_{r2}},\ldots,e^{j\frac{2\pi}{\lambda}dV_{rK}}\right]^T. \quad (24)$$

Since D is less than half wavelength and d is larger than half wavelength, $\mathbf{E}_r$ in (23) can be used to give rough but unambiguous estimates of the direction cosine of r-axis. They are used as coarse references to disambiguate the cyclic phase ambiguities in (24).

It can be simply stated as:

$$\frac{2\pi}{\lambda}DV_{rk}=\angle\mathbf{E}_r(k), \quad (25)$$

$$\frac{2\pi}{\lambda}dV_{rk}=\angle\mathbf{F}_r(k)+2\pi n_r(k). \quad (26)$$

The precise estimates of direction cosine of x-axis and y-axis are obtained from (26).

$$r=x,\quad V_{xk}=\sin\hat{\theta}_k\cos\hat{\phi}_k, \quad (27)$$

$$r=y,\quad V_{yk}=\sin\hat{\theta}_k\sin\hat{\phi}_k. \quad (28)$$

From (27) and (28), the precise estimates of DOA is obtained as:

$$\hat{\theta}_k=\arcsin\sqrt{V_{xk}^2+V_{yk}^2}, \quad (29)$$

$$\hat{\phi}_k=\arctan\sqrt{V_{yk}/V_{xk}}. \quad (30)$$

IV. SIMULATION RESULTS

Computer simulations have been conducted to evaluate the 2D DOA estimation performance of the considered antennas array configurations. Two uncorrelated sources, respectively with DOAs of $(70^\circ,30^\circ)$ and $(20^\circ,40^\circ)$, impinge on this array, where $\mathrm{d}=3\lambda,\mathrm{D}=0.4\lambda$. The number of snapshots is set to 1024 and the SNR (signal-to–noise ration) varies from 0 dB to 45 dB. The total number of element is 11, i.e. $M = 3$. We use the standard bias as the performance measure. The averaged performances over 500 Monte Carlo runs are shown in Fig.2 and Fig.3.

The star curve and the triangle curve plot the DOA, respectively estimated by the uniform L-shaped arrays model and the proposed L-shaped arrays model, at different SNR levels.

In the SNR range (namely, at or above 0 dB) DOA estimation precision based on the proposed L-shaped arrays model is more notable than that of the uniform L-shaped arrays model. The estimation precision at 0 dB based on the proposed L-shaped arrays model has improved larger than 0.7° for elevation angle, 1.8° for azimuth angle, compared with that of the uniform L-shaped arrays model. The enhanced performance is rooted in the sparse embattle of L-shaped arrays model.

In this experiment, the effect of "snapshot number" factor on the estimation accuracy is investigated. The number of elements is set to 11, i.e. $M = 3$ and SNR is 20 dB. The number of snapshots varies from 52 to 1024.The averaged performances over 500 Monte Carlo runs are shown in Fig.4 and Fig.5.

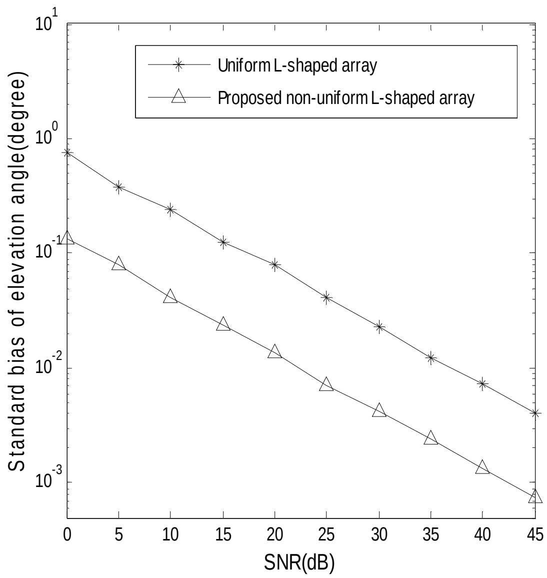

Figure 2. Standard bias of elevation versus SNR

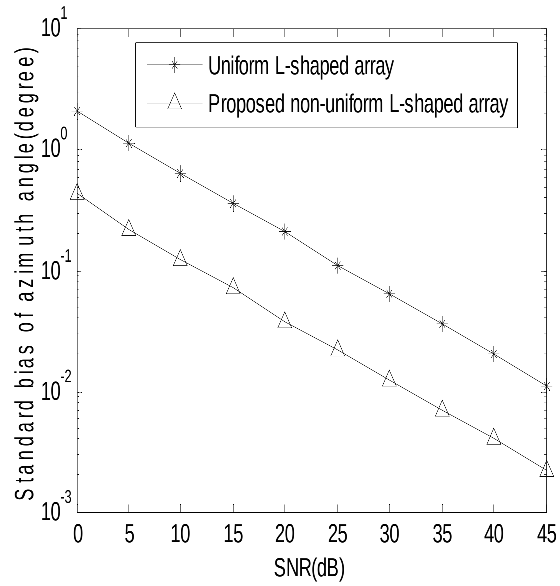

Figure 3. Standard bias of azimuth versus SNR

The star curve and the triangle curve plot the DOA, respectively estimated by the uniform L-shaped arrays model and the proposed L-shaped arrays model, at different snapshot number levels.

In the snapshot number range (namely, at or above 52) DOA estimation precision based on the proposed L-shaped arrays model is more notable than that of the uniform L-shaped arrays model. The estimation precision at 52 based on the proposed L-shaped arrays model has improved larger than 0.3° for elevation angle, 0.7° for azimuth angle, compared with that of the uniform L-shaped arrays model. The enhanced performance is rooted in the sparse embattle of L-shaped arrays model.

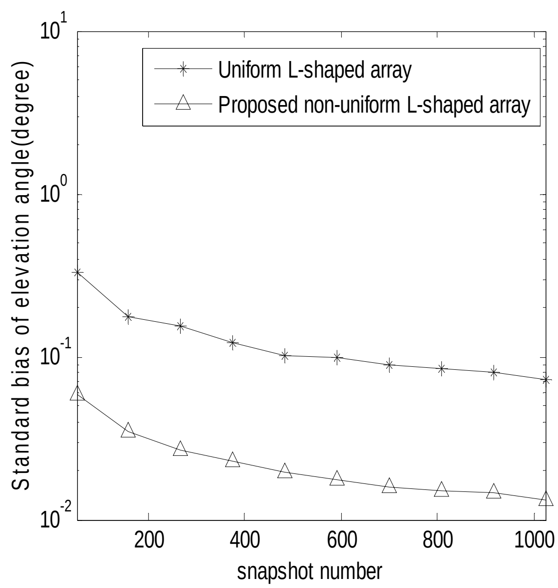

Figure 4. Standard bias of elevation versus snapshot number

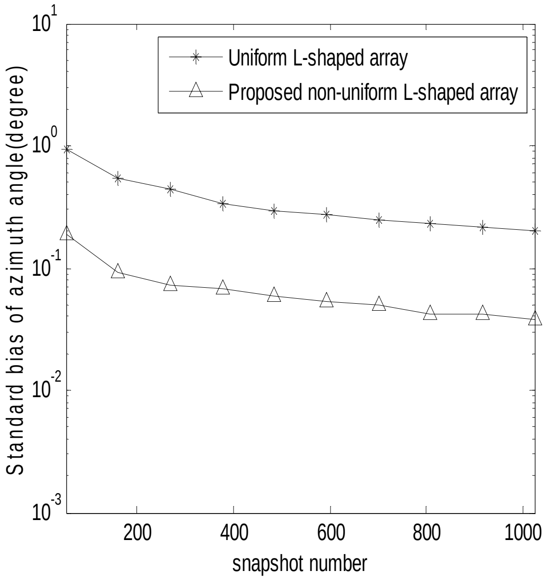

Figure 5. Standard bias of azimuth versus snapshot number

V. CONCLUSION

A closed form estimation of DOA, based on subspace high-resolution ESPRIT algorithm is studied. The precision of estimating parameters is enhanced by aligning the sparse sensor on a nonuniform L-shaped sparse array. The steering vector of the difference of two sub-arrays along x-axis or y-axis is used to give coarse references of DOA to disambiguate the cyclic phase ambiguities between the array elements on sub-array along x-axis or y-axis. Without spectral peak searching, this method has advantages of the small amount of calculation and high precision.

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