Study on Chebyshev FIR Filters in Airborne Gravity Data Processing

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Abstract— **In order to pick up the low-frequency gravity anomaly from airborne gravity data, a finite impulse response (FIR) low-pass digital filter with the Chebyshev approximation theorem was designed, and the filtering effects of three design parameters were studied. Results show that the filter order may amplify the passband section and affect the stopband attenuation; the passband and stopband edge frequency have similar effects on transition band but opposite effects on stopband. Compared with the commercial software results, the standard deviations of the filtered gravity anomalies using the optimal parameters are within 1mGal, and the internal accord accuracy of four survey lines after level adjusting is 0.585mGal.**

Keywords- airborne gravity data; low-pass; Chebyshev approximation; finite impulse response (FIR)

I. INTRODUCTION

Airborne gravimetry is the measurement of Earth's gravity field based on aircrafts or other motional carriers. The appearance of differential GPS in the 1990s makes a great development of airborne gravimetry and its advantages of low-cost, high-efficiency and wide-range are getting obvious. The scalar inertial platform airborne gravimetry system, GT-1A, developed by Russia had taken flight tests in 2001, and the standard deviation of two repeated lines of 100km-long was 0.53mGal(1mGal=10^{-5}m/s^2)[1].China Aero Geophysical Survey and Remote Sensing Center for Land and Resources (AGRS) introduced the GT-1A system in 2007 and integrated the first airborne gravimetry system in the domestic resource exploration to satisfy the small and medium scale regional gravity exploration[2].

The airborne gravity data are measured in the high-frequency vibration environment while the gravity anomaly is low-frequency and usually dozens of milligals, therefore low-pass filtering becomes one of key technologies in airborne gravity data processing. With the character of linear phase and stability, FIR low-pass digital filters are often used for filtering[3-6]. Different from the design of a digital infinite impulse response (IIR) filter with the help of an analog filter, the design of a digital FIR filter is the approximation of an ideal frequency response, and there are three ways for it: window functions, frequency sampling and Chebyshev approximation. The Chebyshev approximation is based on the principle of minimizing the maximum error[8], whose magnitude response curve in the passband and stopband is equiripple, and can precisely determine the passband and stopband corner frequency. Chebyshev FIR low-pass digital filtering experiments on raw gravity anomaly which are calculated from the GT-1A airborne gravity data will be present in this paper, and also the studies on filtering influences with different parameters.

II. CHEBYSHEV FIR LOW-PASS DIGITAL FILTERS

The frequency response of an ideal FIR low-pass digital filter is

$$H_d\left(e^{j\omega}\right)=\begin{cases}1 & 0\le\omega\le\omega_p\\ 0 & \omega_s\le\omega\le\pi\end{cases}, \tag{1}$$

where ω_p is the passband edge angular frequency, ω_s is the stopband edge angular frequency. According to the Chebyshev approximation theorem, if the frequency response $H(e^{j\omega})$ of a designed filter is the best consistent approximation to $H_d(e^{j\omega})$, then equiripple passband and stopband can be obtained as shown in Fig. 1. So this method is also called as equiripple approximation [7]. σ_1 and σ_2 are the peaks of ripples in the passband and the stopband, respectively.

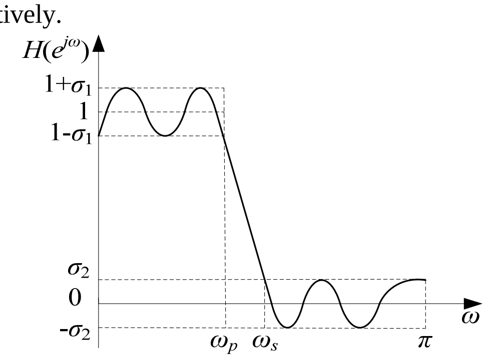

Figure 1. The best consistent approximation of a low-pass digital filter

There are four cases of FIR filters whether the unit impulse response $h(n)$ is odd-symmetric or even-symmetric and its length L is odd or even, and we choose $h(n)$ is even-symmetric and L is odd in this paper, then

$$\begin{aligned}&H\left(e^{j\omega}\right)=e^{-j(L-1)\omega/2}H_a\left(e^{j\omega}\right),\\ &H_a\left(e^{j\omega}\right)=\sum_{n=0}^{M}a\left(n\right)\cos(n\omega), M=(L-1)/2\end{aligned}. \tag{2}$$

Construct a weight function [7]

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$$W\left(e^{j\omega}\right)=\begin{cases}1/K & 0\le\omega\le\omega_p, K=\sigma_1/\sigma_2\\ 1 & \omega_s\le\omega\le\pi\end{cases}, \quad (3)$$

and an error function

$$\begin{aligned}E\left(e^{j\omega}\right)&=W\left(e^{j\omega}\right)\left[H_a\left(e^{j\omega}\right)-H_d\left(e^{j\omega}\right)\right]\\ &=W\left(e^{j\omega}\right)\left[\sum_{n=0}^{M}a\left(n\right)\cos(n\omega)-H_d\left(e^{j\omega}\right)\right]\end{aligned}, \quad (4)$$

then the problem that $H_a(e^{j\omega})$ is the best consistent approximation to $H_d(e^{j\omega})$ can be reported as to find coefficients $a(n)$ making the maximum of $E(e^{j\omega})$ minimized. The general method for this problem is Remez algorithm developed by Parks and McCellan, which is an iteration approach considering the M, ω_p and ω_s invariable but σ_1 and σ_2 variable. Thus the Remez algorithm can accurately confirm the corner frequency in the passband and stopband.

III. FILTERING EXPERIMENTS

The airborne gravity data in this paper was measured for four times along the same survey line by the GT-1A system of AGRS. According to the principle of platform scalar airborne gravimetry, the raw gravity anomaly after several corrections can be acquired as showed in Fig. 2 which illustrates there are three parts of the measurement: preflight, flight and postflight. The left and right relatively stable part of the curve are static measurement at the apron before taking off and after landing of the aircraft, respectively, and the intermediate greatly fluctuating part is the flight of gravity measurement. Fig. 3 is the spectrum of raw gravity anomaly which presents that the useful signal mainly locates within 0.01Hz.

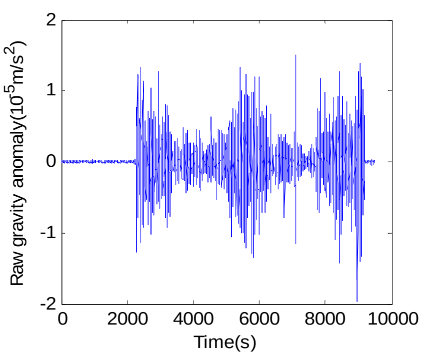


Figure 2. Raw gravity anomaly

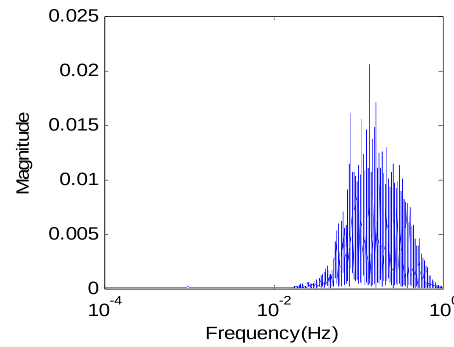


Figure 3. Spectrum of raw gravity anomaly

Parameters of filter order N, passband edge frequency f_p and stopband edge frequency f_s should be determined to design a Chebyshev FIR low-pass digital filter with Matlab. To get the optimal filter parameters, the filtering effects of different parameters are studied and the gravity anomaly extracted by the GT-1A software is taken as the reference.

Setting f_p=0.005Hz, f_s=0.012Hz, do the filtering tests of N= 600, 700, 800. Fig. 4 presents the magnitude responses of Chebyshev filters with different N. There is an attenuation in the passband of the 800-order filter but an increase in the other two filters, and the minimum stopband attenuation (the absolute logarithmic amplitude of the stopband ripple) is getting larger with the increase of N. Table I lists the statistics of the discrepancies between the filtered gravity anomalies and the reference. The standard deviations (STDs) of four survey lines are under 1mGal with the filter order of 800. Due to the amplified magnitude of the passband, the statistics of the lower order filter are worse than the 800-order filter, and insufficient attenuation in the stopband makes small-scale fluctuations of the filtered curves as shown in Fig. 5.

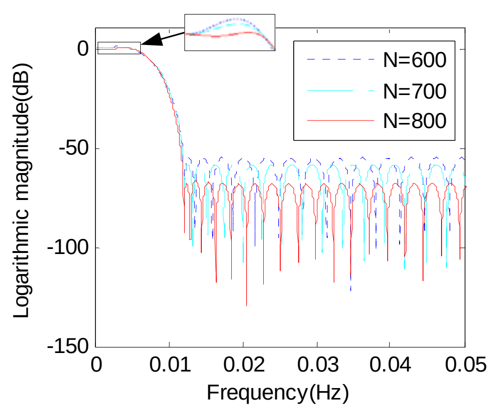


Figure 4. Magnitude responses of Chebyshev FIR filters with different N

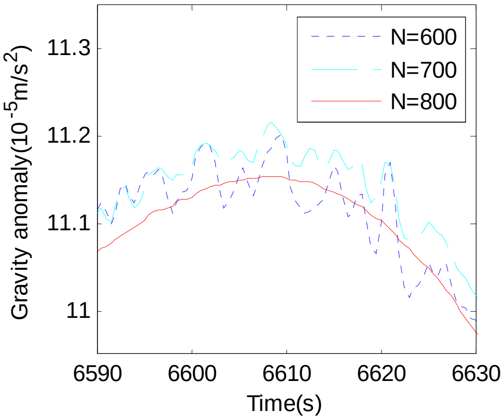


Figure 5. Noise on the filtered curve

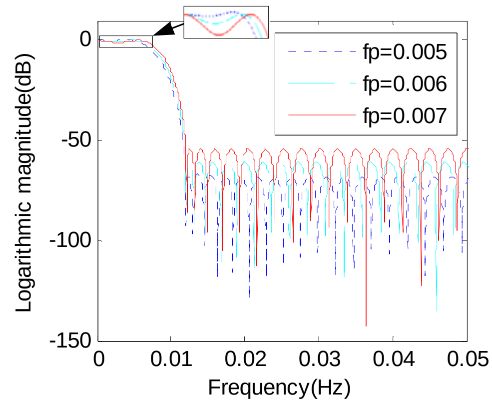


Figure 6. Magnitude responses of Chebyshev FIR filters with different f_p

TABLE I. THE COMPARISON BETWEEN THE FILTERED GRAVITY ANOMALIES AND THE REFERENCE WITH DIFFERENT N

Filter order N		Line 1	Line 2	Line 3	Line 4
600	Max	2.024	2.099	3.216	0.260
	Min	-2.296	-6.110	-2.024	-3.718
	STD	1.168	1.577	0.929	0.765
700	Max	1.725	2.078	2.866	-0.088
	Min	-1.621	-5.386	-1.222	-2.940
	STD	0.900	1.318	0.741	0.542
800	Max	1.515	1.969	2.464	0.197
	Min	-0.716	-3.852	-0.651	-2.157
	STD	0.569	0.943	0.679	0.586

Unit: $10^{-5}m/s^2$.

Setting N=800, f_s=0.012Hz, do the filtering tests of f_p=0.005Hz, 0.006Hz, 0.007Hz. Fig.6 presents the magnitude responses of Chebyshev filters with different f_p. When the passband edge frequency is rising, the passband ripple becomes larger and the minimum stopband attenuation becomes smaller. The transition band moves to higher frequency and gets into the stopband at the same frequency. Table II lists the statistics of the discrepancies between the filtered gravity anomalies and the reference which illustrates the filtered gravity anomaly is getting worse with the increasing f_p. The reason is that the attenuation in the transition band and stopband is smaller with the larger f_p which makes the filtered signal still has some noise.

TABLE II. THE COMPARISON BETWEEN THE FILTERED GRAVITY ANOMALIES AND THE REFERENCE WITH DIFFERENT f_p

Passband edge frequency f_p (Hz)		Line 1	Line 2	Line 3	Line 4
0.005	Max	1.515	1.969	2.464	0.197
	Min	-0.716	-3.852	-0.651	-2.157
	STD	0.569	0.943	0.679	0.586
0.006	Max	1.562	2.263	2.871	0.536
	Min	-1.075	-3.340	-1.420	-2.635
	STD	0.639	1.038	0.912	0.861
0.007	Max	2.710	2.912	3.816	1.335
	Min	-1.872	-4.256	-2.235	-3.323
	STD	0.962	1.378	1.286	1.279

Unit: $10^{-5}m/s^2$.

Setting N=800, f_p=0.005Hz, do the filtering tests of f_s=0.012Hz, 0.013Hz, 0.014Hz. Fig.7 presents the magnitude responses of Chebyshev filters with different f_s. Similar to the f_p, the transition band of the raising f_s moves right but meets in the different f_s and the minimum stopband attenuation becomes greater which is opposite to the rising f_p. Table III lists the statistics of the discrepancies between the filtered gravity anomalies and the reference. The statistic indicators of line 1, line 3 and line 4 are nearly the same, but the STDs of line 2 are getting larger with the increasing f_s which presents that the stopband attenuation of the filter with f_s=0.012Hz is sufficient and the right-moving transition band gently influences filtering result.

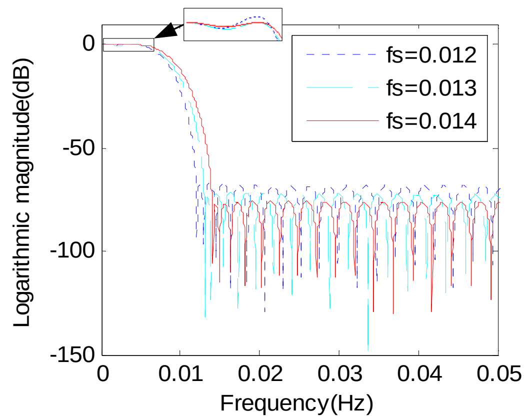


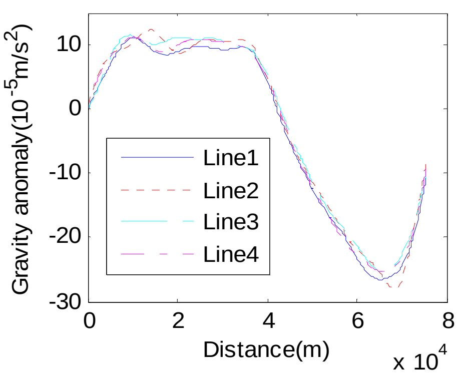
Figure 7. Magnitude responses of Chebyshev FIR filters with different f_s

TABLE III. THE COMPARISON BETWEEN THE FILTERED GRAVITY ANOMALIES AND THE REFERENCE WITH DIFFERENT f_s

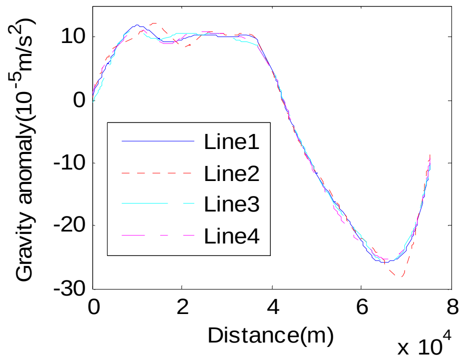
Stopband edge frequency f_s (Hz)		Line 1	Line 2	Line 3	Line 4
0.012	Max	1.515	1.969	2.464	0.197
	Min	-0.716	-3.852	-0.651	-2.157
	STD	0.569	0.943	0.679	0.586
0.013	Max	1.281	2.242	2.489	-0.015
	Min	-0.737	-4.143	-0.881	-2.233
	STD	0.561	1.043	0.709	0.566
0.014	Max	1.296	2.726	2.440	0.344
	Min	-1.159	-4.788	-1.028	-2.264
	STD	0.588	1.240	0.739	0.595

Unit: $10^{-5}m/s^2$.

Based on the statistics above and the smoothness of the filtered curve, parameters of a Chebyshev FIR low-pass digital filter for the raw gravity anomaly are finally chosen as N=800, f_p=0.005Hz, f_s=0.012Hz, and the -3dB frequency is 0.007Hz. The STDs of four survey lines are within 1mGal, the maximum is 2.464mGal in line 3 and the minimum is -3.852mGal in line 2. According to the evaluation method of gravity anomaly along the repeated test lines in [9], the internal accord accuracies of the four survey lines before and after level adjusting are 0.724mGal and 0.585mGal, respectively as shown in Fig. 8.



(a) Before level adjusting



(b) After level adjusting

Figure 8. Internal accord accuracy of four survey lines

IV. CONCLUSIONS

After the comparisons between the filtered results using Chebyshev FIR low-pass digital filters with various parameters and the referenced gravity anomaly, the following conclusions can be drawn.

(1) The filter order may amplify the passband section of the signal which will produce worse result, and the larger order results in the greater stopband attenuation which makes the filtered curve smoother.

(2) With the increasing passband edge frequency, the transition band is moving right and the stopband is moving up which could make the filtered result still containing noise.

(3) Similar to the passband edge frequency, the transition band of the rising stopband edge frequency moves to higher frequency but the start frequency of stopband is different and the minimum stopband attenuation varies oppositely.

(4) Compared with the reference, STDs of the filtered gravity anomalies using the optimal parameters in this paper are under 1mGal, and the internal accord accuracy of four survey lines after level adjusting is 0.585mGal, which proves that the Chebyshev FIR low-pass filter can effectively remove the high-frequency noise of raw gravity anomaly.

REFERENCES

[1] C.D.Zhang, “Several new types of airborne gravimetric systems and airborne gravity gradiometric system,” Geophysical and Geochemical Exploration, vol.29, no.6, pp.471-476, Dec. 2005.(In Chinese)

[2] S.Q.Xiong, “The Present Situation and development of airborne gravity and magnetic survey techniques in China,” Progress in Geophysics, vol. 24, no.1, pp.113-117, Feb. 2009. (In Chinese)

[3] Z.M.Sun, Z.R.Xia, P.Shi, X.T.Wang, Y.Xiao, Y.C.Li, “Filtering and processing for the airborne gravimetry data,” Progress in Geophysics, vol.19, no.1, pp.119-124, Mar. 2004. (In Chinese)

[4] Z.M.Sun, Z.R.Xia, “Design of FIR lowpass differentiator and its applications in airborne gravity,” Chinese Journal of Geophysics, vol.43, no.6, pp.850-855, Nov. 2000. (In Chinese)

[5] Z.H.Guo, F.Luo and Z.F.An, “Experimental researches on FIR lowpass digital filters based on window functions of airborne gravity data,” Geophysical and Geochemical Exploration, vol.31, no.6, pp.568-571, Dec. 2007.(In Chinese)

[6] F.Luo, Z.H.Guo, Y.Luo, M.Wang, “Experimental researches on FIR lowpass filter based on equiripple,” Geophysical and Geochemical Exploration, vol.36, no.5, pp.856-860, Oct. 2012.(In Chinese)

[7] G.S.Hu, *Introduction to Digital Signal Processing*. Tsinghua University Press, Beijing 2005. (In Chinese)

[8] Z.Y.Wu, *Digital Signal Processing*. Higher Education Press, Beijing 2010.(In Chinese)

[9] Z.H.Guo, S.Q.Xiong, J.X.Zhou, X.H.Zhou, “The research on quality evaluation method of test repeat lines in airborne gravity survey,” Chinese Journal of Geophysics, vol.51, no.5, pp.1538-1543, Sep.2008.(In Chinese)